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Adaptive Neural Network Control of Helicopters with Unknown Dynamics

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Abstract— **In this paper, adaptive neural network (NN) tracking control is considered for helicopters in the presence of parametric and functional uncertainties. Based on Lyapunov synthesis, the proposed adaptive NN control ensures that the system outputs track the given bounded reference signals to a small neighborhood of zero, and guarantees semiglobal uniformly ultimate boundedness (SGUUB) of all the closed-loop signals. The effectiveness of the proposed control is illustrated through extensive simulations.**

I. INTRODUCTION

Helicopter systems are characterized by unknown aerodynamical disturbances, which are generally difficult to model accurately. In this context, model-based control may not be the ideal approach since it generally works best when the dynamics are known exactly. The presence of uncertainties and disturbances could disrupt the function of the model-based feedback control and lead to degradation of performance. How to handle the parametric and functional uncertainties is one of the important issues in the control of helicopters. Many techniques have been proposed in the literature for the motion control of helicopters, which range from dynamic inversion to feedback linearization and model reference adaptive control [1], [2], [3], [4], and [5].

Recently, neural networks have been made particularly attractive and promising for applications in modeling and control of nonlinear systems, owing to their universal approximation capabilities, learning and adaptation, parallel distributed structures. The feasibility of applying neural networks to model unknown functions in dynamic systems has been demonstrated in several studies [6], [7]. Advanced stable neural network control approaches have been proposed based on Lyapunov synthesis [8], [9], [10], and [11], which guarantee the stability of the closed-loop systems.

Motivated by the previous works on model-based control of helicopters [2], we present, in this paper, adaptive neural network control to accommodate the presence of parametric and functional uncertainties in the dynamic model of the helicopter and to reduce the dependence on accurate model building.

The paper is organized as follows. In Section II, we describe the helicopter system under study. Adaptive NN control and its stability are discussed in Section III. Finally, simulation results are shown in Section IV, followed by the conclusion in Section V.

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II. PROBLEM FORMULATION AND PRELIMINARIES

In the following study, let $\|\cdot\|$ denote the 2-norm.

Definition 1: [11] The solution $X(t)$ is semiglobally uniformly ultimately bounded (SGUUB) if, for any compact set Ω_0 and all $X(t_0) \in \Omega_0$, there exists an $\mu > 0$ and $T(\mu, X(t_0))$ such that $\|X(t)\| \leq \mu$ for all $t \geq t_0 + T$.

A. Helicopter Dynamics

Consider the following helicopter dynamics described by Lagrangian formulation [2]:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) = B(\dot{q})\tau \quad (1)$$

where $q = [q_1, q_2, q_3]^T = [z, \phi, \gamma]^T$ are the generalized coordinates with z as the altitude ($z > 0$ downwards), ϕ as the yaw angle, and γ as the main rotor position; $D(q) \in R^{3\times3}$ is the inertia matrix; $C(q,\dot{q})\dot{q} \in R^3$ are the Coriolis and centrifugal forces; $F(\dot{q}) \in R^3$ are the friction forces; $G(q) \in R^3$ are the gravitational forces; $B(\dot{q}) \in R^{3\times2}$ is the matrix of control coefficients; and $\tau = [\tau_1, \tau_2]^T \in R^2$ are the control inputs.

The control objective is to design adaptive neural network control that ensures the main rotor angular speed $\dot{q}_3(t)$ is stable, and the tracking errors for the altitude $q_1(t)$ and yaw angle $q_2(t)$ from their respective desired trajectories $q_{1d}(t)$ and $q_{2d}(t)$, are driven to a small neighborhood of zero, i.e.

$$|q_i(t) - q_{id}(t)| \quad \leq \quad \delta$$

where $\delta > 0$, $i = 1, 2$. Ideally, δ should be the threshold of measured noise. At the same time, all closed loop signals are to be kept bounded.

Assumption 1: The desired trajectories $q_{1d}(t)$ and $q_{2d}(t)$ and their time derivatives up to the 3rd order are continuously differentiable and bounded for all $t \geq 0$.

B. Neural Network Approximation

In control engineering, radial basis function neural network (RBFNN) has been successfully used as a linearly parameterized function approximator to solve different problems because of its good capabilities [10], [11]. In this paper, the RBFNN is used to approximate the continuous function $f(Z): R^m \rightarrow R$ as follows:

$$f_{nn}(Z) = W^T S(Z) + \varepsilon(Z) \quad (2)$$

where the input vector $Z \in \Omega_Z \subset R^m$; weight vector $W = [w_1, w_2, ..., w_l]^T \in R^l$, the NNs node number $l > 1$;

$S(Z) = [s_1(Z), ..., s_l(Z)]^T$, with $s_i(Z)$ being chosen as the commonly used Gaussian functions, which have the form

$$s_i(Z) = \exp\left[\frac{-(Z-\mu_i)^T(Z-\mu_i)}{\eta_i^2}\right], \quad i = 1, 2, ..., l$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, ..., \mu_{im}]^T$ is the center of the receptive field and η_i is the width of the Gaussian function; and $\varepsilon(Z)$ is the approximation error which is bounded over the compact set Ω_Z, i.e., $|\varepsilon(Z)| \leq \bar{\varepsilon}$, $\forall Z \in \Omega_Z$ where $\bar{\varepsilon} > 0$ is an unknown constant.

It has been proven that RBFNN (2) can approximate any continuous function $f(Z)$ over a compact set $\Omega_Z \subset R_m$ to arbitrary any degree of accuracy as

$$f(Z) = W^{*T}S(Z) + \varepsilon^*(Z), \quad \forall Z \in \Omega_Z \subset R^m$$

where W^* is ideal constant weights, and $\varepsilon^*(Z)$ is the approximation error for the special case where $W = W^*$.

Assumption 2: On the compact set Ω_Z, the ideal NN weights W^* is bounded by

$$\|W^*\| \leq w_m$$

The ideal weight vector W^* is defined as the value of W that minimizes $|\varepsilon(Z)|$ for all $Z \in \Omega_Z \subset R^m$, i.e.,

$$W^* = \arg\min_W \{\sup_{Z\in\Omega_Z} |f(Z) - W^T S(Z)|\}$$

In general, the ideal weights W^* are unknown and need to be estimated in control design. Let $\hat{W}$ be the estimates of W^* , and the weight estimation errors $\tilde{W} = \hat{W} - W^*$.

Lemma 1: [11] For $e_s \in L_\infty$ and $\hat{W}(0) \in L_\infty$, the learning algorithm

$$\dot{\hat{W}} = -\Gamma_w\left[Se_s + \delta_w(1+|e_s|^m)\hat{W}\right] \quad (3)$$

where $0 \leq m < \infty$, $\Gamma_w = \Gamma_w^T > 0$ and $\delta_w > 0$ are constant design parameters, guarantees that $\hat{W}(t) \in L_\infty$.

Remark 1: Although RBFNN is employed in our control design, it can be replaced by other linearly parameterized function approximators such as high-order neural networks, fuzzy systems, polynomials, splines and wavelet networks without difficulty [11].

III. Control Design

Motivated by the previous work on model-based control of helicopters [2], we will design adaptive NN control to accommodate the presence of uncertainties in system (1), appearing in the functions $D(q)$, $C(q, \dot{q})$, $F(\dot{q})$, and $G(q)$. By exploiting the physical properties of the helicopter, e.g., how the control inputs are distributed to the helicopter dynamics, or the coupling relationship between the states, better performance can be achieved. To this end, we assume partial knowledge of the structure of the dynamics [2], although the functions and parameters involved are unknown:

$$D(q) = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22}(q_3) & d_{23} \\ 0 & d_{23} & d_{33} \end{bmatrix}$$
$$C(q,\dot{q}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & c_{22}(q_3,\dot{q}_3) & c_{23}(q_3,\dot{q}_2) \\ 0 & c_{32}(q_3,\dot{q}_2) & 0 \end{bmatrix}$$
$$F(\dot{q}) = \begin{bmatrix} f_1(\dot{q}_3) \\ 0 \\ f_3(\dot{q}_3) \end{bmatrix} \quad G(q) = \begin{bmatrix} g_1 \\ 0 \\ g_3 \end{bmatrix}$$
$$B(\dot{q}) = \begin{bmatrix} b_{11}(\dot{q}_3) & 0 \\ 0 & b_{22}(\dot{q}_3) \\ b_{31}(\dot{q}_3) & 0 \end{bmatrix} \quad (4)$$

where d_{11}, d_{23}, d_{33}, g_1, g_3 are unknown constants, $d_{22}(q_3)$, $c_{22}(q_3,\dot{q}_3)$, $c_{23}(q_3,\dot{q}_2)$, $c_{32}(q_3,\dot{q}_2)$, $f_1(\dot{q}_3)$, $f_3(\dot{q}_3)$, $b_{11}(\dot{q}_3)$, $b_{22}(\dot{q}_3)$, $b_{31}(\dot{q}_3)$ are unknown functions.

To facilitate control design, the following properties are in order:

Property 1: The vertical inertia is decoupled from the rotational inertia about the vertical axis, leading to the zero entries for d_{12} and d_{13} in $D(q)$.

Property 2: The inertia matrix $D(q)$ is positive definite.

Property 3: The terms d_{11}, $\frac{d_{22}(q_3)d_{33}-d_{23}^2}{2d_{33}}$ and $\frac{d_{22}(q)d_{33}-d_{23}^2}{d_{22}(q_3)}$ are positive.

Property 4: The matrix $(\dot{D} - 2C)$ is skew-symmetric.

Property 5: The following equation $\dot{d}_{22}(q_3) - 2c_{22}(q_3,\dot{q}_3) = 0$ holds.

Assumption 3: There exist positive functions $\bar{b}_{11}(\dot{q}_3,\ddot{q}_3)$ and $\bar{b}_{22}(\dot{q}_3,\ddot{q}_3)$, such that $|\dot{b}_{11}(\dot{q}_3)| \leq \bar{b}_{11}(\dot{q}_3,\ddot{q}_3)$, $|\dot{b}_{22}(\dot{q}_3)| \leq \bar{b}_{22}(\dot{q}_3,\ddot{q}_3)$.

Remark 2: Functions $\bar{b}_{11}(\dot{q}_3,\ddot{q}_3)$ and $\bar{b}_{22}(\dot{q}_3,\ddot{q}_3)$ are introduced for analytical purpose only, and are not needed in actual control system design.

Assumption 4: All the positional variables including $q, \dot{q}, \ddot{q}$, and the control input τ_1 are measurable.

From (1) and (4), we can observe that $\ddot{q}_2$ and $\ddot{q}_3$ are coupled. In this case, control design directly based on (1) is difficult. After some simple manipulations, we can obtain three subsystems: q_1-subsystem (5), q_2-subsystem (6) and q_3-subsystem (7) as follows

$$d_{11}\ddot{q}_1 + f_1(\dot{q}_3) + g_1 = b_{11}(\dot{q}_3)\tau_1 \quad (5)$$

$$\frac{d_{22}(q_3)d_{33} - d_{23}^2}{d_{33}}\ddot{q}_2 + c_{22}(q_3,\dot{q}_3)\dot{q}_2 + c_{23}(q_3,\dot{q}_2)\dot{q}_3 + \frac{d_{23}}{d_{33}}(b_{31}(\dot{q}_3)\tau_1 - c_{32}(q_3,\dot{q}_2)\dot{q}_2 - f_3(\dot{q}) - g_3) = b_{22}(\dot{q}_3)\tau_2 \quad (6)$$

$$\frac{d_{22}(q_3)d_{33} - d_{23}^2}{d_{22}(q_3)}\ddot{q}_3 + c_{32}(q_3,\dot{q}_2)\dot{q}_2 + f_3(\dot{q}_3) + g_3 + d_{23}(b_{22}(\dot{q}_3)\tau_2 - c_{22}(q_3,\dot{q}_3)\dot{q}_2 - c_{23}(q_3,\dot{q}_2)\dot{q}_3) + \frac{d_{23}}{d_{22}(q_3)}(b_{22}(\dot{q}_3)\tau_2 - c_{22}(q_3,\dot{q}_3)\dot{q}_2 - c_{23}(q_3,\dot{q}_2)\dot{q}_3) = b_{31}(\dot{q}_3)\tau_1 \quad (7)$$

In the following, we can analyze and design control law for each subsystem. For clarity, define the tracking errors and the filtered tracking errors as

$$\begin{aligned} e_i &= q_i - q_{id} \\ r_i &= \dot{e}_i + \lambda_i e_i \end{aligned} \quad (8)$$

where λ_i is a positive number, $i = 1, 2$. Then, the boundedness of r_i guarantees the boundedness of e_i and $\dot{e}_i$ [9] [12]. To study the stability of e_i and $\dot{e}_i$, we only need to study the properties of r_i.

In addition, the following computable signals are defined:

$$\begin{aligned} \dot{q}_{ir} &= \dot{q}_{id} - \lambda_i e_i \\ \ddot{q}_{ir} &= \ddot{q}_{id} - \lambda_i \dot{e}_i \end{aligned}$$

A. q_1-subsystem

Since $\dot{q}_1 = \dot{q}_{1r} + r_1$, $\ddot{q}_1 = \ddot{q}_{1r} + \dot{r}_1$, equation (5) becomes

$$\frac{d_{11}}{b_{11}(\dot{q}_3)}\dot{r}_1 = \tau_1 - f_{S1,1} \quad (9)$$

where

$$f_{S1,1} = \frac{1}{b_{11}(\dot{q}_3)}\left[d_{11}\ddot{q}_{1r} + f_1(\dot{q}_3) + g_1\right] \quad (10)$$

Consider the following Lyapunov function candidate

$$V_1(r_1, \tilde{W}_1) = \frac{d_{11}}{2b_{11}(\dot{q}_3)}r_1^2 + \frac{1}{2}\tilde{W}_1^T\Gamma_1^{-1}\tilde{W}_1 > 0 \quad (11)$$

Remark 3: From physics, to lift the helicopter up for flight operation, we need that $\dot{q}_3^2 \geq c_0 > 0$ to overcome the gravity first. As such, we know that $b_{11}(\dot{q}_3) = c_1\dot{q}_3^2 \geq c_{01} > 0$, accordingly, V_1 is a valid Lyapunov candidate. The particular choice of V_1 is to establish the stability for r_1 and $\tilde{W}_1$ only, and we will investigate the stability for q_3 separately later. Similarly, we can choose Lyapunov function candidate (27) for q_2-subsystem in Section III-B.

The time derivative of (11) is given by

$$\begin{aligned} \dot{V}_1 &= \frac{d_{11}r_1}{b_{11}(\dot{q}_3)}\dot{r}_1 - \frac{d_{11}r_1^2}{2}\frac{\dot{b}_{11}(\dot{q}_3)}{b_{11}^2(\dot{q}_3)} + \tilde{W}_1^T\Gamma_1^{-1}\dot{\tilde{W}}_1 \\ &= r_1(\tau_1 - f_{S1,1}) - \frac{d_{11}r_1^2}{2}\frac{\dot{b}_{11}(\dot{q}_3)}{b_{11}^2(\dot{q}_3)} + \tilde{W}_1^T\Gamma_1^{-1}\dot{\tilde{W}}_1 \end{aligned} \quad (12)$$

Adding and subtracting $\frac{d_{11}r_1^2}{2}\frac{\bar{b}_{11}(\dot{q}_3,\ddot{q}_3)}{b_{11}^2(\dot{q}_3)}$ on the right-hand side of (12), we have

$$\begin{aligned} \dot{V}_1 &= r_1(\tau_1 - f_{S1,2}) - \frac{d_{11}r_1^2}{2b_{11}^2(\dot{q}_3)}\left[\bar{b}_{11}(\dot{q}_3,\ddot{q}_3) + \dot{b}_{11}(\dot{q}_3)\right] \\ &\quad + \tilde{W}_1^T\Gamma_1^{-1}\dot{\tilde{W}}_1 \end{aligned} \quad (13)$$

where

$$\begin{aligned} f_{S1,2} &= \frac{1}{b_{11}(\dot{q}_3)}\Big[d_{11}\ddot{q}_{1r} + f_1(\dot{q}_3) + g_1 \\ &\quad - \frac{d_{11}r_1}{2}\frac{\bar{b}_{11}(\dot{q}_3,\ddot{q}_3)}{b_{11}(\dot{q}_3)}\Big] \end{aligned} \quad (14)$$

According to Assumption 3, we know that

$$\bar{b}_{11}(\dot{q}_3,\ddot{q}_3) + \dot{b}_{11}(\dot{q}_3) > 0 \quad (15)$$

From (13) and (15), we can obtain

$$\dot{V}_1 \leq r_1(\tau_1 - f_{S1,2}) + \tilde{W}_1^T\Gamma_1^{-1}\dot{\tilde{W}}_1 \quad (16)$$

The unknown continuous function $f_{S1,2}$ in (16) can be approximated by RBFNN to arbitrary any accuracy as

$$f_{S1,2} = W_1^{*T}S_1(Z_1) + \varepsilon_1(Z_1) \quad (17)$$

where the input vector $Z_1 = [q_1,\ \dot{q}_1,\ \dot{q}_3,\ \ddot{q}_3,\ q_{1d},\ \dot{q}_{1d},\ \ddot{q}_{1d}]^T \in \Omega_{Z1} \subset R^7$; $\varepsilon_1(Z_1)$ is the approximation error satisfying $|\varepsilon_1(Z_1)| \leq \bar{\varepsilon}_1$, where $\bar{\varepsilon}_1$ is an unknown positive constant; W_1^* are unknown ideal constant weights satisfying $\|W_1^*\| \leq w_{1m}$, where w_{1m} is an unknown positive constant; and $S_1(Z_1)$ are the basis functions. By using $\hat{W}_1$ to approximate W_1^*, the error between the actual and the ideal RBFNNs can be expressed as

$$\hat{W}_1^T S_1(Z_1) - W_1^{*T}S_1(Z_1) = \tilde{W}_1^T S_1(Z_1)$$

where $\tilde{W}_1 = \hat{W}_1 - W_1^*$. As W_1^* is a constant vector, we know that

$$\dot{\tilde{W}}_1 = \dot{\hat{W}}_1 \quad (18)$$

Substituting (17) and (18) in (16), we have

$$\dot{V}_1 \leq r_1\left[\tau_1 - W_1^{*T}S_1(Z_1) - \varepsilon_1(Z_1)\right] + \tilde{W}_1^T\Gamma_1^{-1}\dot{\hat{W}}_1 \quad (19)$$

Consider the RBFNN control as

$$\tau_1 = -k_1 r_1 + \hat{W}_1^T S_1(Z_1) \quad (20)$$

where $k_1 > \frac{1}{2}$. Substituting (20) in (19), we have

$$\begin{aligned} \dot{V}_1 &\leq r_1\left[-k_1 r_1 + \tilde{W}_1^T S_1(Z_1) - \varepsilon_1(Z_1)\right] \\ &\quad + \tilde{W}_1^T\Gamma_1^{-1}\dot{\hat{W}}_1 \end{aligned} \quad (21)$$

Consider the following RBFNNs weight adaptation law

$$\dot{\hat{W}}_1 = -\Gamma_1(S_1(Z_1)r_1 + \sigma_1\hat{W}_1) \quad (22)$$

where $\Gamma_1 = \Gamma_1^T > 0$, and small constant $\sigma_1 > 0$ is the σ-modification for the adaptation. Substituting (22) into (21), we have

$$\dot{V}_1 \leq -k_1 r_1^2 - r_1\varepsilon_1(Z_1) - \sigma_1\tilde{W}_1^T\hat{W}_1 \quad (23)$$

Since $2\tilde{W}_1^T\hat{W}_1 \geq \|\tilde{W}_1\|^2 - \|W_1^*\|^2$, (23) becomes

$$\dot{V}_1 \leq -\left(k_1 - \frac{1}{2}\right)r_1^2 + \alpha_1 \quad (24)$$

with

$$\alpha_1 = \frac{1}{2}\bar{\varepsilon}_1^2 + \frac{1}{2}\delta_1\|W_1^*\|^2$$

From (24), we can obtain

$$\dot{V}_1 \leq 0 \quad (25)$$

for $|r_1| \geq \beta_1$, where $\beta_1 = \sqrt{\frac{\alpha_1}{k_1 - \frac{1}{2}}}$, and r_1 will converge to a compact set denoted by

$$\Omega_{r_1} := \{r_1 : |r_1| \leq \beta_1\} \quad (26)$$

According to Lemma 1, we know that $\hat{W}_1(t)$ are bounded for bounded initial weights $\hat{W}_1(0)$. Since the control signal τ_1 is a function of $\hat{W}_1, r_1$, we know that it is also bounded.

B. q_2-subsystem

Similar to Section III-A, to analyze the closed loop stability for the q_2-subsystem, let

$$V_2(r_2,\tilde{W}_2) = -\frac{1}{2b_{22}(\dot{q}_3)}\frac{d_{22}(q_3)d_{33}-d_{23}^2}{d_{33}}r_2^2 + \frac{1}{2}\tilde{W}_2^T\Gamma_2^{-1}\tilde{W}_2 \quad (27)$$

We consider the RBFNN control law and weight adaptation law as

$$\tau_2 = k_2r_2 + \hat{W}_2^TS_2(Z_2) \quad (28)$$
$$\dot{\hat{W}}_2 = \Gamma_2(S_2(Z_2)r_2 - \delta_2\hat{W}_2) \quad (29)$$

where $k_2 > \frac{1}{2}$, $\delta_2 > 0$, $\Gamma_2 = \Gamma_2^T > 0$; the input vector $Z_2 = [\tau_1,\ q_2,\ \dot{q}_2,\ q_3,\ \dot{q}_3,\ \ddot{q}_3,\ q_{2d},\ \dot{q}_{2d},\ \ddot{q}_{2d}]^T \in \Omega_{Z_2} \subset R^9$. Here, we need to mention that $W_2^{*T}S_2(Z_2)$ is to approximate the following function

$$\frac{1}{b_{22}(\dot{q}_3)}\Big[\frac{d_{22}(q_3)d_{33}-d_{23}^2}{d_{33}}\ddot{q}_{2r} + c_{22}(q_3,\dot{q}_3)\dot{q}_{2r} + c_{23}(q_3,\dot{q}_3)\dot{q}_3 + \frac{d_{23}}{d_{33}}(b_{31}(\dot{q}_3)\tau_1 - c_{32}(q_3,\dot{q}_2)\dot{q}_2 - g_3 - f_3(\dot{q}_3)) + \frac{1}{2}r_2^2\frac{d_{22}(q_3)d_{33}-d_{23}^2}{d_{33}}\frac{\bar{b}_{22}(\dot{q}_3,\ddot{q}_3)}{b_{22}(\dot{q}_3)}\Big] \quad (30)$$

where $\varepsilon_2(Z_2)$ is the approximation error satisfying $|\varepsilon_2(Z_2)| \le \bar{\varepsilon}_2$, and $\bar{\varepsilon}_2$ is an unknown positive constant. Similar to Section III-A, we can obtain

$$\dot{V}_2 \le 0 \quad (31)$$

for $|r_2| \ge \beta_2$, where $\beta_2 = \sqrt{\frac{\alpha_2}{k_2-\frac{1}{2}}}$, $\alpha_2 = \frac{1}{2}\bar{\varepsilon}_2^2 + \frac{1}{2}\delta_2\|W_2^*\|^2$; and r_2 will converge to a compact set denoted by

$$\Omega_{r_2} := \{r_2 : |r_2| \le \beta_2\} \quad (32)$$

In addition, we can easily obtain that $\hat{W}_2(t)$ and τ_2 are bounded.

C. q_3-subsystem

Finally, for system (5)–(7) under control laws (20) and (28), the q_3-subsystem (7) can be rewritten as

$$\dot{\eta} = f(\xi,\eta,u) \quad (33)$$

where $\eta = [q_3,\dot{q}_3]^T$, $\xi = [q_1,q_2,\dot{q}_1,\dot{q}_2]^T$, $u = [\tau_1,\tau_2]^T$. Then, the zero dynamics can be addressed as [13]

$$\dot{\eta} = f(0,\eta,u^*(0,\eta)) \quad (34)$$

where $u^* = [\tau_1^*,\tau_2^*]^T$.

Assumption 5: System (33) is hyperbolically minimum-phase, i.e. zero dynamics (34) is exponentially stable. In addition, assume that the control input u is designed as a function of the states (ξ,η) and the reference signal satisfying Assumption 1, and the function $f(\xi,\eta,u)$ is Lipschitz in ξ, i.e., there exist constants L_ξ and L_f for $f(\xi,\eta,u)$ such that

$$\|f(\xi,\eta,u) - f(0,\eta,u_\eta)\| \le L_\xi\|\xi\| + L_f \quad (35)$$

where $u_\eta = u^*(0,\eta)$.

Remark 4: For the general case, it is difficult to demonstrate the validity of Assumption 5. For the specific helicopter model, we can verify it as demonstrated in Section IV-A.

Lemma 2: For the internal dynamics $\dot{\eta} = f(\xi,\eta,u)$ of the system, if Assumptions 1 and 3 are satisfied, then there exist positive constants L_η and T_0, such that

$$\|\eta(t)\| \le L_\eta, \quad \forall t > T_0 \quad (36)$$

Proof: The proof is similar to [13]. For brevity, it is omitted here.

Theorem 1: Consider the system (5)–(7) with Assumptions 1 - 5, under the action of control laws (20), (28) and adaptation laws (22), (29). For each compact set Ω_0, where $\{q_i(0),\dot{q}_i(0),\hat{W}_j(0), i=1,2,3,\ j=1,2\} \in \Omega_0$, all closed loop signals are SGUUB, and the tracking errors e_i converge the compact sets as follows

$$\Omega_{e_i} := \left\{e_i : |e_i| \le \frac{\beta_i}{\lambda_i}\right\}$$

where β_i, λ_i are defined as previously, $i = 1, 2$.

Proof: For each compact set Ω_0, from the previous analysis, we know that $\hat{W}_1, \hat{W}_2, \tau_1, \tau_2$ are bounded and the filtered tracking errors r_1 and r_2 converge to Ω_{r_1} and Ω_{r_2} defined in (26) and (32) respectively. Substituting (8) into (26) and (32), we obtain that

$$-\beta_i \le \dot{e}_i + \lambda_ie_i \le \beta_i$$

where β_i, λ_i are defined as previously, $i = 1, 2$. Solving this inequalities leads to

$$e_i(0)e^{-\lambda_it} - \frac{\beta_i}{\lambda_i}(1-e^{-\lambda_it}) \le e_i \le e_i(0)e^{-\lambda_it} + \frac{\beta_i}{\lambda_i}(1-e^{-\lambda_it})$$

As $t \to \infty$, we have

$$-\frac{\beta_i}{\lambda_i} \le e_i \le \frac{\beta_i}{\lambda_i}$$

i.e.

$$|e_i| \le \frac{\beta_i}{\lambda_i}$$

From Assumption 1 that $q_{1d}, \dot{q}_{1d}, q_{2d}, \dot{q}_{2d}$ are bounded, we know that the states $q_1, \dot{q}_1, q_2, \dot{q}_2$ are bounded. From Lemma 2, we know that $q_3, \dot{q}_3$ is also bounded. Therefore, we have shown that all the closed loop signals are SGUUB. By appropriately choosing the design constants $\lambda_i, k_i, i = 1, 2$, we can achieve the tracking performance of q_1, q_2 to any prescribed accuracy. This completes the proof. ∎

IV. Simulation Study

To illustrate the proposed adaptive neural control algorithms, we consider the VARIO helicopter mounted on a platform [2], with the following dynamic model and parameters

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) = B(\dot{q})\tau \quad (37)$$

where $q = [q_1 \;\; q_2 \;\; q_3]^T = [z, \phi, \gamma]^T$,

$$D(q) = \begin{bmatrix} 7.5 & 0 & 0 \\ 0 & d_{22} & 0.108 \\ 0 & 0.108 & 0.4993 \end{bmatrix}$$

$$C(q, \dot{q}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & c_{22} & c_{23} \\ 0 & c_{32} & 0 \end{bmatrix}$$

$$F(\dot{q}) = \begin{bmatrix} -0.6004\dot{q}_3 \\ 0 \\ -0.0001206\dot{q}_3^2 \end{bmatrix}$$

$$B(\dot{q}) = \begin{bmatrix} 3.411\dot{q}_3^2 & 0 \\ 0 & -0.1525\dot{q}_3^2 \\ 12.01\dot{q}_3 + 10^5 & 0 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} -77.259 \\ 0 \\ -2.642 \end{bmatrix} \quad (38)$$

where $d_{22} = 0.4305 + 0.0003\cos^2(-4.143q_3)$, $c_{22} = 0.0006214\sin(-8.286q_3)\dot{q}_3$, $c_{23} = c_{32} = 0.0006214\sin(-8.286q_3)\dot{q}_2$, and all quantities are expressed in S.I. units.

The control objective is to track the following uniformly bounded desired trajectories (39) [2]:

$$q_{1d} = \begin{cases} -0.2 & 0 \le t \le 50s \\ 0.3[e^{-(t-50)^2/350} - 1] - 0.2 & 50 < t \le 130s \\ 0.1\cos[(t-130)/10] - 0.6 & 130 < t \le 20\pi + 130 \\ -0.5 & t \ge 20\pi + 130 \end{cases}$$

$$q_{2d} = \begin{cases} 0 & t < 50s \\ 1 - e^{-(t-50)^2/350} & 50 \le t < 120s \\ e^{-(t-120)^2/350} & 120 \le t < 180 \\ -1 + e^{-(t-180)^2/350} & t \ge 180 \end{cases} \quad (39)$$

A. Internal Dynamics Stability Analysis

In this subsection, we show that the internal dynamics of the helicopter system in (37) has a stable behavior. In [2], the zero dynamics stability of the model-based control case is discussed in detail. Similarly, for the RBFNN-based case, we substitute (17), (20), (28) and (30) into the q_3-subsystem (7) first. According to the definition of the zero dynamics [13], we set r_1, r_2, $\tilde{W}_1^T$, $\tilde{W}_2^T$, $\varepsilon_1(Z_1)$ and $\varepsilon_2(Z_2)$ to zero, and that the desired trajectories and initial data can be chosen in such a way that terms including $\dot{q}_2^2$, $\ddot{q}_{1d}$, $\ddot{q}_{2d}$ can be neglected [2], we have

$$\ddot{q}_3 = \frac{1}{d_{33}}\left[\frac{b_{31}(\dot{q})}{b_{11}(\dot{q})}(d_1(\dot{q}) + g_1) - f_3(\dot{q}) - g_3\right] \quad (40)$$

We can linearize equation (40) around different equilibrium points $\dot{q}_3$. For each equilibrium point, substituting the term values (38) into (40) and assuming the angular acceleration is zero, we can write

$$4.1137 \times 10^{-4}\dot{q}_3^4 + 1.8011\dot{q}_3^2 - 60968\dot{q}_3 - 7725900 = 0$$

Its solutions are $\dot{q}_3^* = -124.63, -219.5 \pm 468.16i$ and 563.64 rad/s. The first value $\dot{q}_3^* = -124.63$ has a physical meaning for the system. If we linearize equation (40) around the equilibrium point $\dot{q}_3^* = -124.63$, we can obtain an eigenvalue -2.44. Therefore, all initials of $\dot{q}_3$ sufficiently near $\dot{q}_3^* = -124.63$ can converge to -124.63. It then follows that the zero-dynamics of the helicopter system in (37) has a stable behavior. Actually, we can also know, through the simulation results, that the zero dynamics are indeed stable. From Figure 5, we can observe that the main rotor angular speed $\dot{q}_3$ converges to the value -124.63 rad/s for different initial values ranging from -40 rad/s to -150 rad/s, which includes the typical operating values more than sufficiently. These results are expected from the previous stability analysis, and also verifies the results in [2].

B. Comparison of RBFNN-Based Approach and Model-Based Approach

For the RBFNN control laws (20) (28) and adaptation laws (22) (29), the input vectors $Z_1 = [q_1, \dot{q}_1, \dot{q}_3, \ddot{q}_3, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d}]^T \in R^7$, and $Z_2 = [\tau_1, q_2, \dot{q}_2, q_3, \dot{q}_3, \ddot{q}_3, q_{2d}, \dot{q}_{2d}, \ddot{q}_{2d}]^T \in R^9$. Neural networks $\hat{W}_1^T S_1(Z_1)$ contains 2187 nodes (i.e., $l_1 = 2187$), with centers $\mu_l(l = 1, ..., l_1)$ evenly spaced in $[-1.0, 1.0] \times [-0.1, 0.1] \times [-150.0, -40.0] \times [-20.0, 50.0] \times [-1.0, 1.0] \times [-0.1, 0.1] \times [-0.01, 0.01]$, and widths $\eta_l = 1.0(l = 1, ..., l_1)$. Neural networks $\hat{W}_2^T S_2(Z_2)$ contains 19683 nodes (i.e., $l_2 = 19683$), with centers $\mu_l(l = 1, ..., l_2)$ evenly spaced in $[-0.005, 0.005] \times [-10.0, 10.0] \times [-40000, 0.0] \times [-1.0, 1.0] \times [-150.0, -40.0] \times [-20.0, 50.0] \times [-10.0, 10.0] \times [-1.0, 1.0] \times [-0.01, 0.01]$, and widths $\eta_l = 1.0(l = 1, ..., l_2)$. The design parameters of the above controllers are: $k_1 = 26.0$, $\lambda_1 = 1.0$, $k_2 = 26.0$, $\lambda_2 = 1.0$, $\Gamma_1 = \text{diag}\,[600.0]$, $\Gamma_2 = \text{diag}\,[600.0]$, $\delta_1 = 0.001$, $\delta_2 = 0.001$. The initial conditions are: $q_1(0) = -0.2$ m, $\dot{q}_1(0) = 0$ m/s, $q_2(0) = -\pi$ rad, $\dot{q}_2(0) = 0$ rad/s, $q_3(0) = -\pi$ rad, $\dot{q}_3(0) = -120.0$ rad/s, $\tau_1 = -0.0005$ m, $\tau_2 = 0.005$ m, $\hat{W}_1(0) = 0$, $\hat{W}_2(0) = 0$.

The comparison simulation results of RBFNN and model-based can be seen in Figs. 1 – 4. From Figs. 1 and 2, we can see that the tracking performance of RBFNN-based controllers is better than that of model-based controllers, with the presence of the parametric errors ($g_1 = -77.0$ and $g_3 = -1.5$) and functional uncertainties ($f_3 = -0.0001206\dot{q}_3^2$ is unmodelled). Fig. 3 indicates that the RBFNN control actions τ_1 and τ_2 are bounded. The boundedness of the RBFNN weight estimates are indicated in Fig. 4.

V. CONCLUSION

In this paper, adaptive neural network (NN) tracking control is carried out for helicopters in the presence of parametric and functional uncertainties. The good tracking performance and semiglobal uniformly ultimate boundedness (SGUUB) of all the closed-loop signals are guaranteed under the proposed control.

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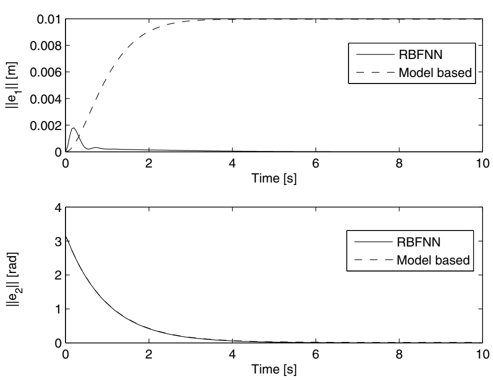

Fig. 1. Tracking error norms with parametric errors

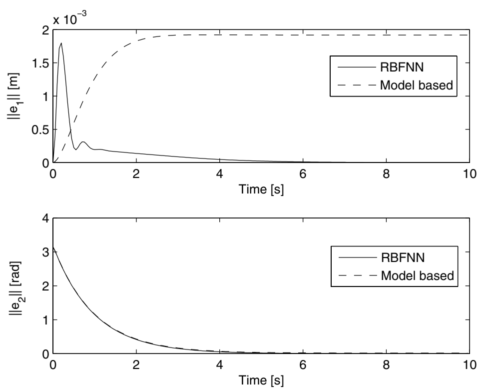

Fig. 2. Tracking error norms with unmodelled dynamics

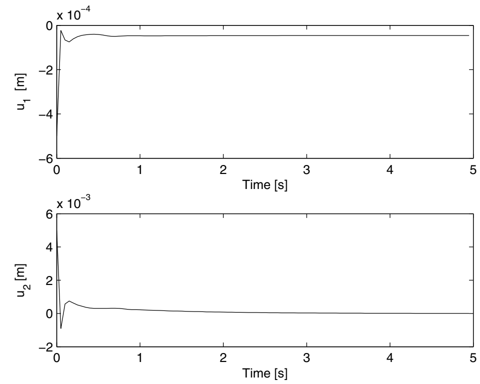

Fig. 3. Control inputs for RBFNN

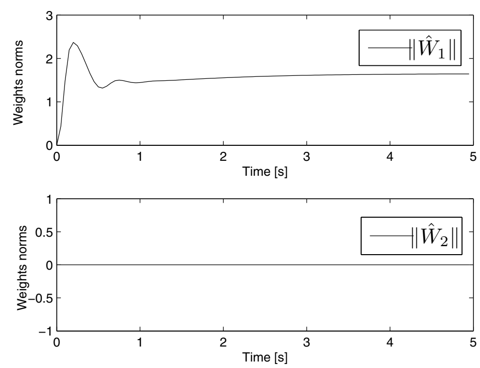

Fig. 4. Weights norms for RBFNN

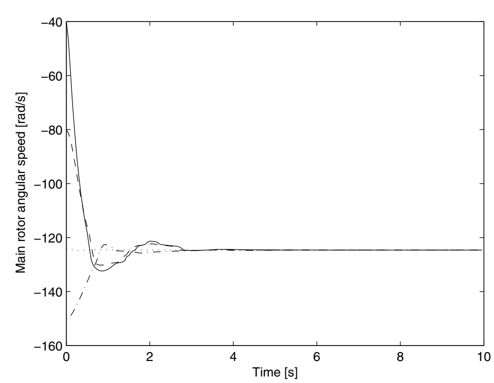

Fig. 5. Main rotor angular speed $\dot{q}_3$ behavior for RBFNN