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Decentralized Cooperative Control for Swarm Agents with High-Order Dynamics

Beibei Ren, Hailong Pei, Zhendong Sun, Shuzhi Sam Ge and Tong Heng Lee

Abstract— **In this paper, decentralized controllers are developed to drive a swarm of mobile agents with high-order ($n > 2$) nonlinear dynamics in strict feedback form into a moving target region while avoiding collisions among themselves. It is important to consider coordination of multiple high-order agent dynamics which generalize the existing simple single-integrator/double-integrator ones because, in practice, we may need to incorporate actuator dynamics into the vehicle dynamics in order to achieve better performance, thus increasing the order of the system dynamics. The control design is based on a fusion of two kinds of new potential functions (target potential functions and collision avoidance potential functions), backstepping technique and Lyapunov synthesis. The presence of parametric uncertainties is handled by adaptive control techniques. The framework does not depend on a fixed ordering of agents, and is robust to individual agent failures. Therefore, flexibility and scalability are improved. Simulation results illustrate the performance of the proposed approach.**

I. Introduction

During the last two decades, the research of multi-agent systems has received a surge of attention of researchers from different disciplines and has been extensively investigated in numerous applications. Various approaches have been proposed for coordination of multi-agent systems, including leader-follower[1]-[3], virtual structure[4][5], behavior-based[6]-[8], navigation functions[9], control Lyapunov functions [10], artificial potentials based [11]-[13].

Among the above seminal works, most of the agent dynamics have been investigated either as simple single-integrator/double-integrator ones, or as vehicle dynamics, that can be converted to double-integrator dynamics via feedback linearization. Notice that, in practice, in order to achieve better performance, we may need to incorporate actuator dynamics into the vehicle dynamics, thus increasing the order of the system dynamics. For example, research on the control of marine vessels has been extended to include consideration of the thruster dynamics. To actively minimize torsional vibrations within the propulsive shafting system, a marine shafting system is modeled as a chained multiple mass-spring system [14][15]. As a result, the whole marine vessel dynamics is described by a high-order nonlinear system in strict feedback form. However, to the best of our knowledge, in most literature about cooperative control of multiple marine vessels, only first-order kinematic models or second-order dynamic models without actuator dynamics were considered [16][17]. This motivates the idea of cooperative control problem for multiple high-order agent systems considering actuator dynamics.

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In this paper, a decentralized multi-agent swarming control with limited sensing ranges is developed for a more general case, where the agent dynamics are represented as high-order nonlinear systems in strict feedback form which is feasible to be handled by backstepping technique. We formalize the concept of a target region for the entire swarm, instead of a target point for each agent. The goal is for all the agents to converge to the target region without collisions, regardless of the exact location for each agent. As such, the flexibility and scalability of the framework are improved. Further contributions include two kinds of potential functions for every agent, i.e., the target potential function and the collision avoidance potential function. By driving both the target potential energy and the collision avoidance potential energy to zero, our target region convergence for all agents and the collisions avoidance between any agents are achieved accordingly. Last, but not least, we incorporate adaptive control into our decentralized control framework based on potential functions and Lyapunov synthesis to accommodate parametric uncertainties in the system model.

II. Problem Formulation and Preliminaries

In this paper, we consider a multi-agent system consisting of N mobile agents, and each of them satisfies the following dynamics in strict feedback form:

$$\begin{aligned} \dot{q}_{i,j} &= \Phi_{i,j}(\bar{q}_{i,j})\theta_i + G_{i,j}(\bar{q}_{i,j})q_{i,j+1} \\ \dot{q}_{i,m} &= \Phi_{i,m}(\bar{q}_{i,m})\theta_i + G_{i,m}(\bar{q}_{i,m})u_i \\ y_i &= q_{i,1} \end{aligned} \quad (1)$$

where $q_{i,j} \in \mathbb{R}^n$, $i = 1, 2, ..., N$, $j = 1, 2, ..., m$ are the states of i-th agent, $\bar{q}_{i,j} = [q_{i,1}^T, q_{i,2}^T, ..., q_{i,j}^T]^T \in \mathbb{R}^{nj}$, $\Phi_{i,j}(\bar{q}_{i,j}) \in \mathbb{R}^{n\times r}$ and $G_{i,j} \in \mathbb{R}^{n\times n}$ are known nonlinear function matrixes, and $\theta_i \in \mathbb{R}^r$ is a vector of uncertain constant parameters, $u_i \in \mathbb{R}^n$ and $y_i \in \mathbb{R}^n$ are the input and output vectors of agent i respectively. We assume that all the agents have the same characteristics and communicate without time delays. Each agent i has a communication range, which is centered at the agent and has a radius R_i. Two agents are considered to be connected if they are within each other's communication range; otherwise, they are considered as disconnected. Moreover, we use $\mathcal{G}_i$ to denote the set of indices for those agents within communication range of agent i. The agent i can only measure the information of itself and its neighbours belonging to $\mathcal{G}_i$.

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Our objective is to ensure that all the agents will converge to a common moving target region, without collisions between any agents in the group. For simplicity, the common moving target region Ω is chosen as a circle centered around the point q_0 with radius r_0 as shown in Fig. 1.

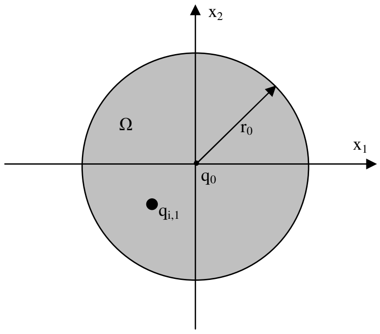

Fig. 1. Illustration of the target region Ω

Definition 1: For agent i, the target region can be expressed as

$$\Omega = \{q_{i,1} \mid f_{i0}(\tilde{q}_{i0}) = \|\tilde{q}_{i0}\|^2 - r_0^2 \leq 0\}, \tag{2}$$

where $\tilde{q}_{i0} = q_{i,1} - q_0$, $q_{i,1}$ and q_0 are the positions of the agent i and the center of the target region respectively, $f_{i0}(\cdot) : \mathbb{R}^2 \to \mathbb{R}$ is the target function of agent i, which satisfies the following properties:

(i) The boundedness of $f_{i0}(\tilde{q}_{i0})$ can assure the boundedness of $\|\tilde{q}_{i0}\|$.
(ii) The target function $f_{i0}(\tilde{q}_{i0})$ is continuous and differentiable with respect to $\tilde{q}_{i0}$.

Definition 2: Define the target potential function $P_{i0}(\tilde{q}_{i0}) : \mathbb{R}^2 \to \mathbb{R}$ for agent i, which satisfies the following properties:

(i) If $q_{i,1} \in \Omega$, then $P_{i0} = 0$; if $q_{i,1} \notin \Omega$, then $P_{i0} > 0$.
(ii) If $q_{i,1} \notin \Omega$, P_{i0} is monotonically increasing with $\|\tilde{q}_{i0}\|$, and $P_{i0} \to \infty$ as $\|\tilde{q}_{i0}\| \to \infty$.
(iii) P_{i0} is continuous and differentiable with respect to $\tilde{q}_{i0}$.
(iv) $\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}} = -\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_0}$.

In this paper, P_{i0} is chosen as follows:

$$P_{i0}(\tilde{q}_{i0}) = \begin{cases} 0, & q_{i,1} \in \Omega \\ \frac{c_i}{2} f_{i0}^2(\tilde{q}_{i0}), & q_{i,1} \notin \Omega \end{cases} \tag{3}$$

where c_i are positive constants and the shape of the target potential function P_{i0} can be illustrated in Fig. 2.

Definition 3: For agent i, we define a safe region Ω_{ij} as the exterior of a circle of radius r_i centered at the position of agent i, which can be expressed as

$$\Omega_{ij} = \{q_{j,1} \mid f_{ij}(\tilde{q}_{ij}) = r_i^2 - \|\tilde{q}_{ij}\|^2 \leq 0,\ j \in \mathcal{G}_i\} \tag{4}$$

where $\tilde{q}_{ij} = q_{i,1} - q_{j,1}$, $q_{i,1}$ and $q_{j,1}$ are the positions of agent i and agent j respectively, $f_{ij}(\tilde{q}_{ij}) : \mathbb{R}^2 \to \mathbb{R}$ is the collision avoidance function of agent i, which satisfies the following properties:

(i) The boundedness of $f_{ij}(\tilde{q}_{ij})$ can assure the boundedness of $\|\tilde{q}_{ij}\|$.

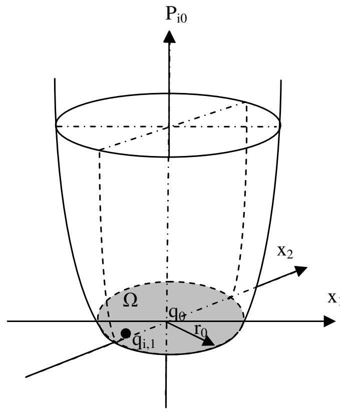

Fig. 2. Illustration of the target potential function P_{i0} of agent i

(ii) The collision avoidance function $f_{ij}(\tilde{q}_{ij})$ is continuous and differentiable with respect to $\tilde{q}_{ij}$.
(iii) $f_{ij}(\tilde{q}_{ij}) = f_{ji}(\tilde{q}_{ji})$ if $r_i = r_j$.

In addition, the region inside the circle is the danger region of agent i, labeled as $\bar{\Omega}_{ij}$.

The safe region Ω_{ij} and the danger region $\bar{\Omega}_{ij}$ of agent i are illustrated in Fig. 3.

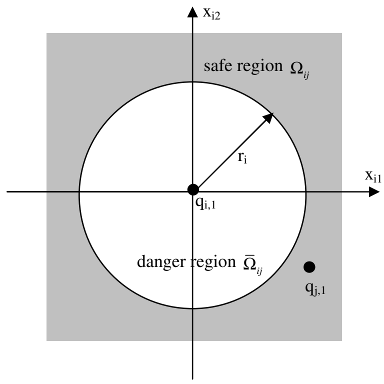

Fig. 3. Illustration of the safe region Ω_{ij} and the danger region $\bar{\Omega}_{ij}$ of agent i

Definition 4: We define the collision avoidance potential function $P_{ij}(\tilde{q}_{ij}) : \mathbb{R}^2 \to \mathbb{R}$ as a function that satisfies the following properties:

(i) If $q_{j,1} \in \Omega_{ij}$, then $P_{ij} = 0$; if $q_{j,1} \in \bar{\Omega}_{ij}$, then $P_{ij} > 0$.
(ii) If $q_{j,1} \in \bar{\Omega}_{ij}$, P_{ij} is monotonically increasing with the decreasing of $\|\tilde{q}_{ij}\|$, and $P_{ij} \to \infty$ as $\|\tilde{q}_{ij}\| \to 0$.
(iii) P_{ij} is continuous and differentiable with respect to $\tilde{q}_{ij}$, $\forall \|\tilde{q}_{ij}\| \in (0, +\infty)$.

In this paper, we choose $P_{ij}(\tilde{q}_{ij})$ as follows:

$$P_{ij}(\tilde{q}_{ij}) = \begin{cases} 0, & q_{j,1} \in \Omega_{ij} \\ \frac{c_{ij}}{2}\left(\log \frac{r_i^2}{\|\tilde{q}_{ij}\|^2}\right)^2, & q_{j,1} \in \bar{\Omega}_{ij} \end{cases} \tag{5}$$

where c_{ij} are positive constants and the shape of $P_{ij}(\tilde{q}_{ij})$ is illustrated in Fig. 4.

Assumption 1: The communication range radius R_i of agent i is larger than its danger region radius r_i, i.e., $R_i > r_i$.

Assumption 2: The target region Ω is big enough to contain all the agents and their danger regions, i.e., $r_0 >$

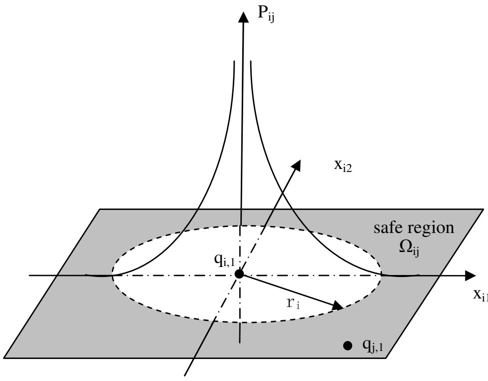

Fig. 4. Illustration of the collision avoidance potential function P_{ij} of agent i with other agents j ($j \neq i$) in the group

$\sum_{i=1}^{N} r_i$. In particular, we also assume that all r_i, $i = 1, 2, ..., N$, are equal. As such, we have that $r_0 > N r_i$.

Assumption 3: The states of the moving target region, $q_0(t)$, $\dot{q}_0(t)$, $\ddot{q}_0(t)$, ..., $q_0^{(m)}(t)$ are known and bounded.

Assumption 4: The control gain matrices $G_{i,j}$, $i = 1, 2, ..., N$, $j = 1, 2, ..., m$ are known and nonsingular.

III. Control Design and Stability Analysis

Denote the error coordinates $z_{i,1} = \tilde{q}_{i0} = q_{i,1} - q_0$ and $z_{i,\rho} = q_{i,\rho} - \alpha_{i,\rho-1}$, $\rho = 2, ..., m$, where $\alpha_{i,\rho-1}$ are stabilizing function vectors to be designed. Consider the following general potential function and Lyapunov function candidates:

$$
\begin{aligned}
V_1 &= \sum_{i=1}^{N} P_{i0}(z_{i,1}) + \sum_{i=1}^{N} \sum_{j \in \mathcal{G}_i} P_{ij}(\tilde{q}_{ij}) + \sum_{i=1}^{N} \frac{1}{2} \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i \\
V_\rho &= V_{\rho-1} + \sum_{i=1}^{N} \frac{1}{2} z_{i,\rho}^T z_{i,\rho}, \quad \rho = 2, ..., m
\end{aligned}
$$

where P_{i0} and P_{ij} are defined in (3) and (5), respectively; $\Gamma_i = \Gamma_i^T > 0$ is a constant matrix, and $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$ is the error between θ_i and its estimate, $\hat{\theta}_i$. By designing the stabilizing functions, control law, and adaptation law as follows

$$
\begin{aligned}
\alpha_{i,1} &= G_{i,1}^{-1}(q_{i,1}) \Big\{ -\Phi_{i,1}(q_{i,1}) \hat{\theta}_i + \dot{q}_0 \\
&\quad -\kappa_{i,1} \Big[\frac{\partial P_{i0}(z_{i,1})}{\partial q_{i,1}} + 2 \sum_{j \in \mathcal{G}_i} \frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}} \Big] \Big\} \qquad (6) \\
\alpha_{i,2} &= G_{i,2}^{-1}(\bar{q}_{i,2}) \Big\{ -\kappa_{i,2} z_{i,2} \\
&\quad -G_{i,1}^T(q_{i,1}) \Big[\frac{\partial P_{i0}(z_{i,1})}{\partial q_{i,1}} + 2 \sum_{j \in \mathcal{G}_i} \frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}} \Big] \\
&\quad -\Big[\Phi_{i,2}(\bar{q}_{i,2}) - \Big(\frac{\partial \alpha_{i,1}}{\partial q_{i,1}} \Big)^T \Phi_{i,1}(q_{i,1}) \Big] \hat{\theta}_i \\
&\quad +\Big(\frac{\partial \alpha_{i,1}}{\partial q_{i,1}} \Big)^T G_{i,1}(q_{i,1}) q_{i,2} \\
&\quad + \sum_{j \in \mathcal{G}_i} \Big(\frac{\partial \alpha_{i,1}}{\partial q_{j,1}} \Big)^T [\Phi_{j,1}(q_{j,1}) \hat{\theta}_j + G_{j,1}(q_{j,1}) q_{j,2}] \\
&\quad + \sum_{j=0}^{1} \Big(\frac{\partial \alpha_{i,1}}{\partial q_0^{(j)}} \Big)^T q_0^{(j+1)} + \Big(\frac{\partial \alpha_{i,1}}{\partial \hat{\theta}_i} \Big)^T \Gamma_i \tau_{i,2} \Big\} \qquad (7) \\
\alpha_{i,\rho} &= G_{i,\rho}^{-1}(\bar{q}_{i,\rho}) \Big\{ -\kappa_{i,\rho} z_{i,\rho} - G_{i,\rho-1}^T(\bar{q}_{i,\rho-1}) z_{i,\rho-1} \\
&\quad -\Big[\Phi_{i,\rho}(\bar{q}_{i,\rho}) - \sum_{k=1}^{\rho-1} \Big(\frac{\partial \alpha_{i,\rho-1}}{\partial q_{i,k}} \Big)^T \Phi_{i,k}(\bar{q}_{i,k}) \Big] \\
&\quad \Big[\hat{\theta}_i - \Gamma_i^T \sum_{l=2}^{\rho-1} \Big(\frac{\partial \alpha_{i,l-1}}{\partial \hat{\theta}_i} \Big) z_{i,l} \Big] \\
&\quad + \sum_{k=1}^{\rho-1} \Big(\frac{\partial \alpha_{i,\rho-1}}{\partial q_{i,k}} \Big)^T G_{i,k}(\bar{q}_{i,k}) q_{i,k+1} \\
&\quad + \sum_{j \in \mathcal{G}_i} \sum_{k=1}^{\rho-1} \Big(\frac{\partial \alpha_{i,\rho-1}}{\partial q_{j,k}} \Big)^T \Big[\Phi_{j,k}(\bar{q}_{j,k}) [\hat{\theta}_j \\
&\quad -\Gamma_j^T \sum_{l=2}^{\rho-1} \Big(\frac{\partial \alpha_{j,l-1}}{\partial \hat{\theta}_j} \Big) z_{j,l}] + G_{j,k}(\bar{q}_{j,k}) q_{j,k+1} \Big] \\
&\quad + \sum_{j=0}^{\rho-1} \Big(\frac{\partial \alpha_{i,\rho-1}}{\partial q_0^{(j)}} \Big)^T q_0^{(j+1)} + \Big(\frac{\partial \alpha_{i,\rho-1}}{\partial \hat{\theta}_i} \Big)^T \Gamma_i \tau_{i,\rho} \Big\}, \\
&\quad \rho = 3, ..., m \qquad (8) \\
u_i &= \alpha_{i,m} \qquad (9) \\
\dot{\hat{\theta}}_i &= \Gamma_i \tau_{i,m} \qquad (10)
\end{aligned}
$$

where $\kappa_{i,\rho}$ are positive constants, and $\tau_{i,\rho}$ is the ρ-th tuning function defined as follows

$$
\begin{aligned}
\tau_{i,1} &= \Phi_{i,1}^T(q_{i,1}) \Big[\frac{\partial P_{i0}(z_{i,1})}{\partial q_{i,1}} + 2 \sum_{j \in \mathcal{G}_i} \frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}} \Big] \qquad (11) \\
\tau_{i,\rho} &= \tau_{i,\rho-1} \\
&\quad + \Big[\Phi_{i,\rho}(\bar{q}_{i,\rho}) - \sum_{k=1}^{\rho-1} \Big(\frac{\partial \alpha_{i,\rho-1}}{\partial q_{i,k}} \Big)^T \Phi_{i,k}(\bar{q}_{i,k}) \Big]^T z_{i,\rho} \\
&\quad - \sum_{j \in \mathcal{G}_i} \sum_{k=1}^{\rho-1} \Phi_{i,k}^T(\bar{q}_{i,k}) \Big(\frac{\partial \alpha_{j,\rho-1}}{\partial q_{i,k}} \Big) z_{j,\rho} \qquad (12)
\end{aligned}
$$

for $\rho = 2, ..., m$.

Then, the derivative of V_m can be written as

$$
\begin{aligned}
\dot{V}_m &= -\sum_{i=1}^{N} \kappa_{i,1} \Big\| \frac{\partial P_{i0}(z_{i,1})}{\partial q_{i,1}} + 2 \sum_{j \in \mathcal{G}_i} \frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}} \Big\|^2 \\
&\quad - \sum_{i=1}^{N} \sum_{j=2}^{m} \kappa_{i,j} \| z_{i,j} \|^2 \qquad (13)
\end{aligned}
$$

Theorem 1: Consider N mobile agent dynamics in (1) with the parameter uncertainties under Assumptions 1-4, decentralized controls in (9) and update laws in (10). Starting at different locations $q_{i,1}(0)$, all the agents will finally converge into the moving target region $\Omega = \{q_{i1} \mid f_{i0}(\tilde{q}_{i0}) = \|\tilde{q}_{i0}\|^2 - r_0^2 \leq 0,\ i = 1, 2, ..., N\}$, without collisions between any agents.

Proof: □ *First, we prove that no collisions occur between any agents.*

From (13), we know that $\dot{V}_m \leq 0$, which implies that $V_m(t) \leq V_m(0)$. With the definition of $V_m(t)$ in (6) and according to Definition 4, the boundedness of $P_{ij}(\tilde{q}_{ij})$ means $\|\tilde{q}_{ij}\| \neq 0$, i.e., there are no collisions among any agents for all $t > 0$.

□ *Next, we will prove that $q_{i1} \in \Omega$, i.e., each agent is located in the moving target region Ω.*

Since $\dot{V}_m$ is negative semidefinite as seen from (13), according to LaSalle Yoshizawa Theorem [18], we know that as $t \to \infty$,

$$C_1: \frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}} + 2\sum_{j \in \mathcal{G}_i} \frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}} = 0, \ i = 1, 2, ..., N \quad (14)$$

which is called an individual condition C_1 each agent must satisfy. Applying summation from $i = 1$ to N on both sides of (14) leads to the other condition C_2, called a group condition:

$$C_2: \quad \sum_{i=1}^{N} \frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}} = 0 \quad (15)$$

To prove that all agents converge into the moving target region Ω, we assume that not all the agents are located in the target region first, which means that there are always some agents located outside the target region. Let us separate N agents into two groups G_{out} and G_{in}. The group G_{out} consists of M $(1 \leq M \leq N)$ agents located outside the target region, and will be marked in grey in the subsequent figures. The group G_{in} consists of $N - M$ agents located inside the target region, and will be marked in black.

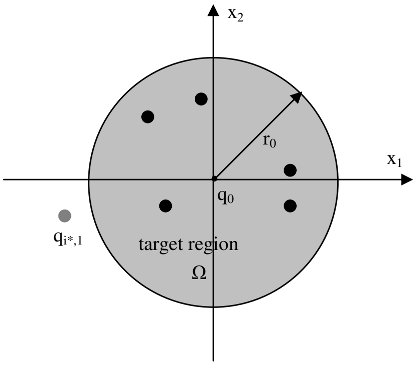

Fig. 5. Illustration of Case I: only one agent i^* is located outside the target region.

Case I ($M = 1$)**:** In this case, the group G_{out} only consists of one agent, labeled as agent i^*, as shown in Fig. 5. According to (3), we have

$$\sum_{i=1}^{N} \frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}} = \frac{\partial P_{i^*0}(\tilde{q}_{i^*0})}{\partial q_{i^*,1}} \neq 0 \quad (16)$$

Obviously, (16) contradicts with the group condition C_2 in (15). As such, Case I does not hold.

Case II ($2 \leq M \leq N - 1$)**:** In this case, the group G_{out} includes M $(2 \leq M \leq N - 1)$ agents and the group G_{in} includes $N - M$ agents. There are two possible cases:

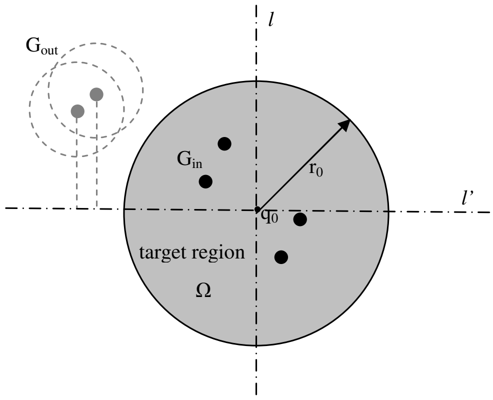

Fig. 6. Illustration of Case II.A: M $(2 \leq M \leq N - 1)$ agents are located outside the target region, and all these M agents are located on one side of the line l passing through the center of the target region.

- **Case II. A:** We can find a line, l, passing through the center of the target region such that all these M agents are located one side of the line as shown in Fig. 6. Then, we can draw another line l' which is also passing through the region center q_0 and perpendicular to the line l. According to (3), we have

$$\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}} = \begin{cases} 0 & i \in G_{in} \\ nonzero & i \in G_{out} \end{cases}$$

Defining the corresponding projection component of every vector $\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}}$ ($i \in G_{out}$) onto the line l', as $proj_{l'}\left[\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}}\right]$, and noticing that in Fig. 6, all the M agents belonging to the group G_{out} are located on the same side of the line l, thus, we know that the signs of all $proj_{l'}\left[\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}}\right]$ ($i \in G_{out}$) are the same. Therefore, we can arrive at

$$\sum_{i=1}^{N} proj_{l'}\left[\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}}\right] = \sum_{i \in G_{out}} proj_{l'}\left[\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}}\right] \neq 0$$

which contradicts with the group condition C_2 in (15), implying Case II.A is impossible too.

Case II. B: We can find such a line l which passes through the center of the target region to separate M agents of the group G_{out} into two disconnected groups, labeled as G_{out1} and G_{out2} respectively, as shown in Fig. 7.

Similar to the previous analysis in Case II. A, we can draw another line l' which is also passing through the region center q_0 and perpendicular to the line l, then project every vector $\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}}$, $i \in G_{out}$ onto the line l'. For both groups G_{out1} and G_{out2}, we have that, $\sum_{i \in G_{out1}} proj_{l'}\left[\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}}\right] \neq 0$, $\sum_{i \in G_{out2}} proj_{l'}\left[\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}}\right] \neq 0$. To make satisfy the group condition C_2 in (15), the above two vectors must

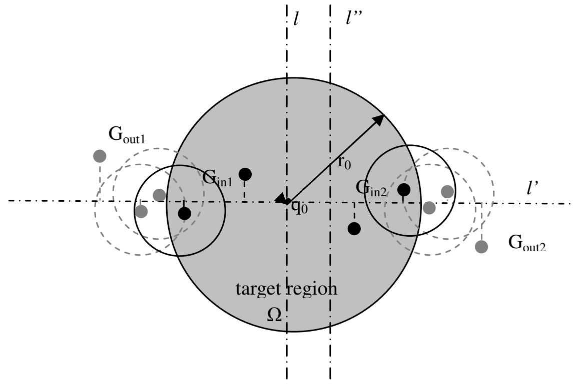

Fig. 7. Illustration of Case II.B: M $(2 \leq M \leq N-1)$ agents are located outside the target region, and all these M agents are separated into two disconnected groups, G_{out1} and G_{out2} respectively, by one line l passing through the center of the target region.

be symmetric about the target region center q_0, i.e.,

$$\begin{aligned}&\sum_{i\in G_{out1}} proj_{l'}\Big[\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}}\Big]\\ =& -\sum_{i\in G_{out2}} proj_{l'}\Big[\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}}\Big]\neq 0 \end{aligned} \quad (17)$$

In addition, every agent i in G_{out1} also needs satisfy the individual condition C_1 in (14), therefore,

$$\begin{aligned}&\sum_{i\in G_{out1}}\left[\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}}+2\sum_{j\in\mathcal{G}_i}\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}}\right]\\ =&\sum_{i\in G_{out1}}\left[\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}}+2\sum_{j\in\mathcal{G}_i\cap G_{out1}}\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}}+\right.\\ &\left. 2\sum_{j\in\mathcal{G}_i\cap G_{out2}}\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}}+2\sum_{j\in\mathcal{G}_i\cap G_{in}}\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}}\right]\\ =&\ 0\end{aligned} \quad (18)$$

Since these two groups G_{out1} and G_{out2} are disconnected, we have

$$\sum_{i\in G_{out1}}\sum_{j\in\mathcal{G}_i\cap G_{out2}}\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}}=0 \quad (19)$$

It is easy to obtain that

$$\sum_{i\in G_{out1}}\sum_{j\in\mathcal{G}_i\cap G_{out1}}\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}}=0 \quad (20)$$

Noting (17), substituting (19)(20) into (18), and projecting onto the line l' and result in

$$\begin{aligned}&2\sum_{i\in G_{out1}}\sum_{j\in\mathcal{G}_i\cap G_{in}} proj_{l'}\Big[\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}}\Big]\\ =& -\sum_{i\in G_{out1}} proj_{l'}\Big[\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}}\Big]\neq 0\end{aligned} \quad (21)$$

which means that in the group G_{in} inside the target region, there are some agents which are connected with some agents in the group G_{out1} outside the target region. Similarly, we also can prove that there are some connected agents in the groups G_{in} and G_{out2}. Therefore, among the $N-M$ agents in the group G_{in} inside the target region, there are at least two agents whose danger regions are intersected with the target region. According to Assumption 2, we can always find a line l'', to separate the $N-M$ agents in the group G_{in} inside the target region into two disconnected groups, labeled as G_{in1} and G_{in2}, which are located on both sides of the line l'', and connected with the groups G_{out1} and G_{out2} respectively. Then, we perform analysis for the group G_{in1} and arrive at the following equation:

$$\begin{aligned}&\sum_{i\in G_{in1}}\left[\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}}+2\sum_{j\in\mathcal{G}_i}\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}}\right]\\ =&\sum_{i\in G_{in1}}\Big[\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}}+2\sum_{j\in\mathcal{G}_i\cap G_{in1}}\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}}\\ &+2\sum_{j\in\mathcal{G}_i\cap G_{in2}}\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}}\\ &+2\sum_{j\in\mathcal{G}_i\cap G_{out1}}\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}}\Big]\end{aligned} \quad (22)$$

It is easy to obtain that

$$\sum_{i\in G_{in1}}\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}} = 0 \quad (23)$$

$$\sum_{i\in G_{in1}}\sum_{j\in\mathcal{G}_i\cap G_{in2}}\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}} = 0 \quad (24)$$

$$\sum_{i\in G_{in1}}\sum_{j\in\mathcal{G}_i\cap G_{in1}}\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}} = 0 \quad (25)$$

From Property (iv) in Definition 4, we have

$$\begin{aligned}&\sum_{i\in G_{in1}}\sum_{j\in\mathcal{G}_i\cap G_{out1}}\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}}\\ =& -\sum_{i\in G_{out1}}\sum_{j\in\mathcal{G}_i\cap G_{in1}}\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}}\end{aligned} \quad (26)$$

Substituting (23)-(26) into (22) results in

$$\begin{aligned}&\sum_{i\in G_{in1}}\left[\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}}+2\sum_{j\in\mathcal{G}_i}\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}}\right]\\ =&\ 2\sum_{i\in G_{in1}}\sum_{j\in\mathcal{G}_i\cap G_{out1}}\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}}\end{aligned} \quad (27)$$

Noting (21), substituting (26) into (27) and projecting onto the line l', we can derive the following equation

$$\begin{aligned}&\sum_{i\in G_{in1}} proj_{l'}\left[\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}}+2\sum_{j\in\mathcal{G}_i}\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}}\right]\\ =& -2\sum_{i\in G_{out1}}\sum_{j\in\mathcal{G}_i\cap G_{in1}} proj_{l'}\left[\frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}}\right]\\ \neq&\ 0\end{aligned} \quad (28)$$

However, according to the individual condition C_1 in (14), we know that

$$\sum_{i \in G_{in1}} proj_{l'} \left[\frac{\partial P_{i0}(\tilde{q}_{i0})}{\partial q_{i,1}} + 2 \sum_{j \in \mathcal{G}_i} \frac{\partial P_{ij}(\tilde{q}_{ij})}{\partial q_{i,1}} \right] = 0 \tag{29}$$

Therefore, (28) contradicts with (29), which means Case II. B does not hold either.

Case III ($M = N$)**:** Following the procedures of Case II, the case does not hold either.

From the above three cases, we can conclude that the assumption, of there being M ($1 \leq M \leq N$) agents located outside of the target region, and $N - M$ agents located inside the target region, cannot hold. Therefore, all the agents should be located in the target region. This completes the proof. ■

IV. Simulation Studies

Consider a group of $N = 4$ identical mobile agents on a $\mathbb{R}^2$ space, i.e. x-y space. All the agents have the same danger region radius $r_i = 0.5$m, communication range radius $R_i = 1.0$m and satisfy the following dynamics:

$$\begin{aligned} \dot{q}_{i,1} &= q_{i,2} \\ \dot{q}_{i,2} &= [q_{i,1} \quad q_{i,2}] \begin{bmatrix} \theta_{i1} \\ \theta_{i2} \end{bmatrix} + u_i \end{aligned} \tag{30}$$

where $\theta_{i1} = 0.1$, $\theta_{i2} = 0.5$, $i \in \{1, 2, 3, 4\}$. The common target region Ω is specified as a circle which is centered at the point q_0 with a radius of $r_0 = 2.5$m and moves along the desired trajectory $q_0 = [t \quad \sin(t)]^T$. The agents are initialized randomly outside the target region with $q_0 = [0.0, 0.0]^T$. Simulation result is shown in Fig. 8. It can be seen that all agents converge into the target region and move together with it. No collisions occur among the agents as well.

V. Conclusion

Due to the importance of considering actuator dynamics into the vehicle dynamics for achieving better performance, decentralized cooperative control have been proposed, in this paper, for multi-agent swarm systems with high-order nonlinear dynamics in strict feedback form by incorporating artificial potentials, adaptive backstepping into Lyapunov synthesis. We have shown that all the agents converge to a common moving target region, without collisions between any agents in the group.

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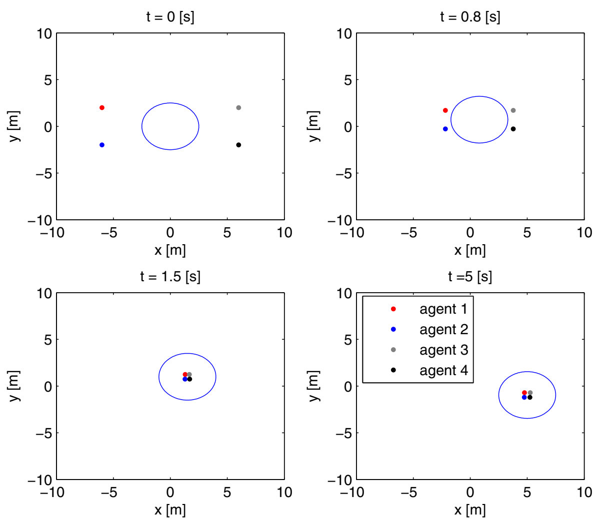

Fig. 8. All agents converging into and moving with the target region