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Synchronized Altitude Tracking Control of Multiple Unmanned Helicopters

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***Abstract*— In this paper, we consider the synchronized altitude tracking control for multiple helicopters, when the desired trajectory is only available to the leaders. By using the weighted average of the neighbors' states as the reference signal, the adaptive neural network (NN) tracking control is designed for each helicopter. It is shown that, the output tracking error of each helicopter converges to an adjustable neighborhood of origin under the proposed NN control, although some of them do not access the desired tracking trajectory directly. Simulation results are provided to demonstrate the effectiveness of the approach presented.**

I. Introduction

Cooperative control of multi-agent systems has drawn significant research interest in recent years [1]–[3]. Multiple helicopters cooperation can accomplish complex tasks such as surveillance, large area search and rescue. One fundamental problem in multi-helicopter cooperation is formation control, in which the helicopters keep a desired formation configuration and at the same time complete the assigned tasks.

Various approaches have been proposed for formation control, including behavioral [4], virtual structure [5], [6], queues and artificial potential trenches [2], [3], and leader-following [7]. In the control of helicopters, one of the main challenges is how to deal with the unknown perturbations to the nominal model, in the presence of parametric and functional uncertainties, unmodeled dynamics, and disturbances from the environment [8]. Much work has been done on this problem: Direct neural dynamic programming for helicopter trimming and tracking was proposed [9]. Adaptive neural network control of helicopters in vertical flight was presented in [8]. Approximation-based control was proposed for uncertain helicopter dynamics in [10], in which both altitude and yaw angle tracking were considered. Robust tracking control was proposed for a single helicopter in [11]. For the formation of helicopters, a non-iterative nonlinear model predictive control was applied to each leader-follower pair [12].

In this paper, we study the synchronized tracking problem of helicopters in vertical flight, in which multiple helicopters track the same desired trajectory, while the desired trajectory is not accessible to all the helicopters in the team. Since the coupling between longitudinal and lateral-directional equations in this flight regime is week, it can be represented by SISO models with zero-dynamics [8], [13]. In the formation group, the desired trajectory is not available to all the helicopters in the team. Then the synchronized tracking control is designed for each helicopter by using the information exchange with its neighbors. The main contributions of this work include: (i) The extended formation graph Laplacian, which contains a spanning tree, with the desired trajectory only available to the root helicopter, is proved to be positive definite, and (ii) The neural approximation based control is designed for each helicopter by using the weighted average of its neighbors' states for the purpose of synchronized tracking.

The remainder of the paper is organized as follows: In Section II, the problem formulation for synchronized tracking of helicopters is presented. In Section III, synchronized tracking is designed by using the neighbors states. Simulation results are presented in Section IV. The paper is concluded in Section V.

II. Problem Formulation

A. Helicopter Dynamics

Consider the following SISO helicopter systems:

$$\begin{aligned} \dot{x}_j &= x_{j+1}, \quad j = 1, \ldots, \rho - 1 \\ \dot{x}_\rho &= f(\eta, x) + g(x, \eta)(u + d) \\ \dot{\eta} &= q(x, \eta) \\ y &= x_1 \end{aligned} \tag{1}$$

where $x = [x_1, \ldots, x_\rho]^T \in \mathbb{R}^\rho$ and $\eta \in \mathbb{R}^{n-\rho}$ are the states; $u,\ y \in \mathbb{R}$ the input and output, respectively; $f: \ \mathbb{R}^n \to \mathbb{R}$ an unknown smooth function; and $q: \ \mathbb{R}^n \to \mathbb{R}$ a partially unknown vector field satisfying certain properties; $g: \ \mathbb{R}^n \to \mathbb{R}$ is an unknown function with certain properties, and d is the external disturbance in the input channel.

Assumption 1: The zero dynamics of system (1), given by $\dot{\eta} = q(x, \eta)$, are exponentially stable. In addition, $q(x, \eta)$ is Lipschitz in x, i.e., there exists positive constants a_q and a_x such that

$$\|q(x, \eta) - q(0, \eta)\| \leq a_x\|x\| + a_q \quad \forall (x, \eta) \in \mathbb{R}^n \tag{2}$$

Under the assumption that the zero dynamics are stable, by the converse Lyapunov theorem, there exists a Lyapunov function $V_0(\eta)$ which satisfies the following inequalities for $(x, \eta) \in \mathbb{R}^n$: (i) $\gamma_1\|\eta\|^2 \leq V_0(\eta) \leq \gamma_2\|\eta\|$, (ii) $\frac{\partial V_0}{\partial \eta} q(0, \eta) \leq -\lambda_a\|\eta\|^2$, and (iii) $\left\|\frac{\partial V_0}{\partial \eta}\right\| \leq \lambda_b\|\eta\|$, where γ_1 , γ_2, λ_a, and λ_b are positive constants.

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Assumption 2: The external disturbance d is uncertain bounded functions $d \in L_\infty$. That is, there exists unknown positive constants ϱ such that $|d(t)| \le \varrho < \infty$, where ϱ can be arbitrarily large.

Assumption 3: There exist smooth functions $\bar{g}(x, \eta)$ and a positive constant $\underline{g} > 0$, such that $\bar{g}(x, \eta) \ge g(x, \eta, u) > \underline{g} > 0$, $\forall (x, \eta) \in \mathbb{R}^n$. Without loss of generality, it is further assumed that the sign of $g(x, \eta, u)$ is positive $\forall (x, \eta) \in \mathbb{R}^n$.

Assumption 4: There exists a positive function $g_0(x, \eta)$ satisfying $|\dot{g}(x, \eta)/2g(x, \eta)| \le g_0(x, \eta)$, $\forall (x, \eta) \in \mathbb{R}^n$.

Remark 1: The general nonlinear SISO helicopter model described as $\dot{x} = f(x, u),\ y = h(x)$ with some assumptions such as it can be input-output linearizable with strong relative degree $\rho < n$, can be described as (1) [8].

B. Formation of Helicopters

We associate the helicopters with nodes in a graph and information exchange with its edges. The following definitions are useful for describing the formation.

Definition 1: [1] A directed graph $\mathcal{G}'$ consists of a non-empty finite set $\mathcal{V}'$ of elements called nodes and a finite set $\mathcal{E}' \subset \mathcal{V}'^2$ of ordered pairs of nodes called arcs, where $e = (v_i, v_j) \in \mathcal{E}'$ and $v_i, v_j \in \mathcal{V}'$. The neighbors set of vertical v_i is defined as $\mathcal{N}_i' = \{v_j \in \mathcal{V}' |\ (v_j, v_i) \in \mathcal{E}'\}$.

For the multiple agents tracking problem, we introduce a virtual agent v_0, whose motion follows the desired trajectory restrictively, and define a non-empty set $\mathcal{V}_0 \subset \mathcal{V}'$, in which the elements can access the desired trajectory, i.e., $v_0 \in \mathcal{N}_j$, iff $v_j \in \mathcal{V}_0$. Then the extended formation graph can be described as $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \mathcal{V}' \cup \{v_0\}$, and $\mathcal{E} = \mathcal{E}' \cup \{(v_0, v_j) | v_0 \in \mathcal{N}_j\}$. For all agents $v_j \in \mathcal{V}_0$, $\mathcal{N}_j = \mathcal{N}_j' \cup \{v_0\}$.

Definition 2: The weighted adjacency matrix of the extended formation graph $\mathcal{G}$, denote $A^*(\mathcal{G})$, is a square matrix of size $|\mathcal{V}|$, with its elements $A^*_{ij} > 0$ if $(v_j, v_i) \in \mathcal{G}$, and is zero otherwise. Define a diagonal matrix $\Delta(\mathcal{G})$ with its elements $\Delta_{jj} = \sum_k A^*_{jk}$, and the normalized Laplacian of the graph is defined as $L = I - A$ with $A = \Delta^{-1} A^*$.

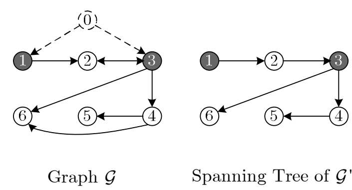
Property 1: [14] For a diagonally dominant matrix with nonzero elements chain $L = (\ell_{ij})$, we have following properties: (i) L is a nonsingular matrix; (ii) If $B = I - D^{-1}L$, where $D = \mathrm{diag}\{\ell_{11}, \ldots, \ell_{nn}\}$, $\ell_{ii} \ne 0$, then $\rho(B) < 1$, where $\rho(B)$ is the spectrum of B; and (iii) If L is real and $\ell_{ij} \le 0$, $\ell_{ii} > 0$, then L is an M-matrix.

Theorem 1: Consider the multiple agent synchronized tracking problem. If the formation graph $\mathcal{G}'$ contains a spanning tree with its root $v_j \in \mathcal{V}_0$, then the normalized adjacent matrix A of the extended formation graph $\mathcal{G}$ is sub-stochastic, and $L = I - A$ is positive definite.

Proof: We know that all the elements of the first row of A are zero. Since $\mathcal{G}'$ has a spanning tree and $v_j \in \{\mathcal{V}_0\}$ is the root, it means that each agent has at least one neighbor, then the sum of any other row of A equals to 1, i.e., A is a sub-stochastic matrix.

It is clear that all the diagonal elements of L are 1, and all the row sum of A is 1 except the first row, that means L is a diagonal dominant matrix with the $J = \{0\}$ (Since the virtual agent is added, we start the row number from 0 corresponding to the label of agents). Let us revisit that $\mathcal{G}'$ has a spanning tree with $v_j \in \mathcal{V}_0$ as the root, it also means that there is a path from v_0 to any agent $v_i \in \mathcal{V}$, then in the matrix L, for every element $i \ne 0$, there exists a sequence of nonzero elements form $\ell_{ii_1}, \ell_{i_1 i_2}, \ldots, \ell_{i_s 0}$. Then L is a diagonally dominant matrix with nonzero elements chain. Since L is real and $\ell_{ij} < 0, i \ne j$, $\ell_{ii} = 1$, According to Property 1, L is a nonsingular M-matrix [14], i.e., L is positive definite. This completes the proof. ∎

Example 1: To explain Theorem 1 clearly, take the sample graph shown in Fig. 1 as an example. Both v_1 and v_3 can access the desired trajectory and $\mathcal{G}'$ contains a spanning tree with node 1 as its root. Take node 5 as an example, we can find that in the laplacian matrix L, there exists a sequence ℓ_{54}, ℓ_{43}, ℓ_{32}, ℓ_{21}, $\ell_{10} \ne 0$.



$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2/3 & 1 & -1/3 & 0 & 0 & 0 \\ -3/4 & 0 & -1/4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1/2 & -1/2 & 0 & 1 \end{bmatrix} \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

Matrix L

Fig. 1. Sample graph and its Laplacian.

In this paper, we studied the synchronized tracking problem of multiple unmanned helicopters as follows:

Considering a group of helicopters, the desired trajectory of the team, $y_d(t)$, and its derivations up to ρ-th order is bounded, and is only available to the helicopters $v_j \in \mathcal{V}_0$. For each helicopter, we design a control by using its own states and its neighbors' states, such that

$$\lim_{t \to \infty} |y_i(t) - y_d(t)| = \bar{\varepsilon}, \quad i = 1, \ldots, N \qquad (3)$$

where $\bar{\varepsilon}$ is a small positive constant.

The desired trajectory $y_d(t)$ is generated by the following reference model:

$$\begin{aligned} \dot{x}_{dj} &= \dot{x}_{di+1}, \quad i = 1, \ldots, \rho - 1 \\ \dot{x}_{d\rho} &= f_d(x_d, t) \end{aligned}$$

where $\rho \ge 2$, $x_d = [x_{d1}, \ldots, x_{d\rho}]^T \in \mathbb{R}^\rho$ are the states of reference system, $y_d = x_{d1} \in \mathbb{R}$ is the system output.

Assumption 5: The reference trajectory $y_d(t)$ and its ρ-th derivatives remain bounded, i.e., $x_d \in \Omega_d \subset \mathbb{R}^\rho$, $\forall t \ge 0$.

Assumption 6: The formation graph $\mathcal{G}'$ has a spanning tree with the root helicopter can access the desired trajectory.

The following lemma is useful for analysis the internal dynamics of the helicopter.

Lemma 1: [15] Denote positive constants $a_1 = (\lambda_b a_x)/\lambda_a$ and $a_2 = (\lambda_b a_q)/\lambda_a$. If Assumptions 1 and 5 satisfied, there exists a positive constant T_0 such that the trajectories $\eta(t)$ of the internal dynamics satisfy $\|\eta\| \le a_1 \|x(t)\| + a_2$.

III. Synchronized Tracking Control Design

In this section, we design the tracking control for each helicopter using the information of itself and its neighbors. Since not all the helicopters can access the information of the desired trajectory, the tracking control is designed based on the relative states with its neighbors. Define the following error variables for the helicopters:

$$z_{i,1} = y_{i,1} - y_{ir}\,, \cdots, \quad z_{i,\rho} := z_{i,1}^{(\rho)} = x_{i,\rho} - y_{ir}^{(\rho)} \tag{4}$$

with

$$y_{ir}(t) = \sum_{j\in\mathcal{N}_i} a_{ij} y_j(t),\ y_{ir}^{(k)}(t) = \sum_{j\in\mathcal{N}_i} a_{ij} y_j^{(k)}(t) \tag{5}$$

where $k = 1, \ldots, \rho - 1$, a_{ij} is the element of the normalized adjacent matrix A of the extended formation graph $\mathcal{G}$.

For each helicopter, we define vectors $\bar{z}_i = [z_{i,1}, \ldots, z_{i,\rho}]^T \in \mathbb{R}^\rho$, $\mathcal{Z}_i = [z_{i,1}, \ldots, z_{i,\rho-1}]^T \in \mathbb{R}^{\rho-1}$, and the filtered tracking error as

$$s_i = [\Lambda^T \quad 1]\bar{z}_i \tag{6}$$

where $\Lambda = [\lambda_1, \lambda_2, \ldots, \lambda_{\rho-1}]^T$ is an appropriately chosen coefficient vector so that $z_{i,\rho} \to 0$ as $s_i \to 0$, i.e., $p^{\rho-1} + \lambda_{\rho-1} p^{\rho-2} + \ldots + \lambda_1$ is Hurwitz. Then we have

$$\dot{\mathcal{Z}}_i = A_p \mathcal{Z}_i + b s_i \tag{7}$$

where $A_p = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \\ -\lambda_1 & -\lambda_2 & \cdots & -\lambda_{\rho-1} \end{bmatrix}$, and $b = [\underbrace{0, \ldots, 0}_{\rho-2}, 1]^T$.

The dynamics of s_i is written as

$$\dot{s}_i = f_i(x_i, \eta_i) + g_i(u_i + d_i) + [0 \quad \Lambda^T]\bar{z}_i - y_{ir}^{(\rho)} \tag{8}$$

Considering $V_{si} = \frac{1}{2g_i} s_i^2$, we have

$$\begin{aligned} \dot{V}_{si} &= -\frac{\dot{g}_i s_i^2}{2g_i^2} + \frac{s_i \dot{s}_i}{g_i} = -\left[g_0 + \frac{\dot{g}_i}{2g_i^2}\right] s_i^2 + s_i(u_i + d_i) \\ &\quad + s_i\big\{f_i(x_i, \eta_i) + [0 \quad \Lambda^T]\bar{z}_i - y_{ir}^{(\rho)} + g_i g_0 s_i\big\}/g_i \end{aligned} \tag{9}$$

Due to the existence of the uncertain items, we use the parameter linearized NN [16] to approximate the unknown nonlinear function $\bar{f}_i(x_i, \eta_i, \bar{z}_i, y_{ir}^{(\rho)}) = \big\{f_i(x_i, \eta_i) + [0 \quad \Lambda^T]\bar{z}_i - y_{ir}^{(\rho)} + g_i g_0 s_i\big\}/g_i$, which can be described as

$$\bar{f}_i(Z_i) = \theta_i^{*T} \varphi_i(Z_i) + \bar{\varepsilon}_i(Z_i) \tag{10}$$

where $\bar{\varepsilon}_i(Z_i)$ is the approximation error which is bounded over the compact set, i.e., $|\varepsilon_i(Z_i)| \le \varepsilon_i^*$, $\forall Z_i \in \Omega_{xi}$ with $\varepsilon_i^* > 0$ is an unknown constant, θ^* is the ideal weighted vector, and $Z_i = [x_i, \eta_i, \bar{z}_i, y_{ir}^{(\rho)}]^T$.

Considering the Lyapunov function candidate

$$V_i = V_{si} + 0.5\tilde{\theta}_i^T \tilde{\theta}_i/\gamma_2 + 0.5\tilde{\varphi}_i^2/\gamma_1 \tag{11}$$

where γ_1 and γ_2 are the positive constants, $\tilde{\theta}_i = \hat{\theta}_i - \theta_i^*$, and $\tilde{\varphi}_i = \hat{\varphi} - \varphi_i^*$, are the estimated errors of parameters and the error bound, where $\hat{\theta}_i$ and $\hat{\varphi}_i$ are the estimation of θ_i^* and $\varphi_i^* = (\varrho_i + \bar{\varepsilon}_i)^2$, respectively.

Then we have

$$\begin{aligned} \dot{V}_i &= -\left(g_0 + \frac{\dot{g}_i}{2g_i^2}\right) s_i^2 + s_i(u_i + d_i) + \frac{1}{\gamma_2}\tilde{\theta}_i \dot{\hat{\theta}}_i \\ &\quad + s_i[\theta_i^{*T}\psi_i(Z_i) + \bar{\varepsilon}_i] + \tilde{\varphi}_i \dot{\hat{\varphi}}_i/\gamma_1 \end{aligned} \tag{12}$$

Remark 2: The NN is constructed to approximate $\bar{f}_i(x_i, \eta_i, \bar{z}_i, y_{ir}^{(\rho)}) = \frac{f_i(x_i,\eta_i) + [0 \quad \Lambda^T]\bar{z}_i - y_{ir}^{(\rho)} + g_i g_0 s_i}{g_i}$ on a whole, which avoids the possible singularity of the direct approximation of g_i.

Select the following control u_i for each helicopter

$$u_i = -\hat{\theta}_i^T \psi_i - k_i s_i - \hat{\varphi}_i s_i/2, \quad i = 1, \ldots, N \tag{13}$$

The update law of parameters are designed as

$$\dot{\hat{\varphi}}_i = \gamma_1\left(s_i^2/2 - \sigma_1 \hat{\varphi}_i\right), \ \dot{\hat{\theta}}_i = \gamma_2\big(\psi_i s_i - \sigma_2 \hat{\theta}_i\big) \tag{14}$$

where σ_1, and σ_2 are the designed positive constants.

By using the Young's inequality, we have $-\sigma_2 \tilde{\theta}_i^T \hat{\theta}_i \le -\sigma_2\|\tilde{\theta}_i\|^2/2 + \sigma_2\|\theta_i^*\|^2/2$, $-\sigma_1 \tilde{\varphi}_i \hat{\varphi}_i \le -\sigma_1 \tilde{\varphi}_i^2/2 + \sigma_1 \varphi_i^{*2}/2$, and $(\varrho_i + \bar{\varepsilon}_i)s_i \le 1/2 + s_i^2 \varphi_i^*/2$. Considering (13) and (14), the time derivation of V_i in the closed-loop trajectory can be written as

$$\begin{aligned} \dot{V}_i &= -(g_0 + \frac{\dot{g}_i}{2g_i^2})s_i^2 - k_i s_i^2 + s_i(\bar{\varepsilon}_i + d_i - \frac{1}{2}\hat{\varphi}_i) \\ &\quad - \sigma_2 \tilde{\theta}_i^T \hat{\theta}_i + s_i \tilde{\varphi}_i s_i/2 - \sigma_1 \tilde{\varphi}_i \hat{\varphi}_i \\ &\le -k_i s_i^2 - \sigma_1 \tilde{\varphi}_i^2/2 - \sigma_2\|\tilde{\theta}_i\|^2/2 + c_{2i} \end{aligned} \tag{15}$$

where $c_{2i} = \sigma_2\|\theta_i^*\|^2/2 + \sigma_1 \varphi_i^{*2}/2 + 1/2$. Now define

$$\Omega_{si} = \left\{ s_i \,\middle|\, |s_i| \le \sqrt{c_{2i}/k_i} \right\}$$

$$\Omega_{\theta_i} = \left\{ (\tilde{\theta}_i, \tilde{\varphi}_i) \,\middle|\, \|\tilde{\theta}_i\| \le \sqrt{2c_{2i}/\sigma_2}, |\tilde{\varphi}_i| \le \sqrt{2c_{2i}/\sigma_1} \right\}$$

$$\Omega_{ei} = \left\{ (s_i, \tilde{\theta}_i, \tilde{\varphi}_i) \,\middle|\, k_i s_i^2 + \frac{1}{2}\sigma_2 \tilde{\theta}_i^T \tilde{\theta}_i + \frac{1}{2}\sigma_1 \tilde{\varphi}_i^2 \le c_{2i} \right\}$$

Since c_{1i}, σ_1, σ_2, and k_i are positive constants, we know that Ω_{si}, Ω_{θ_i} and Ω_{ei} are compact sets. Eq. (15) shows that $\dot{V}_i \le 0$ once the errors are outside the compact set Ω_{ei}. According to the standard Lyapunov theorem, we conclude that s_i, $\tilde{\theta}_i$, and $\tilde{\varphi}_i$ are bounded. From (15) and (III), it can be seen that V_i is strictly negative as long as s_i is outside the compact set Ω_{si}. Therefore, there exsits a constant T_1 such that for $t > T_1$, the filtered tracking error s_i converges to Ω_{si}, that is to say, $s_i \le \beta_{si}(k_i, \gamma_1, \gamma_2, \sigma_1, \sigma_2, \theta_i^*, \varphi_i^*, \varepsilon_i^*) = \sqrt{c_{2i}/k_i}$.

Now, we will show that all the helicopters will track the desired trajectory although only some of them can access the desired trajectory. Define the error between i-th helicopter and the desired trajectory as $\tilde{y}_i(t) = y_i(t) - y_d(t) = y_i(t) - y_0(t)$, and the auxiliary states of each helicopter $\xi_i(t) = [\Lambda^T \quad 1]Y_i$ with $Y_i = [y_i, y_i^{(1)}, \ldots, y_i^{(\rho-1)}]^T$. The filtered error is denoted as $\tilde{\xi}_i(t) = \xi_i(t) - \xi_d(t) = \xi_i(t) - \xi_0(t)$. Using the fact that $s_i(t) = \xi_i(t) - \sum_{j\in\mathcal{N}_i} a_{ij}\xi_j(t)$, we have

$$\tilde{\xi}_i = \xi_i - \xi_0 = \sum_{j\in\mathcal{N}_i} a_{ij}\xi_j + s_i - \xi_0, \quad i = 1, \ldots, N \tag{16}$$

and in the vector form

$$\tilde{\xi} = A\xi + s - \xi_0 \mathbf{1} \tag{17}$$

where $\mathbf{1} = [1, \ldots, 1]^T$, $s = [s_0, s_1, \ldots, s_N]^T$, and A is the normalized adjacency matrix of the extended formation graph. Note $[0, 1, \ldots, 1]^T = A[0, 1, \ldots, 1]^T$, we have

$$\begin{aligned}\tilde{\xi} &= A(\tilde{\xi} + \xi_0 \mathbf{1}) + s + [1, 0 \ldots, 0]^T \xi_0 - \xi_0 \mathbf{1} \\ &= A\tilde{\xi} + [0, 1, \ldots, 1]^T \xi_0 + s + [1, 0 \ldots, 0]^T \xi_0 - \xi_0 \mathbf{1} \\ &= A\tilde{\xi} + s\end{aligned} \tag{18}$$

Under the Assumption 6, we know that that $L = (I - A)$ is an invertible matrix, we have

$$\tilde{\xi} = L^{-1} s \tag{19}$$

Define $\mathcal{Y} = [Y_0^T, \ldots, Y_N^T]^T$, $\tilde{\mathcal{Y}} = [\tilde{Y}_0^T, \ldots, \tilde{Y}_N^T]^T$, $X = [X_0^T, \ldots, X_{\rho-1}^T]^T$, $\tilde{X} = [\tilde{X}_0, \ldots, \tilde{X}_{\rho-1}]^T$, where $X_j = [X_{0,j}, X_{1,j}, \ldots, X_{N,j}]^T$, $\tilde{X}_j = X_j - X_{jd} = X_j - y_0^{(j)} \mathbf{1}$, $\tilde{Y}_i = Y_i - Y_d = Y_i - Y_0$. Then we have

$$\dot{\tilde{\mathcal{Y}}} = \bar{A}_p \tilde{\mathcal{Y}} + \bar{b}\tilde{\xi} \tag{20}$$

where $\bar{A}_p = I_{N+1} \otimes A_p$ and $\bar{b} = I_{N+1} \otimes b$. Symbol "$\otimes$" stands for the Kronecker Product of the matrices.

Considering (18), the error dynamics can be written as

$$\dot{\tilde{\mathcal{Y}}} = \bar{A}_p \tilde{\mathcal{Y}} + \bar{b}\tilde{\xi} = \bar{A}_p \tilde{\mathcal{Y}} + \bar{b} L^{-1} s \tag{21}$$

Lemma 2: For some time constant T_1, define $s_{i,\max} = \sup_{0 \le \tau \le t} |s_i(t)|$, $\beta_{s_i} = \sup_{t > T_1 |s_i(t)|}$, and $s_{\max,i}(t) = \max_i \sup_{0 \le \tau \le t} |s_i(t)|$, then the following equations hold:

$$\|\tilde{\mathcal{Y}}(t)\| \le k_0 e^{-\lambda_0 t} \|\tilde{\mathcal{Y}}(0)\| + \frac{k_0}{\lambda_0} \left[N \lambda_{\max}(L^{-1}) \right] s_{\max,i}(t)$$

$$\|\tilde{\mathcal{Y}}(t)\| \le k_0 e^{-\lambda_0 t} \left(\|\tilde{\mathcal{Y}}(0)\| + \frac{e^{\lambda_0 T_1}}{\lambda_0} \beta_s(T_1) \right) + \frac{k_0}{\lambda_0} \beta_{s_T}$$

where $\beta_s(t) = N\lambda_{\max}(L^{-1}) s_{\max,i}(t)$ and $\beta_{s_T} = N\lambda_{\max}(L^{-1}) \sup_{T_1 \le t} s_{\max,i}(t)$ with constants $\lambda_0 > 0$ and $k_0 > 0$.

Proof: From (21) and the fact that A_p is Hurwiz, we have $\tilde{\mathcal{Y}}(t) = \tilde{\mathcal{Y}}(0) e^{\bar{A}_p t} + \int_0^t e^{\bar{A}_p (t-\tau)} \bar{b} L^{-1} s \, d\tau$, and $\|e^{\bar{A}_p t}\| \le k_0 e^{-\lambda_0 t}$. Then

$$\begin{aligned}&\|\tilde{\mathcal{Y}}(t)\| \le k_0 e^{-\lambda_0 t} \|\tilde{\mathcal{Y}}(0)\| + \int_0^t e^{-\lambda_0 (t-\tau)} \left\| \bar{b} L^{-1} s \right\| \, d\tau \\ &\le k_0 e^{-\lambda_0 t} \|\tilde{\mathcal{Y}}(0)\| + k_0 e^{-\lambda_0 t} \left[N \lambda_{\max}(L^{-1}) s_{\max,i}(t) \right] \int_0^t e^{\lambda_0 \tau} \, d\tau \\ &\le k_0 e^{-\lambda_0 t} \|\tilde{\mathcal{Y}}(0)\| + k_0 e^{-\lambda_0 t} \left[N \lambda_{\max}(L^{-1}) \right] s_{\max,i}(t) \frac{e^{\lambda_0 t} - 1}{\lambda_0} \\ &\le k_0 e^{-\lambda_0 t} \|\tilde{\mathcal{Y}}(0)\| + \frac{k_0}{\lambda_0} \left[N \lambda_{\max}(L^{-1}) \right] s_{\max,i}(t)\end{aligned} \tag{22}$$

where $\lambda_{\max}(\cdot)$ is the maximum eigenvalue of the matrix. Noting the above equation and the fact that $\int_0^t e^{-\lambda_0 (t-\tau)} \|\bar{b}(L^{-1} s)\| d\tau = \int_0^{T_1} e^{-\lambda_0 (t-\tau)} \|\bar{b}(L^{-1} s)\| d\tau + \int_{T_1}^t e^{-\lambda_0 (t-\tau)} \|\bar{b}(L^{-1} s)\| d\tau$. We have (22) as the following:

$$\begin{aligned}\|\tilde{\mathcal{Y}}(t)\| &\le k_0 e^{-\lambda_0 t} \|\tilde{\mathcal{Y}}(0)\| + k_0 e^{-\lambda_0 t} \frac{e^{\lambda_0 T_1} - 1}{\lambda_0} \beta_s(T_1) \\ &\quad + k_0 e^{-\lambda_0 t} \frac{e^{\lambda_0 t_0} - e^{\lambda_0 T_1}}{\lambda_0} \beta_{s_T} \\ &\le k_0 e^{-\lambda_0 t} \left[\|\tilde{\mathcal{Y}}(0)\| + \frac{e^{\lambda_0 T_1}}{\lambda_0} \beta_s(T_1) \right] + \frac{k_0}{\lambda_0} \beta_{s_T}\end{aligned} \tag{23}$$

This completes the proof. ∎

Now we will show that for a proper choice of the control parameters, the trajectories of each vehicle do remain in the compact set. From the fact that $L^{-1}s = ([\Lambda^T \ \ 1] \otimes I_{N+1}) \tilde{\bar{X}}$, where $\tilde{\bar{X}} = [\tilde{X}^T \ \ \tilde{x}_\rho^T]^T$, we can see that $\tilde{x}_\rho = L^{-1}s - (\Lambda^T \otimes I_N)\tilde{X}$. Therefore, $\|\tilde{\bar{X}}\| \le \|\tilde{X}\| + \|\tilde{x}_\rho\| \le (1 + \|\Lambda\|)\|\tilde{X}\| + \|L^{-1}\| \|s\| \le (1 + \|\Lambda\|)\|\tilde{\mathcal{Y}}\| + \lambda_{\max}(L^{-1})\|s\|$. It follows from (23) and the fact that s_i will converge to Ω_{si}, we know that $\|\tilde{\bar{X}}\| \le k_a \|\tilde{\mathcal{Y}}(0)\| + k_b \beta_{s_T} + k_c$, $\forall t \ge T_1$, with $k_a = (1 + \|\Lambda\|)k_0$, $k_b = (k_a/\lambda_0) + 1$ and $k_c = k_a (e^{\lambda_0 T_1}/\lambda_0)\beta_s(T_1)$, Hence $\forall t \ge T_1$

$$\|\bar{X}(t)\| \le \|\tilde{\bar{X}}(t)\| + \|\bar{X}_d(t)\mathbf{1}\| \le k_a \|\tilde{\mathcal{Y}}(0)\| + k_b \beta_{s_T} + k_c + c \tag{24}$$

We now provide the conditions which guarantee $\bar{X} \in \Omega_{\bar{X}}$, $\forall t \ge 0$. Define the compact set

$$\begin{aligned}\Omega_0 \ := \ &\Big\{ \bar{X}(0) \Big| \{ \bar{X} | \|\bar{X}(t)\| < k_a \|\tilde{\mathcal{Y}}(0)\| \} \subset \Omega_{\bar{X}}, \\ &\lambda_{\max}(L^{-1})\|s(0)\| < \beta_{s_T} \Big\}\end{aligned} \tag{25}$$

the positive constant

$$\begin{aligned}c^* \ := \ \sup_{c \in \mathbb{R}^+} &\Big\{ c \Big| \Big\{ \bar{X} | \|\bar{X}\| < k_a \|\tilde{\mathcal{Y}}(0)\| + k_c + c, \\ &\bar{X}(0) \in \Omega_0 \Big\} \subset \Omega_{\bar{X}} \Big\}\end{aligned} \tag{26}$$

We summarize our results in the following theorem.

Theorem 2: Consider a group of helicopters dynamics (1) and the communication graph containing a spanning tree, where the root helicopter can access the desired trajectory, under Assumptions 1–5, the control (13) and parameters update law (14) for each helicopter. For initial conditions $\bar{X}(0), \eta(0)$, $\tilde{\theta}_i(0)$ and $\tilde{\varphi}_i(0)$ starting in any compact set, and the desired trajectory with its derivations up to ρ-th is bounded, all closed signals of the system are semiglobal uniform bound, and the total tracking error of the helicopters $\tilde{\bar{X}}$ converges to a neighborhood of the origin.

Proof: From (24), we know that the overall system state $\bar{X}(t)$ will stay in $\Omega_{\bar{X}}$ for all time. Furthermore, since the NN weights have been proven bounded for any bounded $\hat{\theta}_i(0)$ and $\hat{\varphi}_i(0)$, and from Lemma 1, it can be seen that η_i is bound if x_i is bounded. As a result, the states of the internal dynamics of the helicopter will converges to the compact set $\Omega_{\eta_i} = \{\eta_i \in \mathbb{R}^p | \, \|\eta_i\| \le a_1(\sqrt{2c_2/c_1} + \|X_d\|) + a_2\}$, where $a_1 = \lambda_b a_x/\lambda_a$ and $a_2 = \lambda_b a_q/\lambda_a$ are positive constants. Because the control signal $u_i(t)$ is a function of the weights $\hat{\theta}_i$ and $\hat{\varphi}_i$, the states η_i, x_i, and the filtered tracking error s_i, we know that it is also bounded. Therefore, we have all the closed-loop signals are semiglobal uniform bound. This completes the proof. ∎

IV. SIMULATION STUDY

In this section, we consider the synchronized altitude tracking of six X-cell 50 helicopters which communication graph shown in Fig. 1. The dynamics of the helicopter can be written as follows [17]

$$\begin{aligned}
\dot{\zeta}_1 &= \zeta_2 \\
\dot{\zeta}_2 &= a_0 + a_1\zeta_2 + a_2\zeta_2^2 + (a_3 + a_4\zeta_4 - \sqrt{a_5 + a_6\zeta_4})\zeta_3^2 \\
\dot{\zeta}_3 &= a_7 + a_8\zeta_3 + (a_9 \sin\zeta_4 + a_{10})\zeta_3^2 + a_{\text{th}} \\
\dot{\zeta}_4 &= \zeta_5 \\
\dot{\zeta}_5 &= a_{11} + a_{12}\zeta_4 + a_{13}\zeta_3^2 \sin\zeta_4 + a_{14}\zeta_5 - K_1 u
\end{aligned} \tag{27}$$

The physical meaning of the symbols and the nominal values for constants K_1 and a_i are given in [17].

Let y be the altitude ζ_1. By restricting the throttle input to be constant, we obtain a SISO in which u is the only input variable forcing the output y to track a desired trajectory y_d, which is generated by $y_d = \frac{150.056}{s^4+12.6s^3+64.19s^2+154.35s+150.056} h_{ref}$, where $h_{ref}(t) = 5.5 - 0.5\sin t$.

It can be shown that the system has strong relative degree 4, with the x system given by:

$$\begin{aligned}
\dot{x}_i &= x_{i+1},\ i = 1,2,3 \\
\dot{x}_4 &= b(x) + g(x)u
\end{aligned} \tag{28}$$

where $g(x) = -K_2\zeta_3^2 \left(a_4 - \frac{a_6}{2\sqrt{a_5 + a_6\zeta_4}}\right)$.

The derivation of $b(x)$ in (28) is omitted, and we proceed to verify that the system indeed satisfies the assumptions supported in the control design. It is clear that the system has a strong relative degree $\rho = 4$. To verify Assumption 3, we first note, from a practical standpoint, that the collective pitch angle, ζ_4, is stricter with a range, typically from 0 to 0.44 rad [13]. It can be verified that the bracketed terms in $g(x)$ are virtually constant: they take values in the range $[1.4, 1.5]\times 10^{-3}$. Hence the control coefficient $g(x)$ is always negative. Together with the fact that rotor speed, ζ_3, is nonzero during flight, it can be concluded that there does not exist any control singularities or zero crossings of $g(x)$. Therefore, the first part of Assumption 3 is satisfied.

Remark 3: The second part of the assumption, $g(x) > 0$, does not correspond to this example, there is no less of applicability of the results. The control is still valid as we only need to satisfy $g(x) \neq 0$.

Lastly, it is not difficult to verify the existence of a function

$$g_0(x) = 2\left(|a_8| + |a_9 \sin\zeta_4 + a_{10}||\zeta_3| + \frac{a_6(a_5 + a_6\zeta_4)^{-1.5}}{8\left[a_4 - 0.5a_6(a_5 + a_6\zeta_4)^{-0.5}\right]}\right) > 0, \forall\zeta_3 > 0, \zeta_4 \in [0, 0.44]$$

which fulfill Assumption 4 for the case $g(x) < 0$. Note that this function need not to be known; we only need to show its existence.

The control parameters are chosen as $\Lambda = [64, 48, 12]^T$, $k_i = 3$, while the NN parameters for each helicopter are chosen as $\sigma_1 = 0.05$, $\gamma_1 = 1$, $\sigma_2 = 0.01$, $\gamma_2 = 100$. The saturation limits of the control are $\pm 400 mrad$. The initial conditions are $\zeta_1(0) = [4.3, 0, 95.3567, 0.222, 0]^T$, $\zeta_2(0) = [4.8, 0, 95.3567, 0.3, 0]^T$, $\zeta_3(0) = [5.9, 0, 95.4, 0.22, 0]^T$, $\zeta_4(0) = [6.2, 0, 95.3567, 0.3, 0]^T$, $\zeta_5(0) = [6.8, 0, 95.3567, 0.22, 0]^T$, and $\zeta_6(0) = [7.4, 0, 95.4, 0.21, 0]^T$, and $\hat{\theta}_i(0) = 0$, $\hat{\varphi}_i(0) = 0$, $i = 1, \ldots, 6$. From Fig. 2, we can see that good tracking performance is achieved. The initial errors of all helicopters are sufficiently reduced and the altitude trajectories lie in close proximity of the desired sinusoidal trajectory. Meanwhile, the internal dynamics and the NN weights are all bounded, as shown in Fig. 5 and Fig. 4. From Fig. 3, it can be seen that the control inputs of the helicopters are bounded as well.

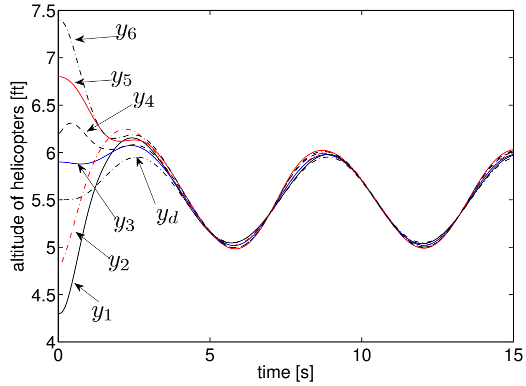
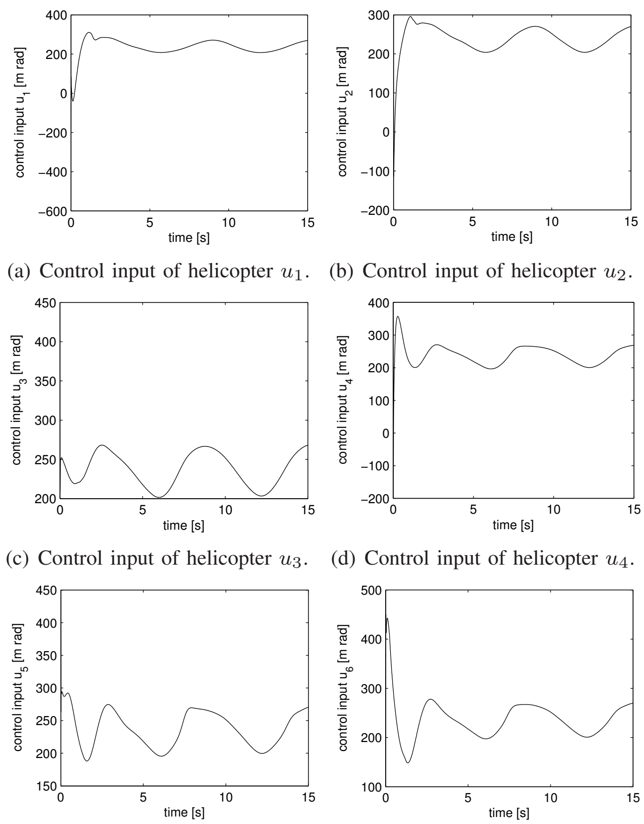


Fig. 2. Altitude synchronization of all helicopters.



(a) Control input of helicopter u_1. (b) Control input of helicopter u_2.

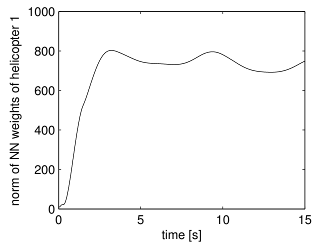
(c) Control input of helicopter u_3. (d) Control input of helicopter u_4.

(e) Control input of helicopter u_5. (f) Control input of helicopter u_6.

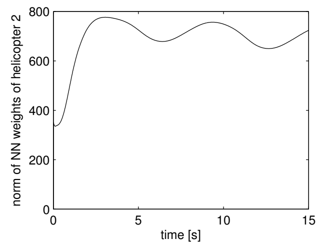
Fig. 3. Control input of helicopters under the proposed control.

V. Conclusion

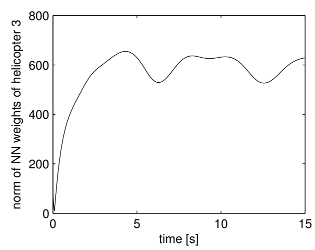
In this paper, we have studied the synchronized tracking problem of multiple helicopters in vertical flight mode. With the condition that the Laplacian matrix of the extended formation graph is positive definite, by using the weighted average of neighbors' states as its reference signal, the adaptive NN tracking control law has been designed for each helicopter. It has been shown that the tracking error of each helicopter converges to an adjustable neighborhood of origin. Simulation results have been obtained to show the effectiveness of the proposed methods.



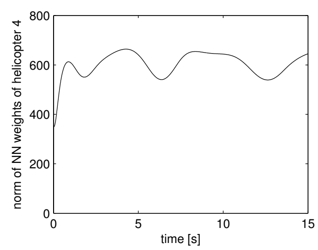
(a) Norm of neural weights, $\|\hat{\theta}_1\|$.



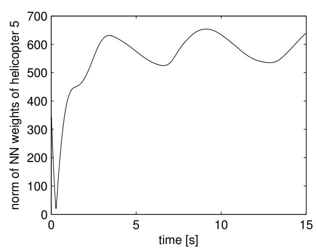
(b) Norm of neural weights, $\|\hat{\theta}_2\|$.



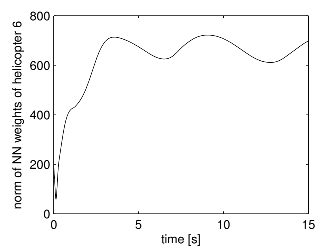
(c) Norm of neural weights, $\|\hat{\theta}_3\|$.



(d) Norm of neural weights, $\|\hat{\theta}_4\|$.



(e) Norm of neural weights, $\|\hat{\theta}_5\|$.



(f) Norm of neural weights, $\|\hat{\theta}_6\|$.

Fig. 4. Norm of neural weights under the proposed control.

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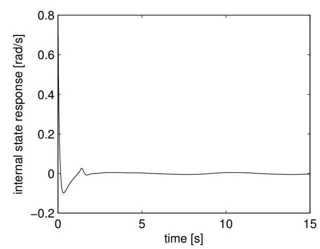
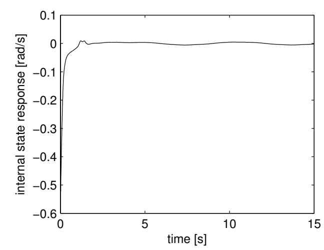
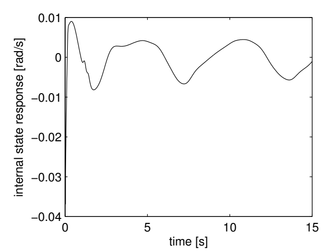


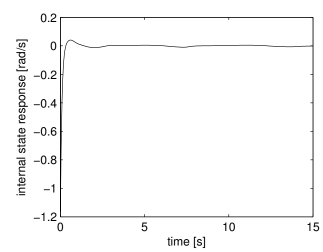
(a) Internal state of helicopter 1.



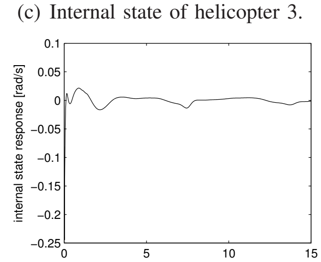
(b) Internal state of helicopter 2.



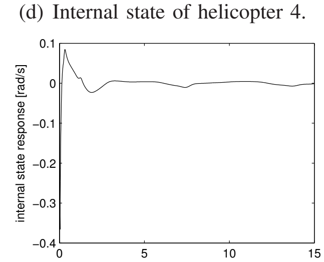
(c) Internal state of helicopter 3.



(d) Internal state of helicopter 4.



(e) Internal state of helicopter 5.



(f) Internal state of helicopter 6.

Fig. 5. Internal state response under the proposed control.