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Synchronized Tracking Control of Multi-Agent System with Limited Information

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Abstract— **In this paper, synchronized tracking control is considered for multiple agents with unknown system dynamics, while the desired trajectory is only available to portion of the team members. Using the weighted average of the neighbors' outputs, adaptive neural network (NN) tracking control is designed for each agent. Rigid mathematical proof was provided for the proposed algorithm based on the Lyapunov analysis. It is shown that, under the proposed NN control, the output tracking error of each agent converges to an adjustable neighborhood of the origin. Simulations of synchronized altitude tracking of multiple unmanned helicopters are provided to demonstrate the effectiveness of the approaches presented.**

I. INTRODUCTION

Control of multi-agent systems with applications to the cooperation of robots, UAVs, AUVs, scheduling of automated highway systems have been intensively studied in recent years [1]–[9]. Various control strategies for multi-agent systems can be roughly assorted into two architectures: centralized and decentralized. In the decentralized control, local control for each agent is designed using locally available information so it requires less computational effort and is more scalable with respect to the swarm size [8], [10]. Much work has been done for the decentralized control of multi-agent system: A multi-agent consensus algorithm was proposed in [2], where the agent is modeled by single-integrator dynamics. Consensus for double-integrate system was studied in [1], where the consensus with a bounded control input and with partial access to a group reference state is considered. Model reference consensus algorithms for high-order system $x^{(\ell)} = u,\ \ell > 3$, was studied in [11]. Consensus algorithms with switching topology and time-delays was proposed in [12], leader-following consensus problem for multi-agent system with fixed and switching topologies was studied in [13], where the agent is modeled by linear system $\dot{x} = Ax + Bu$ with (A, B) a stabilized pair.

The multi-agent systems considered in this paper have the following features: (i) the agent dynamics is high order (≥ 3) and with unknown dynamics; (ii) the desired trajectory is only available to portion of the agents; (iii) each agent can access its neighbors' outputs but not the full states; and (iv) the leadership of the leader itself is unknown to all the others, and the leader can only affect the agents who can sense its outputs. The problem studied is similar to the model reference consensus problem, where each agent is designed to track the desired trajectory by using only neighbor-to-neighbor information exchange among the agents. Most of recent related works are concerned with the multi-agent systems with linear dynamics and is known for control design [10]–[13]. However, for real-world applications, practical systems usually have complicated nonlinear dynamics and there are usually uncertainties in the dynamics. Adaptive control was studied for both consensus problem and leader following problem for a class of multi-agent system in [14]. In [8], hidden layer leader-following problem with discrete uncertain dynamics was considered, where the communication graph should be strongly connected.

In contrast, this paper considers the general case that each agent is of uncertain nonlinear dynamics and only a portion of the agents can access the desired trajectory. Unlike the leader-follower strategy (e.g., [13], [15]), where the information only flows from the leader to the followers, the problem studied in this paper takes into account the general cases where information flow from any agent to any other agent. There are multiple agents who can access the desired trajectory, therefore increase redundancy and robustness for the whole team [2].

In this work, we first revisit our previous results in [7], where the concept of extended formation graph was introduced, and the tracking control was designed based on the neighbors' full states. Then we extend the work to the case with limited information, e.g., using neighbors' outputs only for the control design of each agent.

II. BACKGROUND AND PRELIMINARIES

A. Agent Dynamics

The agent considered in this paper is given by

$$\begin{aligned} \dot{x}_j &= x_{j+1}, \quad j = 1, \ldots, \rho - 1 \\ \dot{x}_\rho &= f(\eta, x) + g(x, \eta)(u + d) \\ \dot{\eta} &= q(x, \eta) \\ y &= x_1 \end{aligned} \tag{1}$$

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where $x = [x_1, \ldots, x_\rho]^T \in \mathbb{R}^\rho$ and $\eta \in \mathbb{R}^{n-\rho}$ are the states of the system; $u,\ y \in \mathbb{R}$ the input and output, respectively; $f:\ \mathbb{R}^n \to \mathbb{R}$ an unknown smooth function; $q:\ \mathbb{R}^n \to \mathbb{R}$ is a partially unknown vector field satisfying certain properties, which will be described shortly; $g:\ \mathbb{R}^n \to \mathbb{R}$ is an unknown function with certain properties, and d is the external disturbance in the input channel.

Assumption 1: The zero dynamics of system (1), given by $\dot{\eta} = q(x,\eta)$, are exponentially stable. In addition, $q(x,\eta)$ is Lipschitz in x, i.e., there exists positive constants a_q and a_x such that $\|q(x,\eta) - q(0,\eta)\| \le a_x\|x\| + a_q \quad \forall (x,\eta) \in \mathbb{R}^n$.

Assumption 2: The external disturbance d is uncertain bounded functions $d \in L_\infty$. That is, there exists unknown positive constants ϱ such that $|d(t)| \le \varrho < \infty$, where ϱ can be arbitrarily large.

Assumption 3: There exist smooth functions $\bar{g}(x,\eta)$ and a positive constant $\underline{g} > 0$, such that $\bar{g}(x,\eta) \ge g(x,\eta) > \underline{g} > 0$, $\forall (x,\eta) \in \mathbb{R}^n$. There exists a positive function $g_0(x,\eta)$ satisfying $|\dot{g}(x,\eta)/2g(x,\eta)| \le g_0(x,\eta)$, $\forall (x,\eta) \in \mathbb{R}^n$ as well. Without loss of generality, it is further assumed that the sign of $g(x,\eta)$ is positive $\forall (x,\eta) \in \mathbb{R}^n$.

Remark 1: $g(x,\eta)$ stands for the control gain of the open-loop system. Assumption 3 is reasonable for many physical systems as follows: (i) there exist lower and upper bounds of the control gain, and (ii) the states of some physical systems cannot change too fast within a small time internal in open-loop due to the 'inertia' of the systems. This does not pose a strong restriction upon the class of systems such as the helicopter altitude control system. The reason is that if the controller is continues, the situation in which a finite input causes an infinity large effect upon the system rarely happen in such systems due to the smoothness of $g(x,\eta)$.

B. Graph Preliminaries

A directed graph $\mathcal{G}$ consists of a node set $\mathcal{V}$ and an edge set $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$. An edge (i,j) in a directed graph denotes that agent j can obtain information from agent i, but not necessarily vice versa [2]. Suppose that there are p nodes in the graph. The weighted adjacent matrix $A = [a_{ij}] \in \mathbb{R}^{p\times p}$ of a weighted directed graph is a square matrix of size $|\mathcal{V}|$, with its elements $a_{ij} > 0$ if $(j,i) \in \mathcal{E}$, and is zero otherwise. Define a diagonal matrix $\Delta(\mathcal{G})$ with its elements $\Delta_{jj} = \sum_k A_{jk}$ and the normalized Laplacian of the graph is defined as $L = I - A^*$ with $A^* = \Delta^{-1}A$.

C. Neural Network Approximation

There are many types of function approximators, neural networks (NN), fuzzy systems, and wavelet networks [16]. In this paper, linearly parametrized NN is used to approximate the unknown continuous function $f_i(Z_i) : \mathbb{R}^q \to \mathbb{R}$ for $i-$th agent, $f_i(Z_i) = \theta_i^T\psi_i(Z_i) + \varepsilon_i(Z_i)$, where the input vector $Z_i^T \in \mathbb{R}^q$, weight vector $\theta_i \in \mathbb{R}^l$, the NN node number $l > 1$, and $\psi_i(Z_i) \in \mathbb{R}^l$. Universal approximation results indicate that, if l is chosen sufficiently large, $\theta_i^T\psi_i(Z_i)$ can approximate any continuous function, $f_i(Z_i)$, to any desired accuracy over a compact set $Z_i^T \in \Omega_{xi}$ to arbitrary any degree of accuracy as $f_i(Z_i) = \theta_i^{*T}\psi_i(Z_i) + \varepsilon_i(Z_i),\ \forall Z_i \in \Omega_{xi} \subset \mathbb{R}^q$, where θ_i^* are the ideal constant weight vector, and $\varepsilon(Z_i)$ is the approximation error which is bounded over the compact set, i.e., $|\varepsilon_i(Z_i)| \le \varepsilon_i^*,\ \forall Z_i \in \Omega_{xi}$ with $\varepsilon_i^* > 0$ is an unknown constant. The ideal weight vector θ_i^* is an artificial quantity required for analytical purposes. θ^* is defined as the value of θ_i that minimizes $|\varepsilon_i|$ for all $Z_i \in \Omega_{xi} \subset \mathbb{R}^q$, i.e., $\theta_i^* := \arg\min_{\theta_i\in\mathbb{R}^l}\left\{\sup_{Z_i\in\Omega_{xi}} |f_i(Z_i) - \theta_i^T\psi_i(Z_i)|\right\}$. In this paper, we choose $\psi_i(Z_i) = \exp\left[-(Z_i-\mu_i)^T(Z_i-\mu_i)/\sigma^2\right],\ i = 1,\ldots,l$, where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{iq}]^T$ is the center of the receptive field and σ is the width of the Gaussian function.

For the Gaussian RBF networks, the following lemma provides an upper bound on the 2-norm of vector $\psi_i(Z_i)$, which is essential in the proofs of our results.

Lemma 1: [17], [18] Consider the Gaussian RBF networks mentioned above. Let $\rho := \frac{1}{2}\min_{i\ne j}\|\mu_i - \mu_j\|$, and let q be the dimension of input Z_i, and σ be the width of Gaussian function. Then we may have $\|\psi_i(Z_i)\| \le \sum_{k=0}^{\infty} 3q(k+2)^{q-1}e^{-2\rho^2k^2/\sigma^2} := m_\psi^*$.

III. Synchronized Tracking Control of Multi-Agent Systems

In this paper, we studied the synchronized tracking problem of multiple agents as follows: Considering a group of agents, the desired trajectory of the team, $y_d(t)$, and its derivations up to ρ-th order is bounded, and is only available to portion of the agents $j \in \mathcal{V}_0$, where $\mathcal{V}_0$ contains all the agent who can access the desired trajectory of the agents. For each agent, we design a control, using its outputs and neighbors outputs, such that the tracking error converges to a neighborhood of zero, i.e.,

$$\lim_{t\to\infty} |y_i(t) - y_d(t)| = \bar{\varepsilon} \tag{2}$$

where $\bar{\varepsilon} > 0$. At the same time, all closed-loop signals are to be kept bounded.

The desired trajectory $y_d(t)$ is generated by the reference model: $\dot{x}_{di} = \dot{x}_{di+1},\ i = 1,\ldots,\rho-1,\ \dot{x}_{d\rho} = f_d(x_d,t)$, where $\rho \ge 2$ is a constant index, $x_d = [x_{d1},\ldots,x_{d\rho}]^T \in \mathbb{R}^\rho$ are the states of reference system, $y_d = x_{d1} \in \mathbb{R}$ is the system output.

Assumption 4: The communication topology of the agents team has a spanning tree with the root agent can access the desired trajectory.

For the multiple agents tracking problem studied in this paper, we introduce a virtual agent, labeled as 0, whose motion follows the desired trajectory restrictively. In accordance with this, for N agents team labeled $1,\ldots,N$, we defined the extended formation graph with adding additional node labeled 0 and edges $(0,k)$ where agents k can access the desired trajectory. The following lemma proposed in [7] gave the positive definiteness of the Laplacian of the extended formation graph, which facilities the subsequent stability proof of the results.

Lemma 2: [7] Consider the multiple agent synchronized tracking problem, if the extended formation graph $\mathcal{G}$ contains a spanning tree with 0 as its root. Then the normalized

adjacent matrix A of the extended formation graph $\mathcal{G}$ is sub-stochastic, and $L = I - A$ is positive definite, whose inverse is given by $L^{-1} = \sum_{l=0}^{\infty} A^l$.
The following lemma is useful for analysis of the internal dynamics of the agent.

Lemma 3: [19] Denote positive constants $a_1 = (\lambda_b a_x)/\lambda_a$ and $a_2 = (\lambda_b a_q)/\lambda_a$. If Assumptions 1 satisfied and the desired trajectory y_d with its up to $\rho - th$ derivatives are all bounded, there exists a positive time constant T_0 such that the trajectories $\eta(t)$ of the internal dynamics satisfy $\|\eta\| \le a_1\|x(t)\| + a_2$, $\forall t > T_0$.

IV. Synchronized Tracking Control Design

In this section, we design the synchronized tracking control for each agent based on its neighbors' full states. Feed-forward approximators are used to compensate for unknown nonlinear functions. Full-state feedback controller, in case that the neighbors' full-states are available for control design, which was proposed in [7] will be briefly revisited first. Based on this, an output feedback controller, in case only the neighbors' outputs are available for control design, will be subsequently designed via certainty equivalence approach for each agent, with the unavailable output derivative estimated with a high-gain observer.

A. Control Design With Full Information

Define the following error variables for the agents:

$$z_{i,1} = y_{i,1} - y_{ir} \ \ldots, \ z_{i,\rho} := z_{i,1}^{(\rho)} = x_{i,\rho} - y_{ir}^{(\rho)} \quad (3)$$

with $y_{ir}(t) = \sum_{j\in\mathcal{N}_i} a_{ij} y_j(t)$, $y_{ir}^{(k)}(t) = \sum_{j\in\mathcal{N}_i} a_{ij} y_j^{(k)}(t)$, $k = 1, \cdots, \rho - 1$, where a_{ij} is the element of the normalized adjacent matrix A of the extended formation graph $\mathcal{G}$.

For each agent, we define vectors $\bar{z}_i$, and $\mathcal{Z}_i$ as $\bar{z}_i = [z_{i,1}, \ldots, z_{i,\rho}]^T \in \mathbb{R}^\rho$, and the filtered tracking error as $s_i = [\Lambda^T \ \ 1]\bar{z}_i$, where $\Lambda = [\lambda_1, \lambda_2, \ldots, \lambda_{\rho-1}]^T$ satisfies $p^{\rho-1} + \lambda_{\rho-1}p^{\rho-2} + \ldots + \lambda_1$ is Hurwitz. Then the dynamics of s_i is written as

$$\dot{s}_i = f_i(x_i, \eta_i) + g_i(u_i + d_i) + [0 \ \ \Lambda^T]\bar{z}_i - y_{ir}^{(\rho)} \quad (4)$$

Define a Lyapunov function candidate for each agent:

$$V_i = \frac{1}{2g_i}s_i^2 + \frac{1}{2\gamma_2}\tilde{\theta}_i^T\tilde{\theta}_i + \frac{1}{2\gamma_1}\tilde{\varphi}_i^2 \quad (5)$$

where γ_1 and γ_2 are the positive constants, $\tilde{\theta}_i = \hat{\theta}_i - \theta_i^*$, and $\tilde{\varphi}_i = \hat{\varphi} - \varphi_i^*$, are the estimated errors of parameters and the error bound, where $\hat{\theta}_i$ and $\hat{\varphi}_i$ are the estimation of θ_i^* and $\varphi_i^* = (\varrho_i + \bar{\varepsilon}_i)^2$, respectively. In case that the neighbors' full states are available, we design the following control:

$$u_i = -\hat{\theta}_i^T\psi_i - k_i s_i - \hat{\varphi}_i s_i/2, \quad i = 1, \ldots, N \quad (6)$$

The update law of parameters are designed as

$$\begin{aligned} \dot{\hat{\varphi}}_i &= -\gamma_1\left[-(1-\varpi_\varphi)s_i^2/2 + \sigma_1\hat{\varphi}_i\right] \\ \dot{\hat{\theta}}_i &= -\gamma_2\big(-\psi_i s_i + \sigma_2\hat{\theta}_i\big) \end{aligned} \quad (7)$$

where $\varpi_{\varphi_i} = 0$ if $|\hat{\varphi}_i| \le M_{\varphi_i}$ with M_{φ_i} is a designed positive constant, or 1 otherwise.

Then we can arrived at [7]

$$\dot{V}_i \le -k_i s_i^2 - \sigma_1\tilde{\varphi}_i^2/2 - \sigma_2\|\tilde{\theta}_i\|^2/2 + c_{2i} \quad (8)$$

where

$$c_{2i} = \sigma_2\|\theta_i^*\|^2/2 + \sigma_1\varphi_i^{*2}/2 + 1/2 \quad (9)$$

According to the standard Lyapunov theorem, if we use the control (6) with parameter updated law (7) for each agent, we can conclude that s_i, $\tilde{\theta}_i$, and $\tilde{\varphi}_i$ are bounded [7], and there exsits a constant T_1 such that for $t > T_1$, the filtered tracking error s_i converges to a compact set, that is to say, $s_i \le \beta_{si}(k_i, \gamma_1, \gamma_2, \sigma_1, \sigma_2, \theta_i^*, \varphi_i^*, \varepsilon_i^*) = \sqrt{c_{2i}/k_i}$.

To show that all the agent will track the desired trajectory, define the error between i-th agent and the desired trajectory as $\tilde{y}_i(t) = y_i(t) - y_d(t) = y_i(t) - y_0(t)$, and the auxiliary states of each agent $\xi_i(t) = [\Lambda^T \ \ 1]Y_i$ with $Y_i = [y_i, y_i^{(1)}, \ldots, y_i^{(\rho-1)}]^T$. The filtered error is denoted as $\tilde{\xi}_i(t) = \xi_i(t) - \xi_d(t) = \xi_i(t) - \xi_0(t)$. Using the fact that $s_i(t) = \xi_i(t) - \sum_{j\in\mathcal{N}_i} a_{ij}\xi_j(t)$, we have

$$\tilde{\xi}_i = \xi_i - \xi_0 = \sum_{j\in\mathcal{N}_i} a_{ij}\xi_j + s_i - \xi_0, \quad i = 1, \ldots, N \quad (10)$$

and in the vector form $\tilde{\xi} = A\xi + s - \xi_0\mathbf{1}$, where $\mathbf{1} = [1, \ldots, 1]^T$, $s = [s_0, s_1, \ldots, s_N]^T$, and A is the normalized adjacency matrix of the extended formation graph. Under Assumption 4, we can have $\tilde{\xi} = L^{-1}s$ [7].

Define vectors $\mathcal{Y} = [Y_0^T, Y_1^T, \ldots, Y_N^T]^T$, $\tilde{\mathcal{Y}} = [\tilde{Y}_0^T, \tilde{Y}_1^T, \ldots, \tilde{Y}_N^T]^T$, $X = [X_0^T, X_1^T, \ldots, X_{\rho-1}^T]^T$, and $\tilde{X} = [\tilde{X}_0, \tilde{X}_1, \ldots, \tilde{X}_{\rho-1}]^T$, where $X_j = [X_{0,j}, X_{1,j}, \ldots, X_{N,j}]^T$, $\tilde{X}_j = X_j - X_{jd} = X_j - y_0^{(j)}\mathbf{1}$, $\tilde{Y}_i = Y_i - Y_d = Y_i - Y_0$. Then we have

$$\dot{\tilde{\mathcal{Y}}} = \bar{A}_p\tilde{\mathcal{Y}} + \bar{b}\tilde{\xi} \quad (11)$$

where $\bar{A}_p = I_{N+1} \otimes A_p$ and $\bar{b} = I_{N+1} \otimes b$. Symbol "$\otimes$" stands for the Kronecker Product of the matrices.

Then the error dynamics can be written as

$$\dot{\tilde{\mathcal{Y}}} = \bar{A}_p\tilde{\mathcal{Y}} + \bar{b}\tilde{\xi} = \bar{A}_p\tilde{\mathcal{Y}} + \bar{b}L^{-1}s \quad (12)$$

And we can get $\|\tilde{\bar{X}}\| \le \big(1 + \|\Lambda\|\big)\|\tilde{\mathcal{Y}}\| + \lambda_{\max}(L^{-1})\|s\|$ [7]. Also, we know that $\|\tilde{\bar{X}}\| \le k_a\|\tilde{\mathcal{Y}}(0)\| + k_b\beta_{s_T} + k_c$, $\forall t \ge T_1$, with $k_a = (1 + \|\Lambda\|)k_0$, $k_b = (k_a/\lambda_0) + 1$ and $k_c = k_a(e^{\lambda_0 T_1}/\lambda_0)\beta_s(T_1)$, Hence

$$\|\bar{X}(t)\| \le k_a\|\tilde{\mathcal{Y}}(0)\| + k_b\beta_{s_T} + k_c + c, \quad \forall t \ge T_1 \quad (13)$$

Then, we summarize our results for the full-state feedback case in the following theorem [7].

Theorem 1: [7] Consider a group of agents dynamics (1), under Assumptions 1–4, control (6) and parameters update law (7) for each agent. For initial conditions $\bar{X}(0), \eta(0)$, $\tilde{\theta}_i(0)$ and $\tilde{\varphi}_i(0)$ starting in any compact set, and the desired trajectory with its derivations up to ρ-th is bounded, all closed signals of the system are semiglobal uniform bound, and the total tracking error of the agents $\tilde{\bar{X}}$ converges to a neighborhood of the origin.

B. Control Design With Limited Information

From the definition (3) of reference states of each agent, we know that only portion of the agents can access the desired trajectory and its derivation. For each agent, its reference output at time t is the weighted average of its neighbors' outputs at the same time, and in the control design, each agent needs to use neighbors states $y_{ir}^{(k)}(t)$, $k = 1, \ldots, \rho$, which are not easy for them to access. In this section, we studied the case that each agent can only access its neighbors output information y_{ir}, and use high gain observer to estimate $y_{ir}^{(k)}(t)$, $k = 1, \ldots, \rho$.

In the following lemma, high gain observer used in [20] is presented, which will be used to estimate the unkown states.

Lemma 4: [19] Consider the following linear system:

$$\begin{aligned} &\epsilon\dot{\pi}_i = \pi_{i+1},\ i = 1, 2, \ldots, \rho - 1 \\ &\epsilon\dot{\pi}_\rho = -\bar{\gamma}_1\pi_\rho - \bar{\gamma}_2\pi_{\rho-1} - \ldots - \bar{\gamma}_{\rho-1}\pi_2 - \pi_1 + \chi(t) \end{aligned} \tag{14}$$

where ϵ is a small positive constant and the parameters $\bar{\gamma}_1$ to $\bar{\gamma}_{\rho-1}$ are chosen such that the polynominal $s^\rho + \bar{\gamma}_1 s^{\rho-1} + \ldots + \bar{\gamma}_{\rho-1}s + 1$ is Hurwitz. Suppose the states $\chi(t)$ and its first n derivatives are bounded, so that $\chi^{(k)} < \varpi_k$ with positive constants ϖ_k. Then the following property holds:

$$\tilde{\chi}^{(k)} := \pi_k/\epsilon^{k-1} - \chi^{(k)} = -\epsilon\zeta^{(k)}, \quad k = 1, 2, \ldots, \rho \tag{15}$$

where $\zeta := \pi_p + \bar{\gamma}_1\pi_{\rho-1} + \ldots + \bar{\gamma}_{\rho-1}\pi_1$ and $\zeta^{(k)}$ denotes the kth derivative of ζ. Furthermore, there exist positive constants h_k and t^* such that for all $t > t^*$ we have $|\zeta^{(k)}| \leq h_k$, $k = 2, 3, \ldots, \rho$.

Note that $\frac{\pi_{k+1}}{\epsilon^k}$ asymptotically converges to $\zeta^{(k)}$, with a small time constant provided that ζ and its k derivatives are bounded. Hence, $\frac{\pi_{k+1}}{\epsilon^k}$ for $k = 1, \ldots, \rho$ is a suitable observer to estimate the output derivatives up to the ρ-th order.

To prevent peaking [21], saturation functions are employed on the observer signals whenever they are outside the domain of interest Ω as follows:

$$\pi_{i,j}^s = \bar{\pi}_{i,j}\phi\left(\pi_{i,j}/\bar{\pi}_{i,j}\right), \quad \bar{\pi}_{i,j} \geq \max_{(\tilde{y}_i, s_i, \tilde{\theta}_i, \tilde{\varphi}_i)\in\Omega}(\pi_{i,j})$$

$\phi(a) = -1$ while $a < -1$, $\phi(a) = a$ while $|a| < 1$, and $\phi(a) = 1$ while $a > 1$.

Now we revisit the control law (6) and adaption law (7). Via the certainty equivalence approach, we modify them by replacing the reference signal $y_{ir}^{(k)}, k = 1, \ldots, \rho$ which depends on the neighbors output with estimates

$$\hat{y}_{ir}^{(k)} = \pi_{i,k}/\epsilon^k, \quad i = 2, \ldots, N \text{ and } k = 1, \ldots, \rho \tag{16}$$

Then, $\hat{z}_{i,k} = x_{i,k} - \hat{y}_{ir}^{(k)}$, and $\hat{s}_i = [\Lambda^T \quad 1]\hat{\bar{z}}_i$ where $\hat{\bar{z}}_i = [z_{i,1}, \hat{z}_{i,2}, \ldots, \hat{z}_{i,\rho}]^T$.

$$\begin{aligned} \tilde{z}_{i,k} &= y_i^{(k-1)} - \hat{y}_i^{(k-1)} = \epsilon\chi_i^{(k-1)} \\ \tilde{y}_{ir}^{(\rho)} &= \hat{y}_{ir}^{(\rho)} - y_{ir}^{(\rho)} \end{aligned} \tag{17}$$

Via the certainty equivalence approach, we modify (6) and (7) by replacing the partially available quantities with their estimates, which can be written as

$$u_i = -\hat{\theta}_i^T\psi_i(\hat{Z}_i) - k_i\hat{s}_i - \hat{\varphi}_i\hat{s}_i/2, \quad i = 1, \ldots, N \tag{18}$$

And the update law of parameters are designed as

$$\begin{aligned} \dot{\hat{\varphi}}_i &= -\gamma_1\left[-(1 - \varpi_\varphi)\hat{s}_i^2/2 + \sigma_1\hat{\varphi}_i\right] \\ \dot{\hat{\theta}}_i &= -\gamma_2\big(-\psi_i\hat{s}_i + \sigma_2\hat{\theta}_i\big) \end{aligned} \tag{19}$$

where γ_1, γ_2, σ_1 and σ_2 are positive constants, and if $|\hat{\varphi}_i| \leq M_{\varphi_i}$, $\varpi_{\varphi_i} = 0$, otherwise and $|\hat{\varphi}_i| = 1$, where M_{φ_i} is a designed positive constant. Consider Lyapunov function candidate

$$V_{ie} = \frac{1}{2}s_i^2 + \frac{1}{2\gamma_2}\tilde{\theta}_i^T\tilde{\theta}_i + \frac{1}{2\gamma_1}\tilde{\varphi}_i^2 \tag{20}$$

And the following lemma is useful for handling the terms containing the estimation errors.

Lemma 5: [19] There exist positive constants F_{ik} which are independent of ϵ_i, such that for $t > t^*$, the estimate $\hat{y}_{ir}^{(k)}$, $i = 1, \ldots, N$, $k = 1, \ldots, \rho$, satisfy $|\tilde{y}_{ir}^{(k)}| = |\hat{y}_{ir}^{(k)} - y_{ir}^{(k)}| \leq \epsilon_i F_{ik}$.

Since s_i is the linear combination of Y_i and Y_j, $j \in \mathcal{N}_i$, we know that there exist positive constants G_{is} which are independent of ϵ_i satisfy $|\tilde{s}_i| \leq \epsilon_i G_{is}$.

Taking the time derivative of V_i along the closed-loop trajectory and using the property $\psi_i(\hat{Z}_i) - \psi_i(Z_i) = \epsilon_i\psi_{ti}$, where ψ_{ti} is a bounded vector function [16], we have

$$\begin{aligned} \dot{V}_{ie} = &-(\frac{\dot{g}_i}{2g_i^2} + g_0)s_i^2 - k_is_i^2 - k_is_i\tilde{s}_i + \frac{1}{\gamma_2}\tilde{\theta}_i\dot{\hat{\theta}}_i + \frac{1}{\gamma_1}\tilde{\varphi}_i\dot{\hat{\varphi}}_i \\ &- s_i\hat{\theta}_i^T\psi_i(\hat{Z}_i) - \frac{1}{2}\hat{\varphi}_is_i\hat{s}_i + s_i(d_i + \bar{\varepsilon}_i) + s_i\theta^{*T}\psi_i(Z_i) \\ \leq &-\frac{k_i}{2}s_i^2 + \frac{k_i}{2}\tilde{s}_i^2 + \frac{1}{2}\left(-\hat{\varphi}_is_i\hat{s}_i + \varphi_is_i^2 + \tilde{\varphi}_i\hat{s}_i^2\right) \\ &- s_i\hat{\theta}_i^T\psi_i(\hat{Z}_i) + s_i\theta^{*T}\psi_i(Z_i) + \hat{s}_i\tilde{\theta}_i^T\psi_i(\hat{Z}_i) \\ &- \sigma_2\tilde{\theta}_i^T\hat{\theta}_i - \sigma_1\tilde{\varphi}_i\hat{\varphi}_i + 1/2 \end{aligned} \tag{21}$$

After long but straightforward calculation, we have $[-\hat{\varphi}_is_i\hat{s}_i + \varphi_is_i^2 + \tilde{\varphi}_i\hat{s}_i^2] \leq s_i^2 + \epsilon_i^2G_{is}^2(\hat{\varphi}_i^2 + \frac{1}{2}\varphi_i^2 + 1)$, and $[-s_i\hat{\theta}_i^T\psi_i(\hat{Z}_i) + s_i\theta^{*T}\psi_i(Z_i) + \hat{s}_i\tilde{\theta}_i^T\psi_i(\hat{Z}_i)] \leq \frac{1}{2}\tilde{\theta}_i^T\tilde{\theta}_i + \frac{1}{2}\epsilon_i^2G_{is}^2m_\psi^{*2} + \frac{1}{2}s_i^2 + \frac{1}{2}\epsilon_i^2\|\psi_{ti}\|^2\|\theta_i^*\|^2$.

Then the time derivation of V_{ie} can be written as

$$\dot{V}_{ie} \leq -(k_i - 2)s_i^2/2 - (\sigma_2 - 1)\tilde{\theta}_i^T\tilde{\theta}_i/2 - \sigma_1\tilde{\varphi}_i^2/2 + c_{2ie} \tag{22}$$

with constant

$$\begin{aligned} c_{2ie} = &\frac{1}{2}\epsilon_i^2G_{is}^2\hat{\varphi}_i^2 + \frac{1}{2}\epsilon_i^2G_{is}^2(m_\varphi^{*2} + 1) + \frac{k_i}{2}\epsilon_i^2 + \frac{1}{2} \\ &+ \frac{\epsilon_i^2\|\psi_{ti}\|^2 + \sigma_2}{2}\|\theta_i^*\|^2 + (\frac{3}{4}\epsilon_i^2G_{is}^2 + \frac{\sigma_1}{2})\varphi_i^{*2} \end{aligned} \tag{23}$$

Now, we choose $k_i > 2$, $\sigma_2 > 1$, and define

$$\Omega_{sie} = \left\{s_i \,\middle|\, |s_i| \leq \sqrt{2c_{2ie}/(k_i - 2)}\right\} \tag{24}$$

$$\Omega_{\theta_{ie}} = \left\{(\tilde{\theta}_i, \tilde{\varphi}_i) \,\middle|\, \|\tilde{\theta}_i\| \leq \sqrt{\frac{2c_{2ie}}{\sigma_2 - 1}}, |\tilde{\varphi}_i| \leq \sqrt{\frac{2c_{2ie}}{\sigma_1}}\right\} \tag{25}$$

$$\Omega_{eie} = \left\{(s_i, \tilde{\theta}_i, \tilde{\varphi}_i) \,\middle|\, \frac{k_i - 2}{2}s_i^2 + \frac{\sigma_2 - 1}{2}\tilde{\theta}_i^T\tilde{\theta}_i + \frac{\sigma_1}{2}\tilde{\varphi}_i^2 \leq c_{2ie}\right\} \tag{26}$$

Since c_{2ie}, σ_1, $\sigma_2 - 1$, and $k_i - 2$ are all positive constants, we know that Ω_{sie}, $\Omega_{\theta_i e}$ and Ω_{eie} are compact sets. Eq. (22) shows that $\dot{V}_{ie} \leq 0$ once the errors are outside the compact set Ω_{ei}. According to the standard Lyapunov theorem, we conclude that s_i, $\tilde{\theta}_i$, and $\tilde{\varphi}_i$ are bounded. From (22) and (24), it can be seen that V_{ie} is strictly negative as long as s_i is outside the compact set Ω_{sie}. Therefore, there exists a constant T_1 such that for $t > T_1$, the filtered tracking error s_i converges to Ω_{sie}, that is to say, $s_i \leq \beta_{sie}$, with $\beta_{sie}(k_i, \gamma_1, \gamma_2, \sigma_1, \sigma_2, \theta_i^*, \varphi_i^*, \epsilon_i) = \sqrt{2c_{2ie}/(k_i - 2)}$.

We can conclude the following theorem.

Theorem 2: Consider a group of agents dynamics (1) and the communication graph containing a spanning tree with the root agent can access the desired trajectory, under Assumptions 1-4, the control law (18), parameters update law (19), and the high gain observer (14), which is turned on at time t^* in advance. For initial conditions $\bar{X}(0)$, $\eta(0)$, $\tilde{\theta}_i(0)$ and $\tilde{\varphi}_i(0)$ staring from any compact set, and the desired trajectory with its derivations up to ρ-th is bounded, then all closed loop signals of the system are semiglobal uniform bound, and the total tracking error of the agents $\tilde{\bar{X}}$ converges to a neighborhood of origin.

Proof: We have conclude that s_i will converge to a compact set Ω_{sie}, then following Lemma 5, it can be concluded that $\|\tilde{\mathcal{Y}}\| \leq k_0 e^{-\lambda_0 t}\left(\|\tilde{\mathcal{Y}}(0)\| + \frac{e^{\lambda_0 T_1}}{\lambda_0}\beta_s(T_1)\right) + \frac{k_0}{\lambda_0}\beta_{s_T}$, and from (13), we can find $\|\bar{X}\|$ is also bounded. Following the same procedure in the full-state feedback control, we can complete the proof. ■

Remark 2: Compare to [8], which requires the communication graph is strong connected, we have relaxed the connectivity condition for the graph and the stability results are obtained due to the merit of Lemma 2.

V. Simulation Study

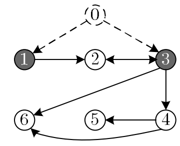
In this section, we consider the synchronized altitude tracking of six X-cell 50 helicopters which communication graph shown in Fig. 1. The dynamics of the helicopter is described by [22]

$$\begin{aligned}
\dot{\zeta}_1 &= \zeta_2 \\
\dot{\zeta}_2 &= a_0 + a_1\zeta_2 + a_2\zeta_2^2 + (a_3 + a_4\zeta_4 - \sqrt{a_5 + a_6\zeta_4})\zeta_3^2 \\
\dot{\zeta}_3 &= a_7 + a_8\zeta_3 + (a_9 \sin\zeta_4 + a_{10})\zeta_3^2 + a_{\text{th}} \\
\dot{\zeta}_4 &= \zeta_5 \\
\dot{\zeta}_5 &= a_{11} + a_{12}\zeta_4 + a_{13}\zeta_3^2 \sin\zeta_4 + a_{14}\zeta_5 - K_1 u \qquad (27)
\end{aligned}$$

where the detailed meaning of each parameters can be found in [22]. As shown in [23], let y be the altitude ζ_1. By restricting the throttle input to be constant, we obtain a SISO in which u is the only input variable forcing the output system, (27) has relative degree 4 with x system give by:

$$\begin{aligned}
\dot{x}_i &= x_{i+1}, \ i = 1, 2, 3 \\
\dot{x}_4 &= b(x) + g(x)u \qquad (28)
\end{aligned}$$

The desired trajectory is generated by $y_d = \frac{150.056}{s^4 + 12.6s^3 + 64.19s^2 + 154.35s + 150.056}h_{\text{ref}}$, where $h_{\text{ref}}(t) = 5.5 - 0.5\sin t$. The control parameters are chosen as



$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2/3 & 1 & -1/3 & 0 & 0 & 0 \\ -3/4 & 0 & -1/4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1/2 & -1/2 & 0 & 1 \end{bmatrix} \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

Graph $\mathcal{G}$ Matrix L

Fig. 1. Extended formation graph and its Laplacian.

$\Lambda = [64, 48, 12]^T$, $k_i = 3$, $i = 1, \ldots, 6$, while the NN parameters for each helicopter are chosen as $\sigma_1 = 0.05$, $\gamma_1 = 1$, $\sigma_2 = 0.01$, $\gamma_2 = 100$. For high gain observer, we choose $\epsilon_i = 0.08$, $\bar{\gamma}_1 = 4$, $\bar{\gamma}_2 = 6$, $\bar{\gamma}_3 = 4$, $\bar{\pi}_2 = 0.1$, $\bar{\pi}_3 = 0.15$, $\bar{\pi}_4 = 0.025$. The saturation limits of the control are ± 400m·rad. The initial conditions are $\zeta_1(0) = [4.3, 0, 95.36, 0.222, 0]^T$, $\zeta_2(0) = [4.8, 0, 95.36, 0.3, 0]^T$, $\zeta_3(0) = [5.9, 0, 95.4, 0.22, 0]^T$, $\zeta_4(0) = [6.2, 0, 95.36, 0.3, 0]^T$, $\zeta_5(0) = [6.8, 0, 95.36, 0.22, 0]^T$, $\zeta_6(0) = [7.4, 0, 95.4, 0.21, 0]^T$, $\hat{\theta}_i(0) = 0$, and $\hat{\varphi}_i(0) = 0$ for each helicopter.

Simulation results are shown in Figs. 2–4. We can see that good tracking performance is achieved for each agent with limited information from Fig. 2. The initial errors of all the agents are sufficiently reduced and the trajectories lie in close proximity of the desired sinusoidal trajectory. The control inputs and the NN weights are shown bounded in Fig. 3 and Fig. 4, respectively.

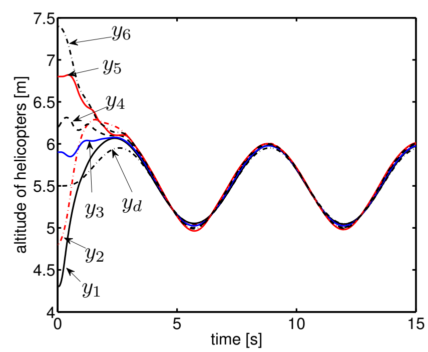


Fig. 2. Synchronization of all the agents with limited information.

VI. Conclusion

In this paper, we have studied the synchronized tracking problem of multi-agent systems with limited information. Under the condition that the Laplacian matrix of the extended formation graph, which contains a spanning tree with the root agent can access the desired trajectory, by using the weighted average of neighbors' outputs as its reference signal, the adaptive NN tracking control law has been designed for each agent. It has been shown that the tracking error of each agent converges to an adjustable neighborhood under proposed control strategy. Simulation results have shown the effectiveness of the proposed methods. One possible future research direction could be the synchronized tracking control with random communication time delays, such as the delays modeled by Markov chains [24].

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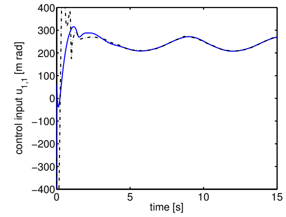
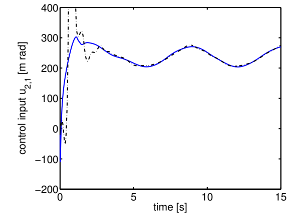
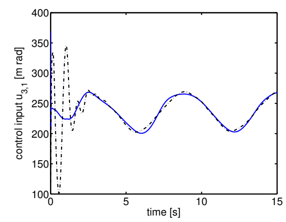


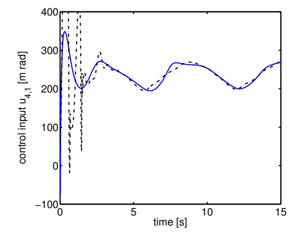
(a) Control input of agent u_1.



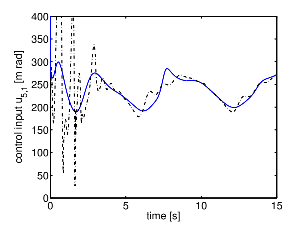
(b) Control input of agent u_2.



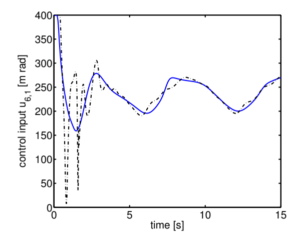
(c) Control input of agent u_3.



(d) Control input of agent u_4.

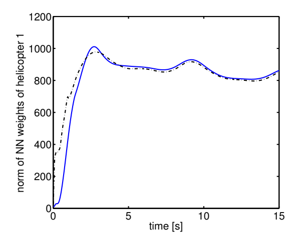


(e) Control input of agent u_5.

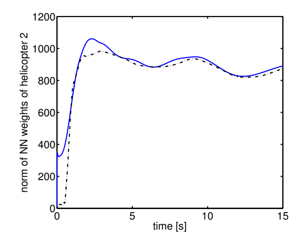


(f) Control input of agent u_6.

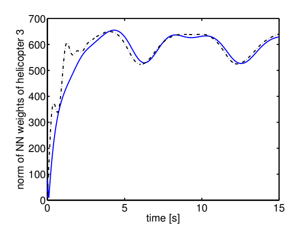
Fig. 3. Control input of helicopters with full-state (dash-dot) and output (solid) feedback control.



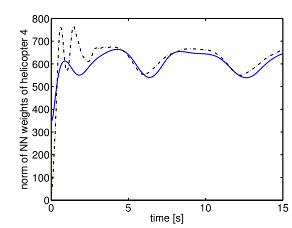
(a) Norm of neural weights, $\|\hat{\theta}_1\|$.



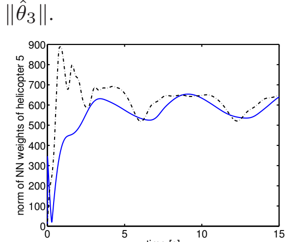
(b) Norm of neural weights, $\|\hat{\theta}_2\|$.



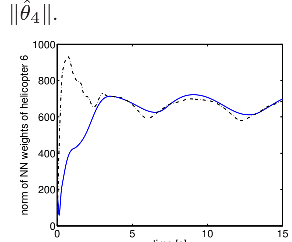
(c) Norm of neural weights, $\|\hat{\theta}_3\|$.



(d) Norm of neural weights, $\|\hat{\theta}_4\|$.



(e) Norm of neural weights, $\|\hat{\theta}_5\|$.



(f) Norm of neural weights, $\|\hat{\theta}_6\|$.

Fig. 4. Norm of neural weights with full-state (dash-dot) and output (solid) feedback control.