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# Boundary Control of a Flexible Marine Installation System

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*Abstract*— **In this paper, boundary control for a flexible marine installation system is developed to position the subsea payload to the desired set-point and suppress the cable's vibration. With the proposed boundary control, uniform boundedness under ocean disturbance can be ensured. The steady state error between the boundary payload and the desired position is proven to converge to a small neighborhood of zero by appropriately choosing design parameters. Simulations are provided to illustrate the applicability and effectiveness of the proposed control.**

## I. INTRODUCTION

Recent years, with the increasing trend towards resources exploitation in deeper and harsher marine environments, accurate positioning control for marine installation operations has gained increasing attention. Due to the requirements for high accuracy and efficiency arising from the modern ocean industry, improving reliability and efficiency of installation operations during oil and gas production in the ocean environment is a challenging research topic in ocean engineering. A typical marine installation system consisting of an ocean surface vessel, a flexible string-type cable and a subsea payload to be positioned for installation on the ocean floor is depicted in Fig. 1. The surface vessel, to which the top boundary of the cable is connected, is equipped with a dynamic positioning system with active thruster. The bottom boundary of the cable is a payload with an end-point thruster attached. This thruster is used for dynamic positioning of the payload. The total marine installation system is subjected to environmental disturbances including ocean current, wave, and wind. A cable that spans a long distance can produce large vibrations under relatively small disturbances, which can degrade the performance of the system and result in a larger offset from the target installation site. Taking into account the unknown time-varying ocean disturbances of the cable leads to the appearance of oscillations, which make the control problem of the marine installation system relatively difficult.

For the purpose of dynamic analysis, the flexible marine installation system with cable, vessel and payload dynamics is represented by one PDE and two ODEs. The dynamics of the flexible mechanical system modeled by PDE is difficult to control due to the infinite dimensionality of the system. Comparing with distributed controllers, boundary control is an economic method to control the distributed parameter system without decomposing the system into finite dimensional space, for example, [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13]. In this paper, our interest is in the boundary control problem involving both dynamic positioning and vibration suppression subjected to ocean disturbance. In the framework of boundary control, we are going to further study the dynamic positioning problem for the string-type model subjected to unknown time-varying ocean disturbance.

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## II. PROBLEM FORMULATION AND PRELIMINARIES

For the marine installation system shown in Fig. 1, frame X-Y is the fixed inertia frame, and frame x-y is the local reference frame fixed along the vertical direction of the surface vessel. The top boundary of the cable is at the vessel and the bottom boundary of the cable is at the underwater payload. Forces from thrusters on vessel and payload are the control inputs of the system, and the boundary position and slope of the cable are used as the feedback signals in the control design. $p(t)$ is the position of the vessel, $w(x,t)$ is the elastic transverse reflection with respect to frame x-y at the position x for time t, and $y(x,t) := p(t) + w(x,t)$ is the position of the cable at the position x for time t. Note that $w(L,t) = 0$ due to the connection between the vessel and the top boundary of the cable. In this paper, we consider the transverse degree of freedom only. We assume that the original position of the vessel is directly above the subsea payload with no horizontal offset, and the payload is filled with seawater.

### A. Dynamic analysis

The kinetic energy of the installation system $E_k$ can be represented as

$$E_k = \frac{1}{2}M\left[\frac{\partial y(L,t)}{\partial t}\right]^2 + \frac{1}{2}\rho\int_0^L \left[\frac{\partial y(x,t)}{\partial t}\right]^2 dx + \frac{1}{2}m\left[\frac{\partial y(0,t)}{\partial t}\right]^2, \quad (1)$$

where $x$ and $t$ represent the independent spatial and time variables respectively, $M$ denotes the mass of the surface vessel, $m$ denotes the mass of bottom payload, $y(L,t) := p(t)$, $\frac{\partial y(L,t)}{\partial t} =: \frac{\partial p(t)}{\partial t}$ and $\frac{\partial^2 y(L,t)}{\partial t^2} =: \frac{\partial^2 p(t)}{\partial t^2}$ are the position, velocity and acceleration of the vessel respectively, $\rho > 0$ is the uniform mass per unit length of the cable, and L is the length of the cable.

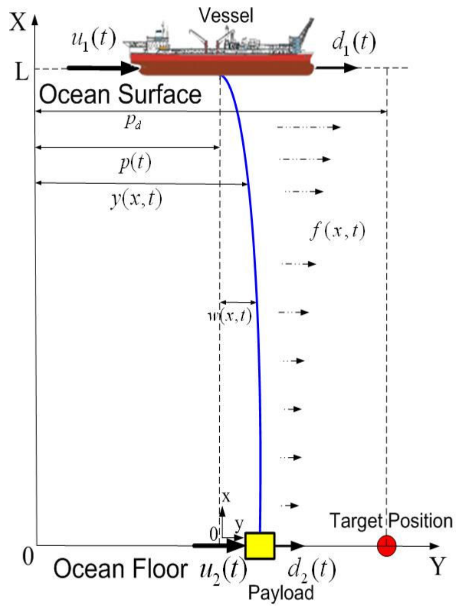


Fig. 1. A typical string-type marine installation system.

The potential energy $E_p$ due to the strain energy of the cable can be obtained from

$$E_p = \frac{1}{2}T\int_0^L \left[\frac{\partial w(x,t)}{\partial x}\right]^2 dx, \quad (2)$$

where $T$ is the tension of the cable. Definition of $y(x,t)$ yields $\frac{\partial y(x,t)}{\partial x} = \frac{\partial w(x,t)}{\partial x}$. Then we have

$$E_p = \frac{1}{2}T\int_0^L \left[\frac{\partial y(x,t)}{\partial x}\right]^2 dx, \quad (3)$$

The virtual work done by ocean current disturbance on the vessel, the cable and the payload is given by

$$\delta W_f = \int_0^L f(x,t)\delta y(x,t)dx + d_1(t)\delta y(L,t) + d_2(t)\delta y(0,t), \quad (4)$$

where $f(x,t)$ is the distributed transverse load on the cable due to the hydrodynamic effects of the ocean current, wave and wind, $d_1(t)$ denotes the environmental disturbances on the vessel, and $d_2(t)$ denotes the environmental disturbances on the paylaod. The virtual work done by damping on the vessel, the cable and the payload is represented by

$$\delta W_d = -\int_0^L c\frac{\partial y(x,t)}{\partial t}\delta y(x,t)dx - d_{s1}\frac{\partial y(L,t)}{\partial t}\delta y(L,t) - d_{s2}\frac{\partial y(0,t)}{\partial t}\delta y(0,t), \quad (5)$$

where $c$ is the distributed viscous damping coefficient of the cable, $d_{s1}$ denotes the damping coefficient of the vessel, and $d_{s2}$ denotes the damping coefficient for the payload. We introduce the control $u_1$ applied to the top boundary of the cable from the thruster attached in the vessel, and the control $u_2$ applied to the bottom boundary of the cable from the thruster attached in the payload. The control is aimed to produce a transverse force for dynamic positioning and vibration suppression. The virtual work done by the boundary control is written as

$$\delta W_m = u_1(t)\delta w(L,t) + u_2(t)\delta w(0,t). \quad (6)$$

Then, we have the total virtual work done on the system as

$$\delta W = \delta W_f + \delta W_d + \delta W_m. \quad (7)$$

The Hamilton's principle [14] is represented by

$$\int_{t_1}^{t_2} \delta(E_k - E_p + W)dt = 0, \quad (8)$$

where $t_1$ and $t_2$ are two time instants, $t_1 < t < t_2$ is the operating interval and $\delta$ denotes the variational operator, $E_k$ and $E_p$ are the kinetic and potential energies of the system respectively, $W$ denotes the virtual work done by nonconservative force acting on the system, including control force, damping and ocean disturbance. The principle states that the variation of the kinetic and potential energy plus the variation of work done by loads during any time interval $[t_1, t_2]$ must equal to zero. Applying the variation operator and integrating Eqs. (1), (3), and (7) by parts respectively and substituting $\delta y(x,t) = 0$ at $t = t_1, t_2$ into the Hamilton's principle Eq. (8), we obtain the governing equations of the system as

$$\rho\ddot{y}(x,t) - Ty''(x,t) + c\dot{y}(x,t) = f(x,t), \quad (9)$$

$\forall(x,t) \in (0,L)\times[0,\infty)$. Setting the terms with single integrals in Eq. (8) equal to zero, we obtain the boundary conditions of the system as

$$u_1(t) + d_1(t) - d_{s1}\dot{y}(L,t) - M\ddot{y}(L,t) - Ty'(L,t) = 0, \quad (10)$$

$$u_2(t) + d_2(t) - d_{s2}\dot{y}(0,t) - m\ddot{y}(0,t) + Ty'(0,t) = 0, \quad (11)$$

$\forall t \in [0,\infty)$.

*Remark 1:* The notations $(*)'$ and $(*)''$ representing the first order and second order derivative of $(*)$ with respect to x respectively, $(\dot{*})$ and $(\ddot{*})$ denoting the first and second order derivative of $(*)$ with respect to time t, respectively, etc. are used for clarity.

### *B. Ocean current disturbance*

The effects of a time-varying surface ocean current $U(x,t)$ on the ocean flexible structures can be modeled as a distributed load [15], [16]. The distributed load on a flexible cable, $f(x,t)$ can be expressed as a combination of a mean drag and an oscillating drag about the mean modeled as

$$f(x,t) = \frac{1}{2}\rho_s C_D(x,t)U(x,t)^2 D + A_D\cos(4\pi f_v(x,t)t + \theta), \quad (12)$$

where $C_D(x,t)$ is the drag coefficient respectively, $f_v$ is the shedding frequency, $\rho_s$ is the sea water density, $\theta$ is the phase angles, and $A_D$ is the amplitude of the oscillatory part of the drag force, typically 20% of the first term in $f(x,t)$ [16]. The non-dimensional vortex shedding frequency can be expressed as

$$f_v(x,t) = \frac{S_t U(x,t)}{D}, \quad (13)$$

where $S_t$ is the Strouhal number and $D$ is the cable outer diameter.

*Assumption 1:* For the distributed load $f(x,t)$ on the cable, the disturbance $d_1(t)$ on the vessel, the disturbance $d_2(t)$ on the payload, we assume that there exist constants $\bar{f} \in R^+$, $\bar{d}_1 \in R^+$ and $\bar{d}_2 \in R^+$, such that $|f(x,t)| \leq \bar{f}$, $\forall(x,t) \in [0,L] \times [0,\infty)$, $|d_1(t)| \leq \bar{d}_1$, $\forall(t) \in [0,\infty)$ and $|d_2(t)| \leq \bar{d}_2$, $\forall(t) \in [0,\infty)$. This is a reasonable assumption as the time-varying disturbances $f(x,t)$, $d_1(t)$ and $d_2(t)$ have finite energy and hence are bounded, i.e., $f(x,t) \in \mathcal{L}_\infty([0,L])$, $d_1(t) \in \mathcal{L}_\infty$ and $d_2(t) \in \mathcal{L}_\infty$.

### C. Preliminaries

For the convenience of stability analysis, we present the following lemmas and properties for the subsequent development.

*Lemma 1:* [17] Rayleigh-Ritz theorem: Let $A \in R^{n\times n}$ be a real, symmetric, positive-definite matrix; therefore, all the eigenvalues of $A$ are real and positive. Let $\lambda_{\min}$ and $\lambda_{\max}$ denote the minimum and maximum eigenvalues of $A$, respectively; then for $\forall x \in R^n$

$$\lambda_{\min}||x||^2 \leq x^T Ax \leq \lambda_{\max}||x||^2, \quad (14)$$

where $||\cdot||$ denotes the standard Euclidean norm.

*Lemma 2:* [1], [18] Let $\phi(x,t) \in R$ be a function defined on $x \in [0,L]$ and $t \in [0,\infty)$ that satisfies the boundary condition, we have

$$\phi(0,t) = 0, \quad \forall t \in [0,\infty), \quad (15)$$

then the following inequalities hold:

$$\phi^2 \leq L\int_0^L [\phi']^2 dx, \quad \forall x \in [0,L]. \quad (16)$$

*Property 1:* [19]:If the kinetic energy of the system (9) - (11), given by Eq. (1) is bounded $\forall t \in [0,\infty)$, then $\dot{y}(x,t)$, $\dot{y}'(x,t)$ and $\dot{y}''(x,t)$ are bounded $\forall(x,t) \in [0,L] \times [0,\infty)$.

*Property 2:* [19]: If the potential energy of the system (9) - (11), given by Eq. (3) is bounded $\forall t \in [0,\infty)$, then $y'(x,t)$ and $y''(x,t)$ are bounded $\forall(x,t) \in [0,L] \times [0,\infty)$.

## III. CONTROL DESIGN

The control objective is to design boundary control to position the subsea payload to the desired set-point position $p_d$ and stabilize the cable at the small neighborhood of its equilibrium position in the presence of the time-varying ocean disturbance. The control forces $u_1(t)$ and $u_2(t)$ are from the thruster in the vessel and the thruster attached in the subsea payload respectively. We propose the following control

$$u_1 = -k_v \dot{y}(L,t) - \mathrm{sgn}[\dot{y}(L,t)]\bar{d}_1, \quad (17)$$

$$u_2 = -k_p(y(0,t) - p_d) - k_s u_a - Ty'(0,t) - m\dot{y}'(0,t) + d_{s2}\dot{y}(0,t) - \mathrm{sgn}(u_a)\bar{d}_2, \quad (18)$$

where $\mathrm{sgn}(\cdot)$ denotes the signum function, $k_s$, $k_p$ and $k_v$ are design positive constants and the auxiliary signal $u_a$ is defined as

$$u_a = \dot{y}(0,t) + y'(0,t). \quad (19)$$

After differentiating the auxiliary signal Eq. (19), multiplying the resulting equation by $m$, and substituting boundary conditions, We have

$$m\dot{u}_a = -k_s u_a - k_p(y(0,t) - p_d) + d_2 - \mathrm{sgn(u_a)\bar{d}_2}. \quad (20)$$

*Remark 2:* The proposed boundary control does not require distributed sensing and all the signals in the boundary control can be measured by sensors or obtained by a backward difference algorithm. $y(L,t)$ and $y(0,t)$ can be sensed by two laser displacement sensors at the top and bottom boundary of the cable. $y'(0,t)$ can be measured by an inclinometer at the bottom boundary of the cable.

*Remark 3:* The control design is based on the distributed parameter model Eqs. (9) to (11), and the spillover problems associated with traditional truncated model-based approaches caused by ignoring high-frequency modes in controller and observer design are avoided.

Consider the Lyapunov function candidate

$$V = V_1 + V_2 + \eta \quad (21)$$

where the energy term $V_1$ and an auxiliary term $V_2$ and a small crossing term $\eta$ are defined as

$$V_1 = \frac{\beta}{2}\rho\int_0^L [\dot{y}]^2 dx + \frac{\beta}{2}T\int_0^L [y']^2 dx + \frac{\beta}{2}M[\dot{y}(L,t)]^2 + \frac{\beta k_p}{2}[y(0,t) - p_d]^2, \quad (22)$$

$$V_2 = \frac{1}{2}mu_a^2, \quad (23)$$

$$\eta = \alpha\rho\int_0^L (x - L)\dot{y}y' dx, \quad (24)$$

where $\alpha$ and $\beta$ are two positive weighting constants.

*Lemma 3:* The Lyapunov function candidate given by (21), can be upper and lower bounded as

$$0 \leq \lambda_1(V_1 + V_2) \leq V \leq \lambda_2(V_1 + V_2), \quad (25)$$

where $\lambda_1$ and $\lambda_2$ are two positive constants defined as

$$\lambda_1 = 1 - \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)}, \lambda_2 = 1 + \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)}. \quad (26)$$

provided

$$\alpha < \frac{\min(\beta\rho, \beta T)}{2\rho L}. \quad (27)$$

**Proof:** Substituting of Lemma 1 in [1] yields

$$|\eta| \leq \alpha\rho L\int_0^L ([y']^2 + [\dot{y}]^2)dx \leq \alpha_1 V_1, \quad (28)$$

where

$$\alpha_1 = \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)}. \quad (29)$$

Then, we obtain

$$-\alpha_1 V_1 \leq \eta \leq \alpha_1 V_1. \tag{30}$$

Considering $\alpha$ is a small positive weighting constant satisfying $0 < \alpha < \frac{\min(\beta\rho,\beta T)}{2\rho L}$, we can obtain

$$\alpha_2 = 1 - \alpha_1 = 1 - \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)} > 0, \tag{31}$$

$$\alpha_3 = 1 + \alpha_1 = 1 + \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)} > 1. \tag{32}$$

Then, we further have

$$0 \leq \alpha_2 V_1 \leq V_1 + \eta \leq \alpha_3 V_1, \tag{33}$$

Given the Lyapunov function candidate in Eq. (21), we obtain

$$0 \leq \lambda_1(V_1 + V_2) \leq V \leq \lambda_2(V_1 + V_2), \tag{34}$$

where $\lambda_1 = \min(\alpha_2, 1) = \alpha_2$ and $\lambda_2 = \max(\alpha_3, 1) = \alpha_3$ are positive constants. ■

*Lemma 4:* The time derivative of the Lyapunov function in (21) can be upper bounded with

$$\dot{V} \leq -\lambda V + \varepsilon, \tag{35}$$

where $\lambda$ and $\varepsilon$ are two positive constants.

**Proof:** Differentiate Eq. (21) with respect to time leads to

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{\eta}. \tag{36}$$

From the first term of the Eq. (36), we have

$$\begin{aligned}
\dot{V}_1 &= \beta T \int_0^L y'\dot{y}' dx + \beta M \ddot{y}(L,t)\dot{y}(L,t) \\
&\quad + \beta\rho \int_0^L \dot{y}\ddot{y} dx + \beta k_p (y(0,t) - p_d)\dot{y}(0,t) \\
&\leq -\frac{\beta T}{2}\left[[\dot{y}(0,t)]^2 + [y'(0,t)]^2\right] + \frac{\beta T}{2} u_a^2 \\
&\quad - \beta(k_v + d_{s1})[\dot{y}(L,t)]^2 - \beta(c - \delta_2)\int_0^L [\dot{y}]^2 dx \\
&\quad + \frac{\beta}{\delta_2}\int_0^L f^2 dx + \frac{\beta k_p}{2\delta_1}[y(0,t) - p_d]^2 \\
&\quad + \frac{\beta k_p \delta_1}{2}[\dot{y}(0,t)]^2,
\end{aligned} \tag{37}$$

where $\delta_1$ and $\delta_2$ are two positive constants. The second term of the Eq. (36) yields

$$\begin{aligned}
\dot{V}_2 &= m u_a \dot{u}_a, \\
&= -k_s u_a^2 - k_p u_a [y(0,t) - p_d] + d_2 u_a - u_a \mathrm{sgn}(\mathrm{u_a})\bar{\mathrm{d}}_2 \\
&\leq -k_s u_a^2 + k_p u_a^2 + k_p [y(0,t) - p_d]^2 \\
&\leq -k_s u_a^2 + k_p u_a^2 - k_p [y(0,t) - p_d]^2 \\
&\quad + 4k_p L \int_0^L [y']^2 dx + 4k_p p_d^2,
\end{aligned} \tag{38}$$

where (16) is employed. The third term of the Eq. (36) yields

$$\begin{aligned}
\dot{\eta} &= \alpha \int_0^L (x - L)y' \left[Ty'' + f - c\dot{y}\right] dx \\
&\quad + \alpha\rho \int_0^L (x - L)\dot{y}\dot{y}' dx \\
&\leq \frac{\alpha T L}{2}[y'(0,t)]^2 - \frac{\alpha T}{2}\int_0^L [y']^2 dx \\
&\quad + \frac{\alpha L}{\delta_3}\int_0^L f^2 dx + \alpha L \delta_3 \int_0^L [y']^2 dx + \frac{\alpha c L}{\delta_4}\int_0^L [\dot{y}]^2 dx \\
&\quad + \alpha c L \delta_4 \int_0^L [y']^2 dx + \frac{\alpha\rho L}{2}[\dot{y}(0,t)]^2 - \frac{\alpha\rho}{2}\int_0^L [\dot{y}]^2 dx.
\end{aligned} \tag{39}$$

where $\delta_3$ and $\delta_4$ are two positive constants. Substituting Eqs. (37), (38) and (39) into Eq. (21), we obtain

$$\begin{aligned}
\dot{V} &\leq -\left(\beta c + \frac{\alpha\rho}{2} - \beta\delta_2 - \frac{\alpha c L}{\delta_4}\right)\int_0^L [\dot{y}]^2 dx \\
&\quad - \left(\frac{\alpha T}{2} - 4k_p L - \alpha L \delta_3 - \alpha c L \delta_4\right)\int_0^L [y']^2 dx \\
&\quad - \beta(k_v + d_{s1})[\dot{y}(L,t)]^2 - \left(k_s - k_p - \frac{\beta T}{2}\right) u_a^2 \\
&\quad - \left(\frac{\beta T}{2} - \frac{\alpha\rho L}{2} - \frac{\beta k_p \delta_1}{2}\right)[\dot{y}(0,t)]^2 \\
&\quad - \left(\frac{\beta T}{2} - \frac{\alpha T L}{2}\right)[y'(0,t)]^2 \\
&\quad - k_p(1 - \frac{\beta}{2\delta_1})[y(0,t) - p_d]^2 \\
&\quad + \left(\frac{\beta}{\delta_2} + \frac{\alpha L}{\delta_3}\right)\int_0^L \bar{f}^2 dx + 4k_p p_d^2 \\
&\leq -\lambda_3(V_1 + V_2) + \varepsilon,
\end{aligned} \tag{40}$$

where the constants $k_s$, $k_v$, $k_p$, $\alpha$, $\beta$, $\delta_1$, $\delta_2$, $\delta_3$ and $\delta_4$ are chosen to satisfy the following conditions:

$$\alpha < \frac{\min(\beta\rho, \beta T)}{2\rho L}, \tag{41}$$

$$\frac{\beta T}{2} - \frac{\alpha\rho L}{2} - \frac{\beta k_p \delta_1}{2} \geq 0, \tag{42}$$

$$\frac{\beta T}{2} - \frac{\alpha T L}{2} \geq 0, \tag{43}$$

$$\sigma_1 = \beta c + \frac{\alpha\rho}{2} - \beta\delta_2 - \frac{\alpha c L}{\delta_4} > 0, \tag{44}$$

$$\sigma_2 = \frac{\alpha T}{2} - 4k_p L - \alpha L \delta_3 - \alpha c L \delta_4 > 0, \tag{45}$$

$$\sigma_3 = \beta(k_v + d_{s1}), \tag{46}$$

$$\sigma_4 = 1 - \frac{\beta}{2\delta_1} > 0, \tag{47}$$

$$\sigma_5 = k_s - k_p - \frac{\beta T}{2} > 0, \tag{48}$$

$$\lambda_3 = \min\left(\frac{2\sigma_1}{\beta\rho}, \frac{2\sigma_2}{\beta T}, \frac{2\sigma_3}{\beta M}, \frac{2\sigma_4}{\beta}, \frac{2\sigma_5}{m}\right) > 0, \tag{49}$$

$$\varepsilon = \left(\frac{\beta}{\delta_2} + \frac{\alpha L}{\delta_3}\right)\int_0^L \bar{f}^2 dx + 4k_p p_d^2 \in \mathcal{L}_\infty. \tag{50}$$

From Ineqs. (34) and (40) we have

$$\dot{V} \leq -\lambda V + \varepsilon, \tag{51}$$

where $\lambda = \lambda_3/\lambda_2$ and $\varepsilon$ are two positive constants. ■

With the above lemmas, the boundary control design for the flexible marine installation system subjected to ocean current disturbance can be summarized in the following theorem.

*Theorem 1:* For the system dynamics described by (9) and boundary conditions (10) - (11), under Assumption 1, and the boundary control (17) and (18), given that the initial conditions are bounded, the transverse reflection $w(x,t)$ of the closed loop system is uniformly bounded, and the system boundary error signal $e(t) = y(0,t) - p_d$ will remain within the compact set $\Omega$ defined by

$$\Omega := \{e \in R| \quad |e| \leq D\} \tag{52}$$

where $D = \sqrt{\frac{2}{\beta k_p \lambda_1}\left(V(0) + \frac{\varepsilon}{\lambda}\right)}$.

**Proof:** Multiplying Eq. (35) by $e^{\lambda t}$ yields

$$\frac{\partial}{\partial t}(V e^{\lambda t}) \leq \varepsilon e^{\lambda t}. \tag{53}$$

Integration of the above inequality, we obtain

$$V \leq \left(V(0) - \frac{\varepsilon}{\lambda}\right) e^{-\lambda t} + \frac{\varepsilon}{\lambda} \leq V(0) e^{-\lambda t} + \frac{\varepsilon}{\lambda} \in \mathcal{L}_\infty. \tag{54}$$

which implies $V$ is bounded. Utilizing Ineq. (16) and Eq. (22), we have

$$\frac{\beta}{2L} T w^2(x,t) \leq \frac{\beta}{2} T \int_0^L [w'(x,t)]^2 dx$$
$$= \frac{\beta}{2} T \int_0^L [y'(x,t)]^2 dx \leq V_1 \leq V_1 + V_2 \leq \frac{1}{\lambda_1} V. \tag{55}$$

Appropriately rearranging the terms of the above Ineq. (55), we obtain $w(x,t)$ is uniformly bounded as follows

$$|w(x,t)| \leq \sqrt{\frac{2L}{\beta T \lambda_1}\left(V(0) e^{-\lambda t} + \frac{\varepsilon}{\lambda}\right)}. \tag{56}$$

Combining Eq. (22) and Ineq. (51) yields

$$\frac{\beta k_p}{2}[y(0,t) - p_d]^2 \leq V_1 \leq V_1 + V_2 \leq \frac{1}{\lambda_1} V \in \mathcal{L}_\infty. \tag{57}$$

$$|y(0,t) - p_d| \leq \sqrt{\frac{2}{\beta k_p \lambda_1}\left(V(0) e^{-\lambda t} + \frac{\varepsilon}{\lambda}\right)}. \tag{58}$$

■

*Remark 4:* In the above analysis, the deflection of the cable $w(x,t)$ can be made arbitrarily small provided that the design control parameters are appropriately selected. By choosing the proper values of $\alpha$ and $\beta$, it is shown that the increase in the control gains $k_v$ and $k_s$ will result in a larger $\sigma_3$ and $\sigma_5$, which will lead a greater $\lambda_3$. Then the value of $\lambda$ will increase, which will reduce the size of $\Omega$ and bring a better vibration suppression performance. Similarly, it is easily seen that the increase in the control gains $k_v$, $k_s$ and $k_p$ will result in a better tracking performance.

## IV. Numerical Simulations

Simulations for a marine installation system under ocean disturbance are carried out to demonstrate the effectiveness of the proposed boundary control Eq. (17) and Eq. (18). The cable, initially at rest, is excited by a distributed transverse disturbance due to ocean current. The corresponding initial conditions of the marine installation system system are given as $y(x,0) = \frac{\partial y(x,0)}{\partial t} = 0$. The system parameters are given in Table 1.

Table 1: parameters of the marine installation system

| Parameter | Description | Value |
|---|---|---|
| $L$ | Length of the cable | $1000m$ |
| $D$ | Diameter of the cable | $0.05m$ |
| $M$ | Mass of the vessel | $9.6 \times 10^7 kg$ |
| $m$ | Mass of the payload | $4 \times 10^5 kg$ |
| $d_{s1}$ | Damping of the vessel | $9 \times 10^7 NS/m$ |
| $d_{s2}$ | Damping of the payload | $2 \times 10^5 NS/m$ |
| $T$ | Tension | $4.00 \times 10^6 N$ |
| $\rho$ | Mass per unit length | $8.02 kg/m$ |
| $\rho_s$ | Sea water density | $1024 kg/m^3$ |
| $c$ | Damping coefficient e | $1 NS/m^2$ |
| $p_d$ | Desired set-point position | $50m$ |

In our simulation experiments, the ocean surface current velocity $U(t)$ is modeled as a mean flow with worst case sinusoidal components to simulate the cable with a mean deflected profile. The sinusoids have frequencies of $\omega_i = \{0.867, 1.827, 2.946, 4.282\}$, for $i = 1$ to 4, corresponding to the four natural modes of vibration of the cable. The surface current $U(t)$ is expressed as

$$U(t) = \bar{U} + U' \sum_{i=1}^{4} \sin(\omega_i t), \quad i = 1, 2, \ldots, 4, \tag{59}$$

where $\bar{U} = 2ms^{-1}$ is the mean flow current and $U' = 0.2$ is the amplitude of the oscillating flow. In the simulation, we assume that the full current load is applied from $x = 1000m$ to $x = 0m$ and thereafter linearly decline to zero at the ocean floor, $x = 0$, to obtain a depth dependent ocean current profile $U(x,t)$. The distributed load $f(x,t)$ is generated by Eq. (12) with $C_D = 1$, $\theta = 0$ and $S_t = 0.2$. The disturbance $d_1(t)$ on the vessel is generated as follows:

$$d_1(t) = [3 + 0.8\sin(0.7t) + 0.2\sin(0.5t) + 0.2\sin(0.9t)] \times 10^6 \tag{60}$$

The disturbance $d_2(t)$ on the payload is given as follows:

$$d_2(t) = [3 + 0.8\sin(0.7t) + 0.2\sin(0.5t) + 0.2\sin(0.9t)] \times 10^4 \tag{61}$$

Displacement of the cable for free vibration, i.e. $u_1(t) = u_2(t) = 0$, exposed to ocean disturbance is shown in Fig. 2. It is clear that the system is unstable and the vibration of the cable is quite large. Displacement of the cable with boundary control Eqs. (17) and (18), by choosing $k_v = 2 \times 10^7$, $k_p = 1.5 \times 10^5$, $k_s = 1 \times 10^6$ under ocean disturbance is shown in Fig. 3. The corresponding boundary control input are shown in Fig. 4. Fig. 3 illustrates that the proposed boundary control is able to bring the subsea payload to the desired position $p_d = 50m$ and stabilize the cable at the small neighborhood of its equilibrium position.

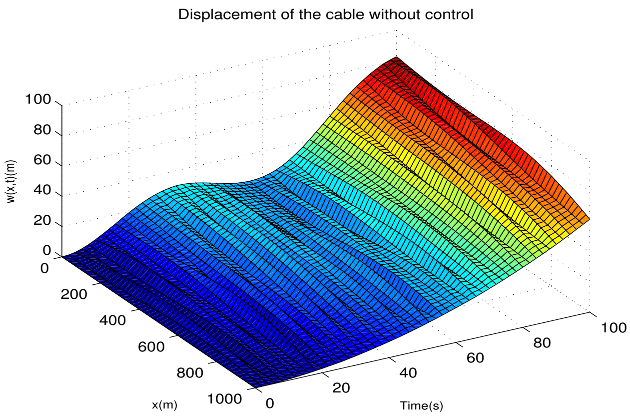


Fig. 2. Displacement of the cable without control.

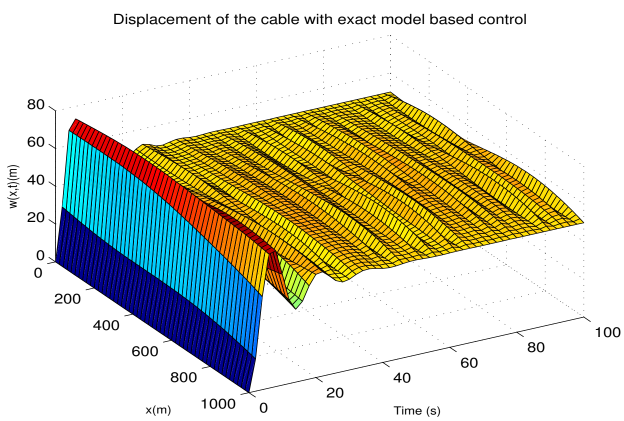


Fig. 3. Displacement of the cable with boundary control.

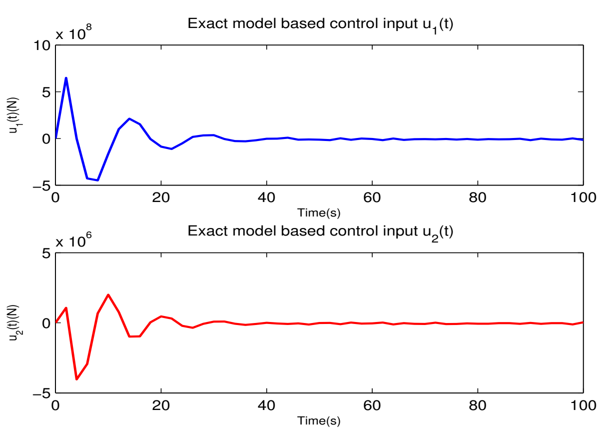


Fig. 4. Model based control input $u_1(t)$ and $u_2(t)$.

## V. Conclusion

In this paper, both positioning control and vibration suppression have been investigated for a flexible marine installation system subjected to the ocean disturbance. The control is designed based on the original infinite dimensional model (PDE), and thus the spillover instability phenomenon is eliminated. The proposed schemes offer implementable design procedures for the control of marine installation systems since all the signals in the control can be measured by sensors or calculated by a backward difference algorithm. The simulation results illustrate that the proposed control is able to position the payload to the desired set-point and suppress the vibration of the cable with a good performance.

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