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Neural Network Control for Non-affine Nonlinear Systems

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Abstract— **Recently, adaptive neural control has been attracting an increasing attention for nonlinear unknown dynamic systems [1][2]. This paper is dedicated to the discussions on a few techniques in the design of adaptive neural network control for non-affine systems which are known to be difficult to control. The techniques include implicit function theorem based neural control for classes of the non-affine systems in Brunovsky form, implicit function theorem with backstepping design for classes of the non-affine systems in pure-feedback form, and pseudo inverse control. This paper is aimed to provide an overview of the state of art of stable control design for non-affine systems using neural network parametrization, and to list the advantages and disadvantages of neural network control.**

I. INTRODUCTION

In the panel discussion, while other colleagues discuss the development of intelligent control, the gap between fuzzy and nonlinear controls, intelligent control of robots, future perspectives of intelligent control, this paper is focused on adaptive neural network control of non-affine nonlinear systems.

Though much research has been dedicated to the control of affine nonlinear systems, there are few research results available in the literature for non-affine nonlinear systems, even though many practical systems, e.g., chemical reactions and PH neutralization, are inherently nonlinear, whose input variables may enter in the systems nonlinearly as described by the general form:

$$\begin{aligned} \dot{x} &= f(x, u) \\ y &= h(x) \end{aligned} \tag{1}$$

where $x \in R^n$ is the state vector, u is the input and y is the output, and $f(\cdot, \cdot)$ and $h(\cdot)$ are unknown smooth vector fields. This paper is dedicated to the discussions of the few techniques in the design of adaptive neural network control for non-affine systems (1), which include implicit function theorem for classes of the non-affine systems in Brunovsky form [3][4][5], implicit function theorem with backstepping design for classes of the non-affine systems in pure-feedback form [6][7], and pseudo inverse control [8][9]. We can see that the designs are only made possible through function approximations, neural networks (NNs) in particular, in this paper.

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II. IMPLICIT FUNCTION THEOREM BASED CONTROL

A. Non-affine Nonlinear Systems in Brunovsky Form

For classes of the non-affine systems in Brunovsky form [3][4][5], an ideal implicit feedback linearization control (IFLC) is established using implicit function theorem under the assumption of non-singularity on the control gain. Then neural networks and mean value theorem are applied to construct this IFLC to realize approximate linearization. Compared with the traditional exact linearization techniques, the proposed adaptive NN controls do not need to search for an explicit controller to cancel the nonlinearities of the systems. In fact, even though functions $f(x, u)$ and $h(x)$ in (1) are known exactly, there does not always exist an explicit controller for feedback linearization. Instead of solving the implicit function to obtain the ideal control, NNs are applied to reconstruct the ideal IFLC for achieving approximation feedback linearization. The closed-loop system is proven to be semi-globally uniformly ultimately bounded (SGUUB), without requirements for persistent excitation (PE) condition and off-line training.

B. Non-affine Nonlinear Systems in Pure-feedback Form

The pure-feedback systems represent a more general class of triangular systems which have no affine appearance of the variables to be used as virtual controls. As indicated in [10], the cascade and non-affine properties make it quite difficult to find the explicit virtual controls and the actual control to stabilize the pure-feedback systems. The main difficulty for adaptive neural control of pure-feedback systems lies in that, when neural networks are used to approximate some desired virtual control α_i^* and desired practical control u^* in the backstepping design, it will generally involve the NN approximation of functions of u and $\dot{u}$. As the NN approximation is one part of control u, this will lead to a circular construction of the practical control. In [6], the circularity problem is avoided for much simpler pure-feedback systems. In particular, implicit function theorem is exploited to assert the existence of the continuous desired virtual controls, then neural network approximators are used to approximate the continuous desired virtual controls and desired practical control. With the help of NN approximation, there is no need to solve the implicit function for the explicit virtual controls and the practical control to cancel the unknown functions in backstepping design. The developed adaptive NN control schemes can make all the signals in the closed-loop achieve SGUUB. In [7], an "ISS-modular" approach for adaptive neural control of the completely non-affine pure-feedback system is presented. By achieving the

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ISS-modularity of the interconnected control module and estimation module, the difficult problem of non-affine pure-feedback system control is resolved by combining adaptive neural design with the backstepping method, ISS analysis and the small-gain theorem. The employment of ISS analysis and the small gain theorem avoids the construction of an overall Lyapunov function for the closed-loop system, and subsequently overcomes the circular design problem in NN control of pure-feedback systems. It provides an effective way for controlling non-affine nonlinear systems.

III. Pseudo Inverse Control

Adaptive output feedback control of non-affine nonlinear systems using neural networks is considered under the assumption that the system is feedback linearizable in [8][9]. The pseudo inverse control includes three parts: (i) the rth derivative of the reference signal, which is introduced as a feed-forward term; (ii) the output of a stabilizing linear dynamic compensator for the linearized dynamics when the modeling error is equal to zero; and (iii) the adaptive control to cancel the modeling error using neural networks. The input vector to the NN is composed of current and past input/output data. The control architecture is adaptive to both parametric uncertainty and unmodeled dynamics. The methodology is applicable to minimum phase observable and stabilizable systems of unknown but bounded dimension, as long as the relative degree is known. The approaches in [8] and [9] share some common features, but differ significantly in the definition of the error signal used by the NN weight adaption laws. The stability analysis is carried out with linearly parameterized NNs and nonlinearly parameterized NNs respectively. Ultimate boundedness of the error signals are shown through Lyapunov's direct method.

IV. Restrictions of Adaptive Neural Control

Though adaptive neural control has many advantages over conventional adaptive control, it also has its own restrictions as detailed below:

(i) Because of the couplings of unknown dynamics and complex neural control, we would not be able to construct design for closed loop nonlinear systems with pre-specified design performance as for the linear systems where we can quantify the overshoot, rising time, settling time, etc., easily.

(ii) There is a trade-off between NN complexity and its good approximation performance. As it is known, the larger the number of NN nodes is, the smaller the approximation error will be. However, NN with larger hidden node number usually makes the control more complex and increase the computation burden for the control system.

(iii) The control performance of adaptive neural control is more conservative than conventional adaptive control in the sense that, only SGUUB results can be obtained for adaptive neural control, because NN approximation is valid only on some compact sets, while for the conventional adaptive control, the parametric representation is valid globally.

(iv) The upper bounds of the NN approximation errors and the ideal NN weights are fictitious and unknown constants for a given NN, which further make control design difficult. For the best control performance, ideally, we should know the ideal weights, and upper bounds of the approximation errors. Otherwise, we are forced to use adaptive control, learning control, or robust control to design much more conservative stable systems with guarantee of good transient responses.

V. Conclusion and Future Work

In conclusion, this paper provided an overview of the state of art of stable control design for non-affine systems using neural network parametrization, and listed the advantages and disadvantages of adaptive neural control over conventional model-based adaptive control. Several techniques, including implicit function theorem based neural control for classes of the non-affine systems in Brunovsky form, implicit function theorem with backstepping design for classes of the non-affine systems in pure-feedback form, and pseudo inverse control, have been discussed to solve problems of nonlinear systems that normally fall outside the capabilities of traditional adaptive control.

Further research will be conducted for multi-inputs and multi outputs (MIMO) systems based on the foundation of [11], and classes of non-affine non-minimum phase systems extended from [5].

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