Region Tracking Control for Multi-Agent Systems with High-Order Dynamics

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***Abstract*— In this paper, decentralized controllers are developed to drive a swarm of mobile agents with high-order ($m > 2$) nonlinear dynamics in strict feedback form into a moving target region while avoiding collisions among themselves. At the same time, the connectivity of the communication graph remains for all time. It is important to consider coordination of multiple high-order agent dynamics which generalize the existing simple single-integrator/double-integrator ones because, in practice, we need to incorporate actuator dynamics into the vehicle dynamics in order to achieve better performance, thus increasing the order of the system dynamics. The control design is based on a fusion of potential functions, backstepping technique and Lyapunov synthesis. The presence of parametric uncertainties is handled by adaptive control techniques. Simulation studies have been carried out to verify the effectiveness of the proposed approach.**

I. Introduction

During the last two decades, the research of multi-agent systems has received a surge of attention of researchers from different disciplines and has been extensively investigated in numerous applications. Various approaches have been proposed for coordination of multi-agent systems, including leader-follower [1], [2], [3], virtual structure [4], [5], behavior-based [6], [7], [8], navigation functions [9], control Lyapunov functions [10], artificial potentials based [11], [12], [13], [14], [15].

Most of the agent dynamics investigated are either simple single/double-integrator ones, or vehicle dynamics, that can be converted to double-integrator dynamics via feedback linearization. In practice, in order to achieve better performance, we need to incorporate actuator dynamics into the vehicle dynamics, thus increasing the order of the system dynamics. For example, to actively minimize torsional vibrations within the propulsive shafting system, a marine shafting system is modeled as a chained multiple mass-spring system [16], [17]. As a result, the whole marine vessel dynamics is described by a high-order nonlinear system in strict feedback form. However, in most literature about cooperative control of multiple marine vessels, only first-order kinematic models or second-order dynamic models without actuator dynamics were considered [18], [19]. This motivates the control of multi-agents with high-order dynamics, such as [20], [21], where multiple high-order linear dynamical agents were treated. In this paper, we formulate the high-order nonlinear agent dynamics in strict feedback form, which represents a more general class of agents and is feasible to be handled by backstepping techniques [22], [23].

Another motivation is from [24], where the region control concept was proposed for individual robots. It has been shown that region reaching tasks consume less energy and result in a faster motion as compared to conventional setpoint control. In [25], region following formation control was developed to achieve that all the robots stay within a moving region as a group. However, the collisions between agents and the limitation of sensing ranges have not been taken into consideration. Motivated by [25], a decentralized multi-agent swarming control with limited sensing ranges was developed based on first-order kinematic models in [26], where all the agents converge into a moving target region, while avoiding collisions among themselves.

In this paper, we extend the work [26] to a more general case where the agent dynamics are represented as nonlinear high-order systems in strict feedback form due to the presence of actuator dynamics. Additionally, parametric uncertainties in the system model are considered as well. The goal is for all the agents to converge to the moving target region without collisions, regardless of the exact location for each agent. Two kinds of potential functions for each agent, i.e., the target potential function and the collision avoidance potential function are included to achieve this objective. Furthermore, to preserve the connectivity of the communication graph for all the time, we introduce the barrier potential functions as well. By ensuring boundedness of the barrier Lyapunov function, it is ensured that the communication graph remains connected for all time if and only if the communication graph is initially connected.

The organization of this paper is as follows. The problem formulation and preliminaries are presented in Section IIIn Section III, the decentralized control to coordination of multiple mobile agents is proposed based on artificial potential functions, backstepping and adaptive control techniques with the presence of parametric uncertainties. The closed-loop system stability is investigated using the LaSalle Yoshizawa Theorem. Extensive simulation studies are shown to demonstrate the effectiveness of the proposed approach in Section IV. Finally, the conclusion is followed in Section V.

II. Problem Formulation and Preliminaries

A. Agent Dynamics

We consider a multi-agent system consisting of N mobile agents and moving on the 2-D space, with similar dynamics

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TABLE I
NOMENCLATURE

$\mathbb{R}$	the field of real numbers;
$\mathbb{R}^n$	the linear space of n-dimensional vectors with elements in $\mathbb{R}$;
$\mathbb{R}^{n\times m}$	the set of $n \times m$-dimensional matrices with elements in $\mathbb{R}$;
$\|x\|$	the Euclidean vector norm of a vector x;
$q_{i,j}$	the states of agent i;
$\bar{q}_{i,j}$	the augmented states of agent i $\bar{q}_{i,j} = [q_{i,1}^T, q_{i,2}^T, ..., q_{i,j}^T]^T \in \mathbb{R}^{nj}$;
$\Phi_j(\cdot)$	the known nonlinear function matrices;
θ_i	the vector of uncertain constant parameters;
$\hat{\theta}_i$	the estimate of θ_i;
$\tilde{\theta}_i$	= $\hat{\theta}_i - \theta_i$;
F_j	smooth function vectors;
G_j	smooth function matrices;
u_i	the input agent i;
y_i	the output vector of agent i;
R	the radius of the communication range of agent;
r	the radius of the danger range of agent;
$\mathbb{G}_i$	the set of indices for those agents within communication range of agent i;
$\mathbb{H}_i$	the set of indices for those agents within the danger range of agent i;
Ω	the common moving target region;
q_0	the position of the center of the target region Ω;
$q_{i,1}$	the position of agent i;
r_0	the radius of the target region Ω;
$\tilde{q}_{i,0}$	the vector from agent i at $q_{i,1}$ to the center of the target region Ω at q_0;
$\tilde{q}_{ij}$	the vector from agent i at $q_{i,1}$ to agent j at $q_{j,1}$;
$f_{i,0}(\cdot)$	the target function of agent i;
$P_{i,0}(\cdot)$	the target potential function of agent i;
$P_{i,j}(\cdot)$	the collision avoidance potential function of agent i with another agent j.
$Q_{i,j}(\cdot)$	the barrier potential function of agent i with another agent j.

in strict feedback form as follows:

$$\begin{aligned} \dot{q}_{i,j} &= F_{i,j}(\bar{q}_{i,j}) + G_{i,j}(\bar{q}_{i,j})q_{i,j+1} \\ \dot{q}_{i,m} &= F_{i,m}(\bar{q}_{i,m}) + G_{i,m}(\bar{q}_{i,m})u_i \end{aligned} \tag{1}$$

where $q_{i,j} \in \mathbb{R}^2$, $i = 1, 2, ..., N$, $j = 1, 2, ..., m$ are the states of i-th agent, $\bar{q}_{i,j} = [q_{i,1}^T, q_{i,2}^T, ..., q_{i,j}^T]^T \in \mathbb{R}^{2j}$, and $q_{i,1} \in \mathbb{R}^2$ is the position vector of i-th agent; $F_{i,j} \in \mathbb{R}^{2\times 1}$ and $G_{i,j} \in \mathbb{R}^{2\times 2}$ are smooth function vectors and matrices respectively; and $u_i \in \mathbb{R}^2$ is the input of agent i. The nonlinear function vectors $F_{i,j}(\bar{q}_{i,j})$ are uncertain and satisfy the following linear-in-the-parameters (LIP) condition:

$$F_{i,j}(\bar{q}_{i,j}) = \Phi_{i,j}(\bar{q}_{i,j})\theta_i \tag{2}$$

where $\Phi_{i,j}(\bar{q}_{i,j}) \in \mathbb{R}^{2\times r}$ are known nonlinear function matrices, and $\theta_i \in \mathbb{R}^r$ is a vector of uncertain constant parameters.

Each agent i has a communication range, which is centered at the agent and has a radius R. Moreover, we use $\mathbb{G}_i$ to denote the set of indices for those agents having communication with agent i. Inter-agent communication is achieved by a communication graph $\mathcal{G}$.

Definition 2.1: The communication graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is an undirected graph that consists of a set of vertices $\mathcal{V} = \{1, ..., N\}$ indexed by the group members, and a set of edges, $\mathcal{E} = \{(i,j) \in \mathcal{V} \times \mathcal{V} \bigm| i \in \mathbb{G}_j\}$ containing pairs of nodes that represent inter-agent communication specifications.

Assumption 2.1: The communication graph $\mathcal{G}$ is a directed graph and connected initially.

Remark 2.1: In this paper, we focus on the region tracking problem for a swarm of mobile agents whose dynamics are governed by nonlinear systems in strict feedback form as (1), motivated by the fact that many practical systems are subjected to this form, such as mobile robots [27] -[32], autonomous underwater vehicles(AUVs)[33]-[35] and unmanned aerial vehicles (UAVs)[36]-[38].

Our objective is to design a decentralized control u_i for each agent i with high-order dynamics such that all the agents will converge to a common moving target region, without collisions between any agents in the group. At the same time, the connectivity of the communication graph remains for all time. It means that if the agents are initially located within the communication zone of an agent, they remain within this area for all time. Therefore, the set $\mathbb{G}_i$ can be defined as the set that agent i can communicate when it is located at its initial position, $q_{i,1}(0)$:

$$\mathbb{G}_i = \{j \in V, j \neq i \bigm| \|q_{i,1}(0) - q_{j,1}(0)\| \leq R\} \tag{3}$$

The common target region Ω is considered as a circle centered around the point q_0 with radius r_0, which can be expressed as

$$\Omega = \{q_{i,1} \in \mathbb{R}^2 \mid f_{i,0}(\tilde{q}_{i,0}) = \|\tilde{q}_{i,0}\|^2 - r_0^2 \leq 0\}, \tag{4}$$

where $\tilde{q}_{i,0} = q_{i,1} - q_0$, $q_{i,1}$ and q_0 are the positions of the agent i and the center of the target region respectively, $f_{i,0}(\cdot) : \mathbb{R}^2 \to \mathbb{R}$ is the target function of agent i.

Assumption 2.2: The target region is big enough to accommodate all agents and their own communication ranges.

B. Potential Functions

1) Target Potential Functions: In this paper, we choose the following target potential function $P_{i,0}(\tilde{q}_{i,0}) : \mathbb{R}^2 \to \mathbb{R}$ for agent i:

$$P_{i,0}(\tilde{q}_{i,0}) = \begin{cases} 0, & q_{i,1} \in \Omega \\ \frac{C_i}{2} f_{i,0}^2(\tilde{q}_{i,0}), & q_{i,1} \notin \Omega \end{cases} \tag{5}$$

where C_i is a positive constant.

Property 1: The target potential function $P_{i,0}(\cdot)$ in (5) satisfies the following properties:

(i) If $q_{i,1} \in \Omega$, then $P_{i,0} = 0$; if $q_{i,1} \notin \Omega$, then $P_{i,0} > 0$.
(ii) If $q_{i,1} \notin \Omega$, $P_{i,0}$ is monotonically increasing with $\|\tilde{q}_{i,0}\|$, and $P_{i,0} \to \infty$ as $\|\tilde{q}_{i,0}\| \to \infty$.
(iii) $P_{i,0}$ is continuously differentiable with respect to $\tilde{q}_{i,0}$.

2) Collision Avoidance Potential Functions: To achieve the collision avoidance among agents, we define a danger range for each agent, which is centered at the agent and has a radius r, where $0 < r < R$. We use $\mathbb{H}_i$ to denote the set of indices for those agents within the danger range of agent i. Since $0 < r < R$, we know that $\mathbb{H}_i \subset \mathbb{G}_i$. Hence,

$$\mathbb{H}_i = \{j \in \mathbb{G}_i \;\Big|\; \|\tilde{q}_{i,j}\| \le r\} \tag{6}$$

where $\tilde{q}_{i,j} = q_{i,1} - q_{j,1}$, $q_{i,1}$ and $q_{j,1}$ are the positions of agent i and agent j respectively.

Then, we choose the following collision avoidance potential function $P_{i,j}(\tilde{q}_{i,j}) : \mathbb{R}^2 \to \mathbb{R}$ for agent i:

$$P_{i,j}(\tilde{q}_{i,j}) = \begin{cases} 0, & \|\tilde{q}_{i,j}\| > r \\ \frac{C_{i,j}}{2}\left(\log \frac{r^2}{\|\tilde{q}_{i,j}\|^2}\right)^2, & \|\tilde{q}_{i,j}\| \le r \end{cases} \tag{7}$$

where $C_{i,j} = C_{j,i}$ is a positive constant.

Property 2: The collision avoidance potential function $P_{i,j}(\cdot)$ in (7) satisfies the following properties:

(i) If $\|\tilde{q}_{i,j}\| > r$, then $P_{i,j} = 0$; if $\|\tilde{q}_{i,j}\| \le r$, then $P_{i,j} > 0$.
(ii) If $\|\tilde{q}_{i,j}\| \le r$, $P_{i,j}$ is monotonically increasing with the decreasing of $\|\tilde{q}_{i,j}\|$, and $P_{i,j} \to \infty$ as $\|\tilde{q}_{i,j}\| \to 0$.
(iii) $P_{i,j}$ is continuously differentiable with respect to $\tilde{q}_{i,j}$, $\forall \|\tilde{q}_{i,j}\| \in (0, +\infty)$.

3) Barrier Potential Functions: To preserve the connectivity of the communication graph for all the time, i.e., if the communication graph is initially connected, then it remains connected for all time, we define the barrier potential functions as follows:

$$Q_{i,j} = \frac{C'_{i,j}}{2} \log \frac{R^2}{R^2 - \|\tilde{q}_{i,j}\|^2} \tag{8}$$

where $C'_{i,j} = C'_{j,i}$ is a positive constant, $\|\tilde{q}_{i,j}\| \in [0, R)$, and $\log(\cdot)$ denotes the natural logarithm of $\cdot$.

Property 3: The barrier potential function $Q_{i,j}(\cdot)$ in (8) satisfies the following properties:

(i) $Q_{i,j} = 0$, when $\|\tilde{q}_{i,j}\| = 0$.
(ii) $Q_{i,j}$ is monotonically increasing with $\|\tilde{q}_{i,j}\|$ on $\|\tilde{q}_{i,j}\| \in [0, R)$. And $Q_{i,j} \to \infty$ as $\|\tilde{q}_{i,j}\| \to R$.
(iii) Q_{ij} is continuous and differentiable with respect to $\tilde{q}_{i,j}$, $\forall \|\tilde{q}_{i,j}\| \in [0, R)$.

Assumption 2.3: The states of the moving target region, $q_0(t)$ and its time derivatives up to the mth order are continuous and bounded.

Assumption 2.4: The control gain matrices $G_{i,j}$, $i = 1, 2, ..., N$, $j = 1, 2, ..., m$ are known and nonsingular.

III. Control Design and Stability Analysis

In this section, we will design the decentralized control u_i for each agent i with the dynamics in strict feedback form (1) to ensure that all the agents can converge to a common moving target region, without collisions between any agents in the group. Adaptive backstepping techniques are adopted to accommodate parametric uncertainty in the nonlinear function vectors $F_{i,j}(\bar{q}_{i,j})$. By employing target potential functions, collision avoidance potential functions and barrier potential functions in the first step of backstepping, we can guarantee region convergency without collisions and the connectivity of the communication graph for all the time. Subsequent steps are based on quadratic Lyapunov functions and follow the standard backstepping procedures in [22]. Since adaptive backstepping design is standard, the detailed procedures are omitted here for concise presentation. Interested readers are referred to [22]. Denote the error coordinates $z_{i,1} = \tilde{q}_{i,0} = q_{i,1} - q_0$ and $z_{i,\rho} = q_{i,\rho} - \alpha_{i,\rho-1}$, $\rho = 2, ..., m$, where $\alpha_{i,\rho-1}$ is a stabilizing function vector to be designed. Consider the following general potential function and Lyapunov function candidates:

$$\begin{aligned} V_1 &= \sum_{i=1}^{N} P_{i,0}(z_{i,1}) + \sum_{i=1}^{N} \sum_{j\in\mathbb{H}_i} P_{i,j}(\tilde{q}_{i,j}) \\ &\quad + \sum_{i=1}^{N} \sum_{j\in\mathbb{G}_i} Q_{i,j}(\tilde{q}_{i,j}) + \sum_{i=1}^{N} \frac{1}{2}\tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i \\ V_\rho &= V_{\rho-1} + \sum_{i=1}^{N} \frac{1}{2} z_{i,\rho}^T z_{i,\rho}, \quad \rho = 2, ..., m-1 \\ V_m &= V_{m-1} + \sum_{i=1}^{N} \frac{1}{2} z_{i,m}^T z_{i,m} \end{aligned} \tag{9}$$

where $\Gamma_i = \Gamma_i^T > 0$, and $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$ is the error between θ_i and its estimate, $\hat{\theta}_i$. Consider the stabilizing functions, control law, and adaptation law as follows

$$\begin{aligned} \alpha_{i,1} &= G_{i,1}^{-1}(q_{i,1})\Big\{-\Phi_{i,1}(q_{i,1})\hat{\theta}_i + \dot{q}_0 \\ &\quad -\kappa_{i,1}\Bigg[\frac{\partial P_{i,0}(z_{i,1})}{\partial q_{i,1}} + 2\sum_{j\in\mathbb{H}_i} \frac{\partial P_{i,j}(\tilde{q}_{i,j})}{\partial q_{i,1}} \\ &\quad +2\sum_{j\in\mathbb{G}_i} \frac{\partial Q_{i,j}(\tilde{q}_{i,j})}{\partial q_{i,1}}\Bigg]\Big\} \end{aligned} \tag{10}$$

$$\begin{aligned} \alpha_{i,2} &= G_{i,2}^{-1}(\bar{q}_{i,2})\Big\{-\kappa_{i,2} z_{i,2} - G_{i,1}^T(q_{i,1})\Bigg[\frac{\partial P_{i,0}(z_{i,1})}{\partial q_{i,1}} \\ &\quad +2\sum_{j\in\mathbb{H}_i} \frac{\partial P_{i,j}(\tilde{q}_{i,j})}{\partial q_{i,1}} + 2\sum_{j\in\mathbb{G}_i} \frac{\partial Q_{i,j}(\tilde{q}_{i,j})}{\partial q_{i,1}}\Bigg] \\ &\quad -\Big[\Phi_{i,2}(\bar{q}_{i,2}) - \Big(\frac{\partial \alpha_{i,1}}{\partial q_{i,1}}\Big)^T \Phi_{i,1}(q_{i,1})\Big]\hat{\theta}_i \\ &\quad +\Big(\frac{\partial \alpha_{i,1}}{\partial q_{i,1}}\Big)^T G_{i,1}(q_{i,1}) q_{i,2} \\ &\quad +\sum_{j\in\mathcal{G}_i} \Big(\frac{\partial \alpha_{i,1}}{\partial q_{j,1}}\Big)^T [\Phi_1(q_{j,1})\hat{\theta}_j + G_1(q_{j,1}) q_{j,2}] \\ &\quad +\sum_{j=0}^{1} \Big(\frac{\partial \alpha_{i,1}}{\partial q_0^{(j)}}\Big)^T q_0^{(j+1)} + \Big(\frac{\partial \alpha_{i,1}}{\partial \hat{\theta}_i}\Big)^T \Gamma_i \tau_{i,2}\Big\} \end{aligned} \tag{11}$$

$$\begin{aligned} \alpha_{i,\rho} &= G_{i,\rho}^{-1}(\bar{q}_{i,\rho})\Big\{-\kappa_{i,\rho} z_{i,\rho} - G_{i,\rho-1}^T(\bar{q}_{i,\rho-1}) z_{i,\rho-1} \\ &\quad -\Big[\Phi_{i,\rho}(\bar{q}_{i,\rho}) - \sum_{k=1}^{\rho-1}\Big(\frac{\partial \alpha_{i,\rho-1}}{\partial q_{i,k}}\Big)^T \Phi_{i,k}(\bar{q}_{i,k})\Big] \\ &\quad \Big[\hat{\theta}_i - \Gamma_i^T \sum_{l=2}^{\rho-1}\Big(\frac{\partial \alpha_{i,l-1}}{\partial \hat{\theta}_i}\Big) z_{i,l}\Big] \end{aligned}$$

$$
\begin{aligned}
&+\sum_{k=1}^{\rho-1}\Big(\frac{\partial \alpha_{i,\rho-1}}{\partial q_{i,k}}\Big)^T G_{i,k}(\bar{q}_{i,k})q_{i,k+1} \\
&+\sum_{j\in\mathcal{G}_i}\sum_{k=1}^{\rho-1}\Big(\frac{\partial \alpha_{i,\rho-1}}{\partial q_{j,k}}\Big)^T\Big[\Phi_{j,k}(\bar{q}_{j,k})[\hat{\theta}_j \\
&-\Gamma_j^T\sum_{l=2}^{\rho-1}\Big(\frac{\partial \alpha_{j,l-1}}{\partial \hat{\theta}_j}\Big)z_{j,l}] + G_{j,k}(\bar{q}_{j,k})q_{j,k+1}\Big] \\
&+\sum_{j=0}^{\rho-1}\Big(\frac{\partial \alpha_{i,\rho-1}}{\partial q_0^{(j)}}\Big)^T q_0^{(j+1)} \\
&+\Big(\frac{\partial \alpha_{i,\rho-1}}{\partial \hat{\theta}_i}\Big)^T\Gamma_i\tau_{i,\rho}\Big\}, \\
&\rho = 3,...,m \qquad (12)
\end{aligned}
$$

$$u_i = \alpha_{i,m} \qquad (13)$$

$$\dot{\hat{\theta}}_i = \Gamma_i\tau_{i,m} \qquad (14)$$

where $\kappa_{i,\rho}$ are positive constants, and $\tau_{i,\rho}$ is the ρ-th tuning function defined as follows

$$
\begin{aligned}
\tau_{i,1} &= \Phi_{i,1}^T(q_{i,1})\Bigg[\frac{\partial P_{i,0}(z_{i,1})}{\partial q_{i,1}} + 2\sum_{j\in\mathbb{H}_i}\frac{\partial P_{i,j}(\tilde{q}_{i,j})}{\partial q_{i,1}} \\
&+2\sum_{j\in\mathbb{G}_i}\frac{\partial Q_{i,j}(\tilde{q}_{i,j})}{\partial q_{i,1}}\Bigg] \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
\tau_{i,\rho} &= \tau_{i,\rho-1} \\
&+\Big[\Phi_{i,\rho}(\bar{q}_{i,\rho}) - \sum_{k=1}^{\rho-1}\Big(\frac{\partial \alpha_{i,\rho-1}}{\partial q_{i,k}}\Big)^T\Phi_{i,k}(\bar{q}_{i,k})\Big]^T z_{i,\rho} \\
&-\sum_{j\in\mathcal{G}_i}\sum_{k=1}^{\rho-1}\Phi_{i,k}^T(\bar{q}_{i,k})\Big(\frac{\partial \alpha_{j,\rho-1}}{\partial q_{i,k}}\Big)z_{j,\rho} \qquad (16)
\end{aligned}
$$

for $\rho = 2,...,m$.

Then, the derivative of V_m defined in (9) can be written as

$$
\begin{aligned}
\dot{V}_m &= -\sum_{i=1}^{N}\kappa_{i,1}\Bigg\|\frac{\partial P_{i,0}(z_{i,1})}{\partial q_{i,1}} \\
&+2\sum_{j\in\mathbb{H}_i}\frac{\partial P_{i,j}(\tilde{q}_{i,j})}{\partial q_{i,1}} + 2\sum_{j\in\mathbb{G}_i}\frac{\partial Q_{i,j}(\tilde{q}_{i,j})}{\partial q_{i,1}}\Bigg\|^2 \\
&-\sum_{i=1}^{N}\sum_{j=2}^{m}\kappa_{i,j}\|z_{i,j}\|^2 \qquad (17)
\end{aligned}
$$

Theorem 1: Consider N mobile agents with similar dynamics in (1) under Assumptions 2.1-2.4, decentralized controls (13) and update laws (14). Starting at different locations $q_{i,1}(0)$, all the agents will finally converge into the moving target region Ω in (4), without collisions between any agents. At the same time, the connectivity of the communication graph remains for all time.

Proof: $\square$ *First, we prove that no collisions occur between any agents.*

From (17), we know that $\dot{V}_m \leq 0$. Integrating both sides in the interval $[0, t]$, $\forall t > 0$, we obtain that $V_m(t) \leq V_m(0)$. With the definition of $V_m(t)$ in (9), we have $\sum_{i=1}^{N}\sum_{j\in\mathbb{H}_i} P_{i,j}(\tilde{q}_{i,j}) \leq V_m(0)$. According to Property 2 (ii), the boundedness of $P_{i,j}(\tilde{q}_{i,j})$ means $\|\tilde{q}_{i,j}\| \neq 0$, i.e., there are no collisions among any agents for all $t > 0$.

$\square$ *Next, we will prove that the connectivity of the communication graph remains for all time.*

According to Assumption 2.1, the communication graph $\mathcal{G}$ is a directed graph and connected initially. It means that there are always some agents which are initially located within the communication range of an agent i, i.e., the set $\mathbb{G}_i = \{j \in V, j \neq i \,\big|\, \|q_{i,1}(0) - q_{j,1}(0)\| \leq R\}$ defined in (3) exists. Since $V_m(t) \leq V_m(0) < \infty$ for $\forall t > 0$, we know the boundedness of $Q_{i,j}(\tilde{q}_{i,j})$. According to the Property 3 (ii) of $Q_{i,j}(\tilde{q}_{i,j})$, we obtain that agents which are initially located within distance R from each other will remain within this distance for all time. Therefore, the connectivity of the communication graph is preserved for all time.

$\square$ *Finally, we will prove that $q_{i,1} \in \Omega$, i.e., each agent is located in the moving target region Ω.*

Since $\dot{V}_m$ is negative semidefinite as seen from (17), according to LaSalle Yoshizawa Theorem [22], we know that as time tends to infinity, $\dot{V}_m$ tends to 0. From (17), we can obtain that

$$
\begin{aligned}
&\frac{\partial P_{i,0}(z_{i,1})}{\partial q_{i,1}} + 2\sum_{j\in\mathbb{H}_i}\frac{\partial P_{i,j}(\tilde{q}_{ij})}{\partial q_{i,1}} \\
&+2\sum_{j\in\mathbb{G}_i}\frac{\partial Q_{i,j}(\tilde{q}_{ij})}{\partial q_{i,1}} = 0 \qquad (18)
\end{aligned}
$$

as $t \to \infty$, $i \in \{1, 2, ..., N\}$. Applying summation from $i = 1$ to N on both sides of (18) results in

$$
\begin{aligned}
&\sum_{i=1}^{N}\Big\{\frac{\partial P_{i,0}(\tilde{q}_{i,0})}{\partial q_{i,1}} + 2\sum_{j\in\mathbb{H}_i}\frac{\partial P_{i,j}(\tilde{q}_{i,j})}{\partial q_{i,1}} \\
&+2\sum_{j\in\mathbb{G}_i}\frac{\partial Q_{i,j}(\tilde{q}_{i,j})}{\partial q_{i,1}}\Big\} = 0 \qquad (19)
\end{aligned}
$$

According to Properties 2, 3 and the fact that the interactions between agents are bi-directional and they can cancel each other, we have

$$\sum_{i=1}^{N}\sum_{j\in\mathbb{H}_i}\frac{\partial P_{i,j}(\tilde{q}_{i,j})}{\partial q_{i,1}} = 0 \qquad (20)$$

$$\sum_{i=1}^{N}\sum_{j\in\mathbb{G}_i}\frac{\partial Q_{i,j}(\tilde{q}_{i,j})}{\partial q_{i,1}} = 0 \qquad (21)$$

Substituting (20) and (21) into (19) leads to :

$$\sum_{i=1}^{N}\frac{\partial P_{i,0}(\tilde{q}_{i,0})}{\partial q_{i,1}} = \sum_{i=1}^{N}\frac{\partial P_{i,0}(\tilde{q}_{i,0})}{\partial \|q_{i,0}\|^2}q_{i,0} = 0 \qquad (22)$$

To prove that all agents converge into the moving target region Ω, we assume that not all the agents are located in the target region first. Then we seek to arrive at some contradiction results, which will mean that all agents are located in the target region.

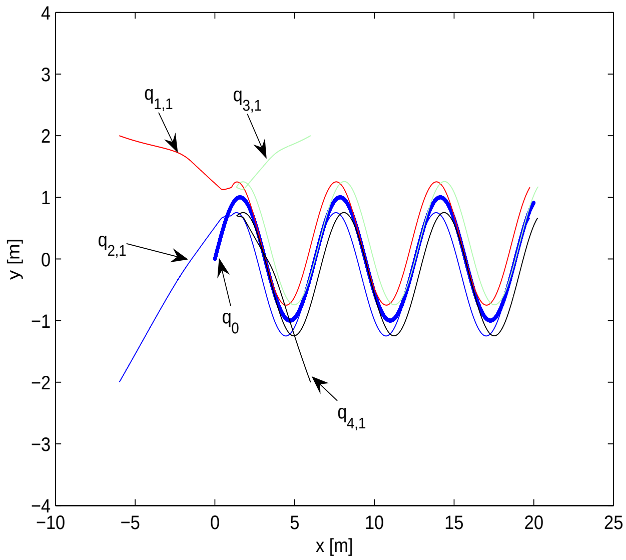

Fig. 1. Trajectories of agents and the center of the target region

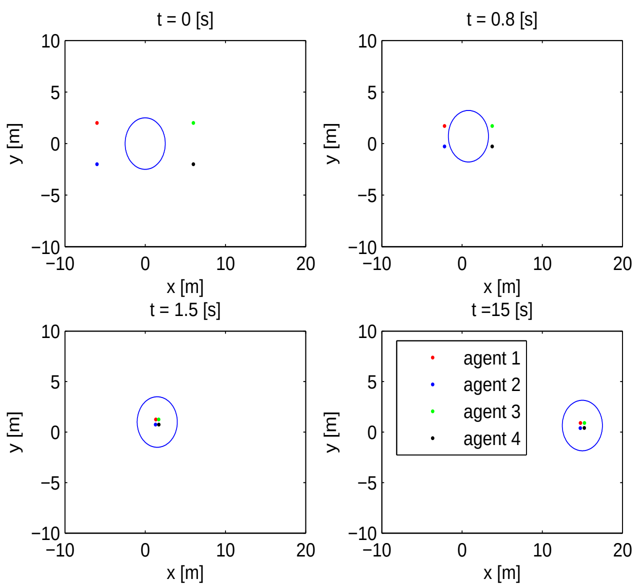

Fig. 2. All agents converging into and moving with the target region

Case (i): If the agents outside the target region are one side, the vector $\frac{\partial P_{i,0}(\tilde{q}_{i,0})}{\partial \|q_{i,0}\|^2} q_{i,0}$ in (22), have the same sign along one axis, and thus, they cannot cancel each other. This contradicts with (22).

Case (ii): If the agents outside the target region are on the opposite sides, we separate them into two subgroups, A and B. Due to Assumption 2.2, there is no interaction between these two subgroups. According to (18), for each subgroup, e.g. subgroup A, we have

$$\sum_{i \in A} \frac{\partial P_{i,0}(\tilde{q}_{i,0})}{\partial q_{i,1}} = \sum_{i \in A} \frac{\partial P_{i,0}(\tilde{q}_{i,0})}{\partial \|q_{i,0}\|^2} q_{i,0} = 0 \tag{23}$$

However, since all the agents in the subgroup A are located on one side of the target region, the vector $\frac{\partial P_{i,0}(\tilde{q}_{i,0})}{\partial \|q_{i,0}\|^2} q_{i,0}$ in (23) have the same sign along one axis, and thus, they cannot cancel each other. Therefore, it contracts with (23). Similar conclusion could be made to subgroup B.

From the above Case (i) and Case (ii), we can conclude that all agents converge into the moving target region Ω. This completes the proof. ■

IV. Simulation Studies

We consider a group of $N = 4$ mobile agents on a $\mathbb{R}^2$ space, i.e. x-y space, with the danger region radius $r = 0.5$m, communication range radius $R = 1.0$m and the following dynamics:

$$\begin{aligned} \dot{q}_{i,1} &= q_{i,2} \\ \dot{q}_{i,2} &= q_{i,3} \\ \dot{q}_{i,3} &= q_{i,1}\theta_{i1} + q_{i,2}\theta_{i2} + q_{i,3}\theta_{i3} + u_i \end{aligned} \tag{24}$$

where $q_{i,j} \in \mathbb{R}^2$, $i \in \{1,2,3,4\}$, $j \in \{1,2,3\}$, and $\theta_{i1} = 0.1$, $\theta_{i2} = 0.5$, $\theta_{i3} = 0.5$. The common target region Ω is specified as a circle which is centered at the point q_0 with a radius of $r_0 = 2.5$m and moves along the desired trajectory $q_0 = [t \;\; \sin(t)]^T$. The agents are initialized randomly outside the target region with $q_0 = [0.0, 0.0]^T$. Simulation results are shown in Figs. 1-3. Fig. 1 shows the trajectories of all agents and the center of target region. From Fig. 2, we observe that all agents converge into the target region and move together with it. This is also seen in the top sub-figure of Figure 3, where all the corresponding target potential energies $P_{i,0}$ are driven to zero. At the same time, the collision avoidance capabilities of the agents are verified in the bottom sub-figure of Fig. 3, where all the corresponding collision avoidance potential energies $P_{i,j}$ are driven to zero.

V. Conclusion

Due to the importance of considering actuator dynamics into the vehicle dynamics for achieving better performance, decentralized cooperative control has been proposed, in this paper, for multi-agent systems with high-order dynamics in strict feedback form by incorporating artificial potentials and adaptive backstepping into Lyapunov synthesis. We have shown that all the agents converge to a common moving target region, without collisions between any agents in the group. At the same time, the connectivity of the communication graph remains for all time.

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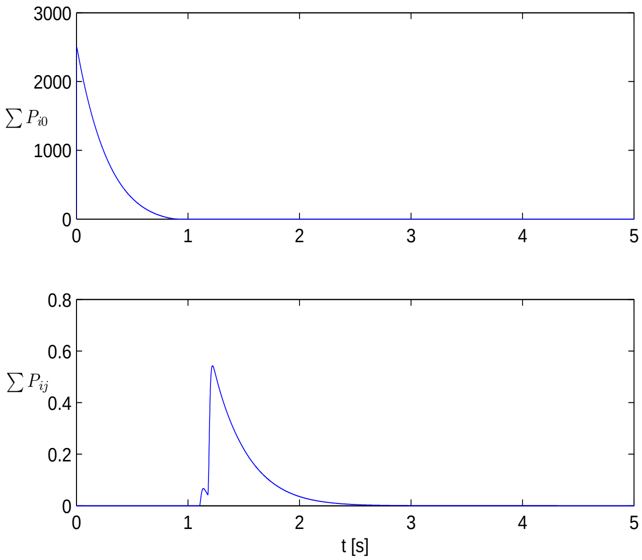

Fig. 3. Sum of agent target potential energies (Top) and sum of agent collision avoidance potential energies (Bottom)

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