UDE-BASED ROBUST CONTROL FOR A CLASS OF NON-AFFINE NONLINEAR SYSTEMS

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ABSTRACT

In this paper, the UDE (uncertainty and disturbance estimator) based robust control is investigated for a class of non-affine nonlinear systems in a normal form. Control system design for non-affine nonlinear systems is one of the most difficult problems due to the lack of mathematical tools. This is also true even for the exact known non-affine systems because of the difficulty in explicitly constructing the control law. It is shown that the proposed UDE-based robust control strategy leads to a stable system. The most important features of the approach are that (i) by adding and subtracting the control term u, the original non-affine form is transformed into a semi-affine form, which not only simplifies the control design procedure, but also avoids the singularity problem of the controller; (ii) the employment of UDE makes the estimation of the lumped uncertain term which is a function of control input, states and disturbances possible, rather than states alone; and (iii) it does not require any knowledge (e.g., bounds) about the uncertainties and disturbances, except the information about the bandwidth, during the design process. The stability of the closed-loop system is established. Effectiveness of the proposed approach is demonstrated through application to the hard disk driver control problem.

1 Introduction

Adaptive control of nonlinear systems using feedback linearization has been extensively studied for affine nonlinear systems, i.e., the model is linear in the input variables and the nonlinearities are linearly parameterized by unknown parameters, and many significant developments have been achieved. In the early stage of the research, several important results on adaptive nonlinear control were developed for systems with matching conditions [1][2], extended matching condition, or growth conditions on system nonlinearities, and for systems in strict-feedback form (unmatched condition) [3, 4, 5, 6, 7].

However, many practical systems, e.g., chemical reactions and PH neutralization, are inherently nonlinear, and the input variables may not be expressed in an affine form. Indeed, control of non-affine nonlinear systems is not only academically challenging but also of practical interest, though it is extremely difficult and challenging due to the lack of mathematical tools. In fact, it is impossible to handle the control problem of the non-affine nonlinear system directly because, in general, even if it is known that the inverse exists, it is impossible to construct it analytically. Consequently, no control system design is possible along the lines of conventional model-based control.

For controller design of non-affine nonlinear systems, several researchers have suggested to use neural networks (NNs) as emulators of inverse systems [8, 9]. The main idea is that for a system with a finite relative degree, the mapping between a system input and the system output is one-to-one, thus allowing the construction of a "left-inverse" of the nonlinear system using NN. Hence, if the controller is an "inverse operator" of the nonlinear system, the reference input to the controller would produce a control input to the plant, which would in turn produce an output identical to the reference input. Based on the implicit function theory, the NN control methods proposed in [8, 10, 11, 12] have been used to emulate the "inverse controller" to achieve tracking control objectives. Nevertheless, no rigorous stability proofs of the closed-loop systems were given for on-line adaptive NN control due to the difficulties in analysis of discrete-time nonlinear adaptive systems. In [13, 14], stable adaptive NN control was constructed for non-affine nonlinear systems using the Implicit Function Theorem and the Mean Value Theorem that are not usually associated with neural network control theory. In [15], under the assumption of the existence of an linearizing control, approximated linearizing feedback control is constructed for a class of non-affine nonlinear systems with the lower bound of the estimate of the control effectiveness being explicitly given.

In this paper, we propose a novel robust control strategy for a class of single-input-single-output (SISO) non-affine nonlinear systems in the normal form. The basic idea is that by adding and subtracting the control term u we transform the original non-affine system into a semi-affine form which allows the application of the existing feedback linearization control methods. For the control design, we employ the uncertainty and disturbance estimator (UDE) to estimate the lumped uncertainty term which is a function of the control input, states and disturbances, rather than the states only. This is achievable due to the special structure of the UDE. The UDE control algorithm, which was proposed in [16] as a replacement of the time-delay control (TDC) in [17], is based on the assumption that a signal can be approximated and estimated using a filter with the right bandwidth. The two-degree-of-freedom nature of the UDE-based control has been revealed in [18], which enables the decoupled design of the reference model and the filters. The UDE-based control strategy has been successfully applied to robust input-output linearization [19] and combined with sliding-mode control [20][21], and further extended to uncertain systems with state delays, for both linear systems [22] and nonlinear systems [23].

Compared with other approaches for control of non-affine nonlinear systems, our approach has the following advantages:

(i) By adding and subtracting the control term u, the original non-affine form is transformed into a semi-affine form, which not only simplifies the control design procedure, but also avoids the singularity problem of the controller.
(ii) The employment of UDE makes the estimation of the lumped uncertain term which is a function of control input, states and disturbances possible, rather than states alone.
(iii) It does not require any knowledge (e.g., bounds) about the uncertainties and disturbances, except the information about the bandwidth of the uncertainties and disturbances, during the design process.

2 Problem formulation and preliminaries

Consider a class of single-input-single-output (SISO) non-affine nonlinear systems described by the following normal form

$$\begin{cases} \dot{x}_i &= x_{i+1},\ i=1,2,...,n-1 \\ \dot{x}_n &= f(x,u)+d(t) \\ y &= x_1 \end{cases} \tag{1}$$

where $x(t)=[x_1,x_2,...,x_n]\in R^n$ is the state vector, $n\geq 1$ is the system order; $u(t)\in R$, $y(t)\in R$ are the system input and output, respectively. The mapping $f(\cdot,\cdot):R^{n+1}\rightarrow R$ is an unknown smooth nonlinear function and $d(t)$ is the external unknown disturbance.

Assumption 1: The nonlinear function $\frac{\partial[f(x,u)]}{\partial u}\neq 0$, for all $(x,u)\in R^n\times R$.

Assumption 1 is reasonable as $\frac{\partial[f(x,u)]}{\partial u}$ can be viewed as the control gain of system (1) and $\frac{\partial[f(x,u)]}{\partial u}$ being away from zero is a controllability condition for system (1).

Consider the following reference model

$$\begin{cases} \dot{x}_{d,i} &= x_{d,i+1},\ i=1,2,...,n-1 \\ \dot{x}_{d,n} &= f_d(x_d) \\ y_d &= x_{d,1} \end{cases} \tag{2}$$

where $x_d=[x_{d,1},x_{d,2},...,x_{d,n}]^T\in R^n$ is the state vector of the reference system, $y_d\in R$ is the reference system output, $f_d(\cdot,\cdot):R^{n+1}\rightarrow R$ is a known smooth nonlinear function. The initial condition is denoted as $x_d(0)$.

Assumption 2: The states of the reference model remain uniformly bounded.

The objective is to design a controller such that $y(t)$ asymptotically tracks a reference output trajectory $y_d(t)$, i.e., the tracking error $y-y_d$ exponentially converges to zero when t tends to infinity.

The main difficulty of this control design lies in:

(i) the function $f(x,u)$ in (1) is unknown;
(ii) the system input $u(t)$ does not appear linearly in $f(x,u)$, which makes the direct feedback linearization difficult/impossible; and
(iii) the disturbance $d(t)$ is unknown.

3 Control design based on uncertainty and disturbance estimator

Define the following variables

$$\begin{aligned} e &= x-x_d \\ &= [x_1-x_{d,1},x_2-x_{d,2},\cdots,x_n-x_{d,n}]^T \\ &= [e_1,e_2,\cdots,e_n]^T \end{aligned}$$

and a filtered tracking error as

$$e_s=[\Lambda^T 1]e \tag{3}$$

where $\Lambda=[\lambda_1,\lambda_2,\cdots,\lambda_{n-1}]^T$ is an appropriately chosen coefficient vector such that the polynomial $s^{n-1}+\lambda_{n-1}s^{n-2}+\cdots+\lambda_1$

is Hurwitz. According to (1)(2), the derivative of the filtered tracking error e_s in (3) with respect to time t is given by

$$\dot{e}_s = [0\ \Lambda^T]e - f_d(x_d) + d(t) + f(x,u) \tag{4}$$

Since $\partial([0\ \Lambda^T]e - f_d(x_d) + d(t))/\partial u = 0$, according to Assumption 1, it is clear that

$$\frac{\partial([0\ \Lambda^T]e - f_d(x_d) + d(t) + f(x,u))}{\partial u} \neq 0,\ \forall (x,u) \in R^n \times R.$$

Using the implicit function theorem [24], it follows that there exists a smooth implicit desired control input u^* such that

$$[0\ \Lambda^T]e - f_d(x_d) + d(t) + f(x,u^*) = 0. \tag{5}$$

Since the function $f(x,u^*)$ in (5) is unknown and non-affine, it is impossible to get the solution of u^* directly and analytically.

To facilitate the control design, add and subtract u on the right hand side of (4)

$$\dot{e}_s = [0\ \Lambda^T]e - f_d(x_d) + D(x,u,t) + u. \tag{6}$$

where

$$D(x,u,t) = f(x,u) + d(t) - u \tag{7}$$

represents the lumped uncertainty term of the system, which is a function of input, states and disturbance.

Note that (6) is transformed into the so-called semi-affine form as the control u in the last term is expressed in an affine form, though u also exists in the uncertainty term $D(x,u,t)$. This semi-affine form allows the application of the existing feedback linearization control methods.

Consider the following controller

$$u = v + u_D \tag{8}$$

where v is the stabilizing term to be designed using the standard linear control techniques and u_D is the robustifying term to cancel the uncertainty $D(x,u,t)$ in (6).

If the term $D(x,u,t)$ in (6) is known, then the robustfying term u_D can be chosen as

$$u_D = -D(x,u,t) \tag{9}$$

and the stabilizing term v is considered as

$$v = -ke_s - [0\ \Lambda^T]e + f_d(x_d) \tag{10}$$

where k is a positive design constant. Then (8) becomes

$$u = -ke_s - [0\ \Lambda^T]e + f_d(x_d) - D(x,u,t) \tag{11}$$

Substituting (11) into (6) results in

$$\dot{e}_s = -ke_s. \tag{12}$$

It shows that the closed-loop system is stable and $e_s = e^{-kt}e_s(0)$ will exponentially converge to zero when t tends to infinity. Therefore, it can be concluded from (3) that the tracking error $y - y_d$ also exponentially converges to zero when t tends to ∞.

However, the term $D(x,u,t)$ in (6) is unknown due to the uncertainty and disturbance, so the robustfying term u_D in (9) needs to be redesigned again. One of the promising candidate is the uncertainty and disturbance estimator (UDE), which was proposed in [16] as a replacement of the time-delay control (TDC) in [17]. The UDE algorithm is based on the assumption that a signal can be approximated and estimated using a filter with the right bandwidth. So if the filter has a broad enough bandwidth, the estimator is able to estimate the lumped uncertainty term accurately and quickly.

Substituting (8)(10) into (6) leads to

$$\dot{e}_s = -ke_s + D(x,u,t) + u_D \tag{13}$$

and solving $D(x,u,t)$ gives

$$D(x,u,t) = \dot{e}_s + ke_s - u_D. \tag{14}$$

It indicates that the unknown dynamics and the disturbances can be observed by the system states and the control signal. However, it cannot be used in the control law directly. The UDE technique [16] adopts an estimation of this signal. Following the procedure given in [16], assume that $G_f(s)$ is a strictly proper low-pass filter with unity steady-state gain, whose passband contains the frequency contents of $D(x,u,t)$. Then $D(x,u,t)$ can be accurately approximated by

$$\hat{D}(x,u,t) = g_f(t) * (\dot{e}_s + ke_s - u_D), \tag{15}$$

where $\hat{D}(x,u,t)$ is an estimate of $D(x,u,t)$, "$*$" is the convolution operator, $g_f(t) = L^{-1}\left\{G_f(s)\right\}$, and $L^{-1}\{\cdot\}$ is the inverse Laplace transform operator. This is called the UDE. Therefore, $\hat{D}(x,u,t) = D(x,u,t)$, if the bandwidth of the filter is chosen broad enough.

Selecting $u_D = -\hat{D}(x,u,t)$ and using (15) lead to

$$u_D = -g_f(t) * (\dot{e}_s + ke_s - u_D). \tag{16}$$

Solving (16) for u_D results in

$$u_D = -L^{-1}\left\{\frac{G_f(s)}{1-G_f(s)}\right\} * (\dot{e}_s + ke_s). \quad (17)$$

Substituting (17) and (10) into (8) leads to the UDE controller

$$u = -ke_s - [0\ \Lambda^T]e + f_d(x_d) - L^{-1}\left\{\frac{G_f(s)}{1-G_f(s)}\right\} * (\dot{e}_s + ke_s) \quad (18)$$

It is clear that the control law is formed by the system tracking error, the reference model, the low-pass filter and the feedback gain.

Under the assumption of $\hat{D}(x,u,t) = D(x,u,t)$, when the low-pass filter is chosen with unity steady-state gain and broad enough bandwidth, substituting the control law in (18) into (6) results in the same error dynamics as given in (12), thus eliminating the effect of uncertain and disturbances.

Assume that the frequency range of the system dynamics and the external disturbance is limited by w_f. Then, $G_f(s)$ can be chosen as the following first order low-pass filter

$$G_f(s) = \frac{1}{1+\tau s} \quad (19)$$

where $\tau = 1/w_f > 0$ is a time constant. It is worth noting that

$$\frac{G_f(s)}{1-G_f(s)} = \frac{1}{\tau s}$$

which means that an integral action is included in the controller. In this case, (18) can be further simplified as

$$\begin{aligned} u &= -ke_s - [0\ \Lambda^T]e + f_d(x_d) - \frac{1}{\tau}(e_s + k\int_0^t e_s dt) \\ &= -(k+\frac{1}{\tau})e_s - \frac{k}{\tau}\int_0^t e_s dt - [0\ \Lambda^T]e + f_d(x_d) \end{aligned} \quad (20)$$

The first three terms introduce a proportional-integrative (PI) control law. The last term imposes the desired dynamics with the reference model.

4 Stability Analysis

In this section, we will investigate the closed-loop performance using the UDE-based controller (18). Our main result is summarized in the following theorem.

Theorem 1. *Consider the closed-loop system consisting of the non-affine nonlinear system (1) satisfying Assumption 1, the reference model (2) satisfying Assumption 2, and the UDE-based controller (18). Assume that the filter $G_f(s)$ is chosen appropriately with the enough bandwidth and order. Then the closed-loop system is stable and the tracking error $y - y_d$ exponentially converges to zero.*

Proof: Selecting $u_D = -\hat{D}$ and substituting into (13) give

$$\dot{e}_s = -ke_s + \tilde{D}. \quad (21)$$

where $\tilde{D} \triangleq D - \hat{D}$. Since the low-pass filter is chosen with unity steady-state gain and broad enough bandwidth to cover the frequency range of the system dynamics and the external disturbance, $\hat{D}(x,u,t)$ is able to estimate $D(x,u,t)$ accurately and quickly. If $G_f(s)$ is chosen as the first order filter in (19), the choice of the filter time constant, τ, affects the uncertainty estimation error accuracy. The small value of τ , which represents broader bandwidth, leads to a smaller estimation error. When τ is chosen small enough, $\tilde{D} \to 0$. Therefore, it can be concluded that the closed-loop system is stable and the error dynamics e_s will exponentially converge to zero when t tends to infinity. Thus the tracking error $y - y_d$ also exponentially converges to zero when t tends to infinity.

5 Application to hard disk drives control

To demonstrate the effectiveness, results of the control of hard disk drives using the proposed approach are presented in this section. The objective is to reject the external time varying disturbance and to make the position error signal (PES) of the read/write (R/W) head converges to zero in the tracking mode. The hard disk under consideration is a Hitachi GST 1.8-inch hard disk drive with model number HTC426020G7AT00 and spindle motor rotational speed 4200 RPM [25].

Figure 1 shows the frequency response of voice-coil-motor (VCM) actuator positioner of the 1.8-inch disk drive. By curve fitting to the measured frequency response in Figure 1, the model plant G of the hear disk drive is obtained as follows

$$\begin{aligned} G &= G_P G_{R1} G_{R2}, \\ G_P &= \frac{5.527\times 10^7}{s^2 + 502.7s + 3.948\times 10^5}, \\ G_{R1} &= \frac{0.954s^2 + 1031s + 6.964\times 10^8}{s^2 + 1056s + 6.964\times 10^8}, \\ G_{R2} &= \frac{2.527\times 10^9}{s^2 + 6032s + 2.527\times 10^9}, \end{aligned}$$

where G_P is the nominal plant without resonance modes, while G_{R1} and G_{R2} are the resonance modes at 4.2kHz and 8kHz respectively. In implementation, to deal with the above resonance modes, we use two corresponding notch filters at 4.2kHz and 8kHz respectively as follows

$$C_{N1} = \frac{s^2 + 2639s + 6.964 \times 10^8}{s^2 + 5278s + 6.964 \times 10^8},$$
$$C_{N2} = \frac{s^2 + 1.005 \times 10^4 s + 2.527 \times 10^9}{s^2 + 5.027 \times 10^4 s + 2.527 \times 10^9}.$$

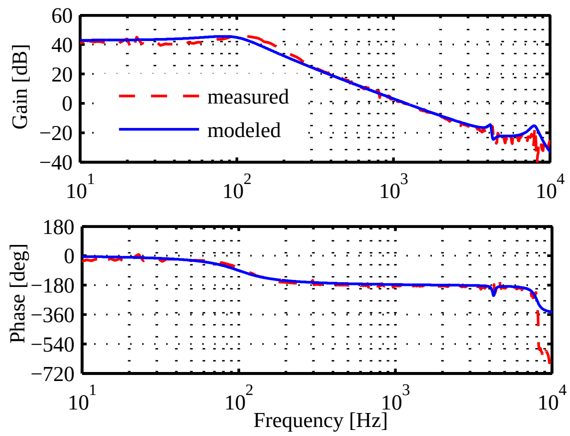

Figure 1. Frequency response of the VCM actuator .

The nominal plant G_P can be transformed into the form of system (1) as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = f(x,u) + d(t) \\ y = x_1 \end{cases} \tag{22}$$

where $f(x,u) = -3.948 \times 10^5 x_1 - 502.7 x_2 + 5.527 \times 10^7 u$, and $d(t)$ is the external disturbance which is acting on the actuator. It is noticed that the nominal plant of the hard disk drives in (22) is an affine system, which is a special class of the non-affine system described in (1). However, to verify the effectiveness of our proposed control strategy, the information about $f(x,u)$ and $d(t)$ are assumed unknown and are not utilized in the control design. For the purpose of simulation, we assume that the external disturbance is chosen as in Figure 2, which consists of a sum of sinusoids and a zero mean random noise in the low-frequency range.

The step response with the magnitude of 5 μm is investigated for this study. This is a regulation problem with $f_d(x_d) = 0$ and $x_{d,1} = 5$ μm , so $y_d = 5$ μm. Note that the set-point for the system is embedded into the initial value of the reference model. The other design parameters in the controller (20) are chosen as $k = 1 \times 10^{-6}$, $\lambda = 1 \times 10^{-4}$, $\tau = 1 \times 10^{-3}$. The position error signal (PES) and the control signal are shown in Figures 3 and 4

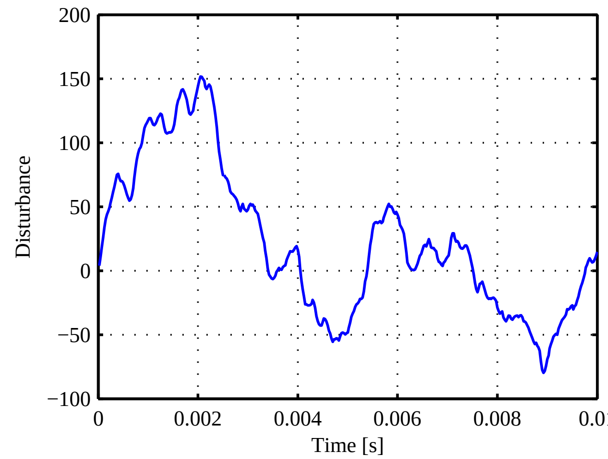

Figure 2. The disturbance signal $d(t)$.

respectively. It can be seen that the good tracking performance is achieved and the proposed control is able to handle the external disturbances described in Figure 2.

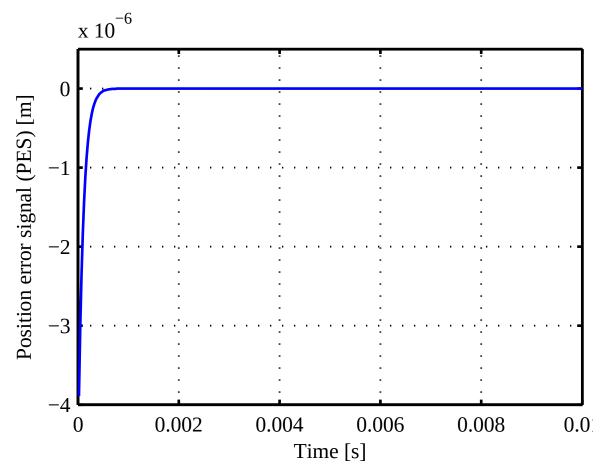

Figure 3. The position error signal (PES) of the read/write head.

6 Conclusion

This paper presented a new framework of robust control for a class of non-affine nonlinear systems. By adding and subtracting the control term u, the original non-affine form was transformed into a semi-affine form, which not only simplifies the control design procedure, but also avoids the singularity problem of the controller. Then the uncertainty and disturbance estimator (UDE) was employed to estimate the lumped uncertain term which is a function of control input, states and disturbances. The common assumptions about the uncertainties and disturbances are relaxed in our paper. It has been shown that UDE-based control can be implemented to achieve very good performance.

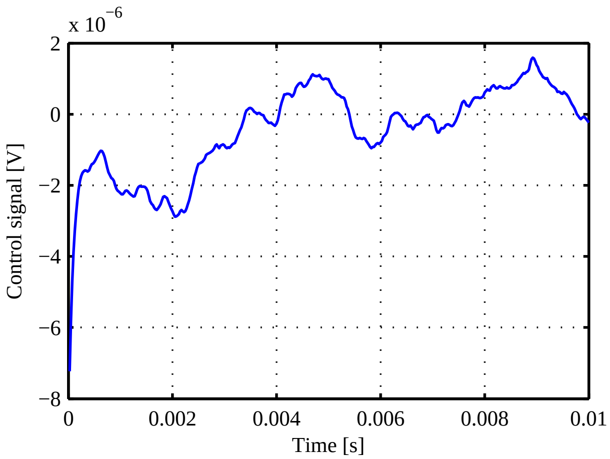

Figure 4. The control signal.

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