ADAPTIVE NN CONTROL FOR A CLASS OF NON-AFFINE NONLINEAR SYSTEMS WITH UNKNOWN CONTROL GAIN

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Abstract: In this paper, adaptive neural network (NN) control is presented for a class of non-affine nonlinear systems with unknown control gain. Using the Mean Value Theorem and backstepping method, we propose a constructive approach for adaptive NN control design. The Nussbaum function is used to relax the requirement on control direction. The semi-global uniformly ultimately boundedness (SGUUB) of the closed-loop system is achieved. In addition, the simulation study results are given to demonstrate the effectiveness of the proposed control.

Keywords: Neural networks, adaptive, non-affine, Nussbaum function

1. INTRODUCTION

In the past decades, interest in non-affine nonlinear system control has been ever increasing and some developments have been achieved in the literature. In these non-affine nonlinear systems, input variables may enter into the systems nonlinearly as described by the general form:

$$\begin{aligned} \dot{x} &= f(x, u) \\ y &= h(x) \end{aligned} \qquad (1)$$

where $x \in R^n$ is the state vector, u is the input and y is the output, and $f(\cdot, \cdot)$ and $h(\cdot)$ are unknown smooth vector fields.

To deal with unknown functional uncertainties, neural networks (NNs) are often used as online approximators, owing to their universal approximation capabilities, learning and adaptation, parallel distributed structures (Narendra and Parthasarathy, 1990; Lewis *et al.*, 1999; Ge *et al.*, 2002; Hovakimyan *et al.*, 2002). In (Ge *et al.*, 1999), an ideal implicit feedback linearization control (IFLC) was first established for classes of the non-affine systems in Brunovsky form, using implicit function theorem under the assumption of non-singularity on the control gain. In (Wang *et al.*, 2006), adaptive neural control of the completely non-affine pure-feedback system was presented based on "ISS-modular" approach. Adaptive output feedback control of non-affine nonlinear systems using pseudo inverse control was considered under the assumption that the system was feedback linearizable in (Calise *et al.*, 2001; Hovakimyan *et al.*, 2002).

The foregoing works require the sign of the control gain. For example, the assumption, that the sign of control gain is positive without loss of generality, is always made, which makes the control design easier. When the sign of control gain is unknown, the problem becomes more complex. To counteract the lack of a prior knowledge of control direction, the most often adopted method is the Nussbaum function, which was first introduced in (Nussbaum, 1983). For the affine nonlinear systems, in (Ye and Jiang, 1998) and

(Ye, 1999), adaptive control combining backtepping with Nussbaum function was proposed for n-order linear-in-parameter strict feedback nonlinear systems, respectively with unknown constant control gains and unknown time-varying control gains. Perturbed strict feedback systems with unknown functions and unknown constant control gain were considered using neural networks and Nussbaum function in (Ge and Wang, 2003). In (Liu and Huang, 2006), global robust output regulation for nonlinear systems in output feedback form was considered without the knowledge of the sign of the high-frequency gain. For the non-affine nonlinear systems, in (Du *et al.*, 2006), adaptive neural network control was designed for the SISO low-triangular-structured non-affine nonlinear systems with a novel Nussbaum gain function, which can deal with the unknown functional control gains.

In our work, adaptive neural network control combining with backstepping design are presented for a general class of SISO non-affine nonlinear systems(1), which can be transformed to a normal form with zero dynamics after a diffeomorphism transformation. To the best of our knowledge, there are no such works dealing with such a kind of systems in the literature at present stage. With the employment of Nussbaum function, the resultant adaptive NN control overcomes the restriction on the known sign of control gain, which is both required for control design in early works (Ge *et al.*, 1999; Ge and Zhang, 2003).

The organization of this paper is as follows. Section 2 presents some preliminary knowledge, include some notations, assumptions and RBFNN, which are used in the later adaptive neural control design. In Section 3, we present the control design methodology based on adaptive NN backstepping, together with the use of the Mean Value Theorem and Nussbaum functions to handle the non-affine nonlinearities and the unknown control directions. Results of simulation studies are shown in Section 4, before conclusions are drawn in Section 5.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1 Mathematics Preliminaries

In the following study, let $\|\cdot\|$ denote the 2-norm, $\|\cdot\|_F$ denote the Frobenius norm and $|\cdot|_1$ denote 1-norm, i.e. given $A = [a_{ij}] \in R^{m\times n}$, $\|A\|_F^2 = tr\{A^TA\} = \sum_{i,j} a_{i,j}^2$, and $|A|_1 = \sum_{i,j} |a_{i,j}|$.

Definition 1. The solution of (1) is Semi-Globally Uniformly Ultimately Bounded (SGUUB) if, for any compact set Ω_0, there exists an $S > 0$ and $T(S, X(t_0))$ such that $\|X(t)\| \le S$ for all $X(t_0) \in \Omega_0$ and $t \ge t_0 + T$.

Lemma 1. (Mean Value Theorem) (Apostol, 1974) Assume that $f(x, y) : R^n \times R \to R$ has a derivative (finite or infinite) at each point of an open set $R^n \times (a, b)$, and assume also that it is continuous at both endpoints $y = a$ and $y = b$. Then there is a point $\xi \in (a, b)$ such that $f(x, b) - f(x, a) = f^{'}(x, \xi)(b - a)$.

Lemma 2. (Ge and Wang, 2004) Let function $V(t) \ge 0$ be a continuous function defined $\forall t \in R^+$ and $V(0)$ bounded, and $\rho(t) \in L_\infty$ be real-valued function. If the following inequality holds:

$$\dot{V}(t) \le -c_1 V(t) + c_2\rho(t) \qquad (2)$$

where $c_1 > 0$, c_2 are constants, then we conclude that $V(t)$ is bounded.

2.1.1. Nussbaum Functions A function $N(\zeta)$ is called a Nussbaum-type function if it has the following properties(Nussbaum, 1983):

$$\lim_{s\to\infty} \sup \frac{1}{s}\int_0^s N(\zeta)d\zeta = \infty \qquad (3)$$

$$\lim_{s\to\infty} \inf \frac{1}{s}\int_0^s N(\zeta)d\zeta = -\infty \qquad (4)$$

Commonly used Nussbaum functions include: $k^2\cos(k)$, $k^2\sin(k)$, and $\exp(k^2)\cos((\pi/2)k)$ (Ilchmann, 1993). In this paper, an even Nussbaum function $\exp(k^2)\cos((\pi/2)k)$ is exploited.

Lemma 3. (Ge *et al.*, 2004) Let $V(\cdot)$, $\zeta(\cdot)$ be smooth functions defined on $[0, t_f)$ with $V(t) \ge 0$, $\forall t \in [0, t_f)$, and $N(\cdot)$ be an even smooth Nussbaum-type function. If the following inequality holds:

$$V(t) \le c_0 + e^{-c_1 t}\int_0^t g(x(\tau))N(\zeta)\dot{\zeta}e^{c_1\tau}d\tau + e^{-c_1 t}\int_0^t \dot{\zeta}e^{c_1\tau}d\tau, \quad \forall t \in [0, t_f)$$

where c_0 represents some suitable constant, c_1 is a positive constant, and $g(x(\tau))$ is a time-varying parameter which takes values in the unknown closed intervals $I = [l^-, l^+]$, with $0 \notin I$, then $V(t)$, $\zeta(t)$, $\int_0^t g(x(\tau))N(\zeta)\dot{\zeta}d\tau$ must be bounded on $[0, t_f)$.

According to Proposition 2 (Ryan, 1991), if the solution of the resulting closed-loop system is bounded, then $t_f = \infty$.

2.1.2. Neural Networks In this paper, the following RBFNN (Haykin, 1999) is used to approximate the continuous function $h(Z): R^q \to R$,

$$h_{nn}(Z) = W^T S(Z) \tag{5}$$

where the input vector $Z \in \Omega \subset R^q$, weight vector $W = [w_1, w_2, ..., w_l]^T \in R^l$, the NN node number $l > 1$; and $S(Z) = [s_1(Z), ..., s_l(Z)]^T$, with $s_i(Z)$ being chosen as the commonly used Gaussian functions, which have the form

$$s_i(Z) = \exp\left[\frac{-(Z-\mu_i)^T(Z-\mu_i)}{\eta^2}\right] \tag{6}$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, ..., \mu_{iq}]^T$ is the center of the receptive field and η is the width of the Gaussian function and $i = 1, 2, ..., l$.

It has been proven that network (5) can approximate any continuous function over a compact set $\Omega_Z \subset R^q$ to arbitrary any accuracy as

$$h(Z) = W^{*T} S(Z) + \varepsilon(Z), \ \forall Z \in \Omega_Z \tag{7}$$

where W^* is ideal constant weights, and $\varepsilon(Z)$ is the approximation error ($\varepsilon(Z)$ is denoted as ε to simplify the notation).

Assumption 1. There exist ideal constant weights W^* such that $|\varepsilon| \leq \varepsilon^*$ with constant $\varepsilon^* > 0$ for all $Z \in \Omega_Z$. Moreover, W^* is bounded by $\|W^*\| \leq w_m$ on the compact set Ω_Z.

It is clear that W^* is usually unknown and need to be estimated in function approximation. Let $\hat{W}$ be the estimates of W^*, and the weight estimation error be $\tilde{W} = \hat{W} - W^*$.

Remark 1. Although RBFNN is employed in our control design, it can be replaced by other linearly parameterized function approximators such as high-order neural networks, fuzzy systems, polynomials, splines and wavelet networks without difficulty (Ge *et al.*, 2002).

2.2 Problem Formulation

Consider a class of SISO general nonlinear system described by (1), our control objective is output tracking of a desired reference trajectory such that the tracking error converges to a neighbourhood of zero, i.e. $|y(t) - y_d(t)| \leq \delta$, where $\delta > 0$. At the same time, all closed loop signals are to be kept bounded.

The reference trajectory $y_d(t)$ is generated by the following reference model:

$$\begin{aligned} \dot{\xi}_{di} &= \xi_{di+1}, \quad 1 \leq i \leq \rho - 1, \\ \dot{\xi}_{d\rho} &= f_d(\xi_d), \\ y_d &= \xi_{d1}, \end{aligned} \tag{8}$$

where $\rho \geq 2$ is a constant index; $\xi_d = [\xi_{d1}, \xi_{d2}, \cdots, \xi_{d\rho}]^T \in R^\rho$ are the states of the reference system; $y_d \in R$ is the system output; and $f_d : R^\rho \to R$ is a known function.

Assumption 2. The reference trajectory $y_d(t)$ and its ρ derivatives remain bounded, i.e., $\xi_d \in \Omega_d \subset R^\rho$, $\forall t \geq 0$, where ρ is the relative degree of (1).

Assumption 3. System (1) is input-output linearizable with strong relative degree $\rho < n$.

Define $\phi_j(x) = L_f^{j-1} h$ for $j = 1, ..., \rho$, where $L_f h$ denotes the Lie derivative of the function $h(x)$ with respect to the vector field $f(x, u)$. Due to Assumption 3, it was shown in (Isidori, 1995) that there exist other $n - \rho$ functions $\phi_{\rho+1}, ..., \phi_n$ independent of u, such that the mapping $\Phi(x) = [\phi_1(x), \phi_2(x), ..., \phi_n(x)]^T$ has a Jacobian matrix which is nonsingular for all $x \in \Omega_x$. Therefore, $\Phi(x)$ is a diffeomorphism on Ω_x. By setting $\xi = [\phi_1(x), \phi_2(x), ..., \phi_\rho(x)]^T$ and $\eta = [\phi_{\rho+1}(x), \phi_{\rho+2}(x), ..., \phi_n(x)]^T$, system (1) can be expressed in the normal form:

$$\begin{aligned} \dot{\eta} &= q(\xi, \eta) \\ \dot{\xi}_j &= \xi_{j+1}, \quad j = 1, ..., \rho - 1 \\ \dot{\xi}_\rho &= b(\xi, \eta, u) \\ y &= \xi_1 \end{aligned} \tag{9}$$

where $b(\xi, \eta, u) = L_f^\rho h$; $q(\xi, \eta) = [L_f \phi_{\rho+1}(x), L_f \phi_{\rho+2}(x), ..., L_f \phi_n(x)]^T$; $x = \Phi^{-1}(\xi, \eta)$, for $(\xi, \eta, u) \in \bar{U} := \{(\xi, \eta, u) | (\xi, \eta) \in \Phi(\Omega_x); u \in \Omega_u\}$.

Assumption 4. The zero dynamics of system (9), given by $\dot{\eta} = q(0, \eta)$ are exponentially stable. In addition, the function $q(\xi, \eta)$ is Lipshitz in ξ, i.e.

$$\|q(\xi, \eta) - q(0, \eta)\| \leq a_\xi \|\xi\| + a_q, \quad \forall (\xi, \eta) \in \Phi(\Omega_x) \tag{10}$$

Under Assumption 4, by the converse Lyapunov theorem, there exists a Lyapunov function $V_0(\eta)$ which satisfies the following inequalities:

$$\gamma_1 \|\eta\|^2 \leq V_0(\eta) \leq \gamma_2 \|\eta\|^2 \tag{11}$$

$$\frac{\partial V_0}{\partial \eta} q(0, \eta) \leq -\lambda_a \|\eta\|^2 \tag{12}$$

$$\left\| \frac{\partial V_0}{\partial \eta} \right\| \leq \lambda_b \|\eta\| \tag{13}$$

where γ_1, γ_2, λ_a, and λ_b are positive constants.

For ease of notation, define $g(\xi,\eta,u) = \frac{\partial b(\xi,\eta,u)}{\partial u}$. According to Assumption 3, we know that $g(\xi,\eta,u) \neq 0$, $\forall(\xi,\eta,u) \in \bar{U}$.

Assumption 5. The sign of $g(\xi,\eta,u)$, i.e. control direction, is unknown. and there exist constants $\underline{g}$ and $\bar{g}$ such that $0 < \underline{g} \leq |g(\xi,\eta,u)| \leq \bar{g} \leq \infty$. In addition, the constants $\underline{g}$ and $\bar{g}$ are used to handle the stability analysis only.

3. CONTROL DESIGN

In this section, the adaptive NN control scheme is constructed, based on backstepping and the use of Mean Value Theorem and Nussbaum function to handle the non-affine functions. We employ backstepping for the ξ subsystem, and then make use of the exponential stability of the zero dynamics to show that the overall closed loop system is stable and that output tracking is achieved.

Step 1: Let $z_1(t) = \xi_1(t) - y_d(t)$ and $z_2(t) = \xi_2(t) - \alpha_1(t)$, where $\alpha_1(t)$ is a virtual control function to be determined. Define a quadratic function $V_1 = \frac{1}{2}z_1^2$, and choose the virtual control α_1 as

$$\alpha_1 = -k_1 z_1 + \dot{y}_d, \tag{14}$$

where positive constant $k_1 > \frac{1}{4}$. We can show that

$$\dot{V}_1 = -k_1 z_1^2 + z_1 z_2 \leq -\gamma_1 V_1 + z_2^2 \tag{15}$$

where $\gamma_1 = 2(k_1 - \frac{1}{4})$. From (15), we know that if z_2 can be can be regulated such that it is bounded, then, according to Lemma 2, V_1 must be bounded in finite time, such that the boundedness of z_1 can also achieved. The regulation of z_2 will be dealt with in the following steps.

Step i ($\mathbf{i = 2, ..., \rho - 1}$): Let $z_i(t) = \xi_i(t) - \alpha_{i-1}(t)$, and $\alpha_i(t)$ be a virtual control function, which is defined as

$$\alpha_i = -k_i z_i + \dot{\alpha}_{i-1}, \tag{16}$$

where the derivative can be written as

$$\dot{\alpha}_{i-1} = \frac{\partial \alpha_{i-1}}{\partial \bar{\xi}_{i-1}} \dot{\bar{\xi}}_{i-1} + \sum_{k=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(k)}} y_d^{(k+1)} \tag{17}$$

where $\bar{\xi}_i := [\xi_1, ..., \xi_i]^T$. Define quadratic function $V_i = \frac{1}{2}z_i^2$. As the procedure in Step 1, it can be shown that

$$\dot{V}_i \leq -\gamma_i V_i + z_{i+1}^2 \tag{18}$$

Similarly, according to Lemma 2, the regulation of z_i will be guaranteed by the regulation of z_{i+1}, which will be dealt with in the subsequent step.

Step ρ: This is the final step where the actual control law u will be designed. Define $z_\rho = \xi_\rho - \alpha_{\rho-1}$, it derivative is

$$\begin{aligned}\dot{z}_\rho &= \dot{\xi}_\rho - \dot{\alpha}_{\rho-1} \\ &= b(\xi,\eta,u) - \dot{\alpha}_{\rho-1}\end{aligned} \tag{19}$$

Using the Mean Value Theorem in Lemma 1, there exists $\lambda(0 < \lambda < 1)$ such that

$$b(\xi,\eta,u) = b(\xi,\eta,u_0) + g_\lambda(u - u_0) \tag{20}$$

where $g_\lambda := g(\xi,\eta,u_\lambda)$, and $u_\lambda = \lambda u + (1-\lambda)u_0$. By choosing $u_0 = 0$, (20) is expressed as

$$b(\xi,\eta,u) = b(\xi,\eta,0) + g_\lambda u \tag{21}$$

Substituting (21) into (19), we obtain

$$\begin{aligned}\dot{z}_\rho &= b(\xi,\eta,0) - \dot{\alpha}_{\rho-1} + g_\lambda u \\ &= b'(\xi,\eta,\dot{\alpha}_{\rho-1}) + g_\lambda u\end{aligned} \tag{22}$$

where $b'(\xi,\eta,\dot{\alpha}_{\rho-1}) = b(\xi,\eta,0) - \dot{\alpha}_{\rho-1}$ is also an unknown function, which can be approximated by RBFNNs as

$$b'(\xi,\eta,\dot{\alpha}_{\rho-1}) = W^{*T}S(Z) + \varepsilon \tag{23}$$

where $Z = [\xi,\eta,\dot{\alpha}_{\rho-1}]^T \in \Omega \subset R^{n+1}$, W^* denotes the vector of ideal constant weights, and $|\varepsilon| \leq \varepsilon^*$ is the approximation error with constant $\varepsilon^* > 0$. Since W^* is unknown, we use $\hat{W}$ to estimate W^*, and the weight estimation error be $\tilde{W} = \hat{W} - W^*$.

Consider the following quadratic function

$$V_\rho = \frac{1}{2}z_\rho^2 + \frac{1}{2}\tilde{W}^T\Gamma^{-1}\tilde{W} \tag{24}$$

where $\Gamma = \Gamma^T > 0$. Its derivative along (22) and (23) is

$$\dot{V}_\rho = z_\rho[W^{*T}S(Z) + \varepsilon + g_\lambda u] + \tilde{W}^T\Gamma^{-1}\dot{\hat{W}} \tag{25}$$

Consider the following actual control law and adaptation law

$$u = N(\zeta)[z_\rho + \hat{W}^T S(Z)] \tag{26}$$

$$\dot{\zeta} = z_\rho^2 + z_\rho \hat{W}^T S(Z) \tag{27}$$

where $N(\cdot)$ is an even smooth Nussbaum-type function. Substituting (26) into (25), we have

$$\begin{aligned}\dot{V}_\rho = &-z_\rho^2 + (g_\lambda N(\zeta) + 1)[z_\rho^2 + z_\rho \hat{W}^T S(Z)] \\ &+ \tilde{W}^T\Gamma^{-1}[\dot{\hat{W}} - z_\rho \Gamma S(Z)] + z_\rho\varepsilon\end{aligned} \tag{28}$$

Consider the follow NN weight adaptation law

$$\dot{\hat{W}} = \Gamma(z_\rho S(Z) - \sigma\hat{W}) \quad (29)$$

where $\sigma > 0$ is a constant design parameter. Then, substituting (27) and (29) into (28) yields

$$\dot{V}_\rho = -z_\rho^2 + (g_\lambda N(\zeta) + 1)\dot{\zeta} - \sigma\tilde{W}^T\hat{W} + z_\rho\varepsilon \quad (30)$$

By completion of squares and Young's inequality, the following inequalities hold

$$-\sigma\tilde{W}^T\hat{W} \le -\frac{\sigma}{2}\|\tilde{W}\|^2 + \frac{\sigma}{2}\|W^*\|^2 \quad (31)$$

$$z_\rho\varepsilon \le \frac{1}{4}z_\rho^2 + \varepsilon^2 \quad (32)$$

Substituting (31) and (32) into (30), we have

$$\dot{V}_\rho \le -\gamma_\rho V_\rho + (g_\lambda N(\zeta) + 1)\dot{\zeta} + c_\rho \quad (33)$$

where γ_ρ and c_ρ are positive constants, which are defined as

$$\gamma_\rho = \min\{\frac{3}{2}, \frac{\sigma}{\lambda_{max}(\Gamma^{-1})}\}, \quad c_\rho = \frac{\sigma}{2}w_m^2 + \varepsilon^{*2}$$

Multiplying (33) by $e^{\gamma_\rho t}$ and integrating over $[0, t]$, we have

$$V_\rho \le V_\rho(0) + \frac{c_\rho}{\gamma_\rho} + e^{-\gamma_\rho t}\int_0^t (g_\lambda N(\zeta) + 1)e^{\gamma_\rho \tau}\dot{\zeta}d\tau$$

According to Lemma 3, V_ρ is bounded, hence z_ρ, $\hat{W}$ are uniformly ultimately bounded. Then, applying Lemma 2 $\rho - 1$ times backward, it can be seen that V_i, z_i $i = 1, ..., \rho - 1$ and hence x_i are bounded.

The following lemma is useful for stability analysis of the internal dynamics.

Lemma 4. Given that Assumptions 2 and 4 are satisfied, there exist positive constants a_1, a_2 and T_0 such that the trajectories $\eta(t)$ of the internal dynamics satisfy

$$\|\eta(t)\| \le a_1(\|z(t)\| + \|\xi_d(t)\|) + a_2, \forall\ t > T_0 \quad (34)$$

Proof: The proof is very similar to that given in (Ge and Zhang, 2003). It is omitted here for space limitation. ■

We summarize our results in the following theorem.

Theorem 1. Consider the general nonlinear system (1) satisfying Assumptions 2-5, with control law (26) and adaptation laws (29). For initial conditions $\xi(0)$, $\eta(0)$, $\tilde{W}(0)$ belonging to any compact set Ω_0, all closed loop signals are Semi-Globally Uniformly Ultimately Bounded (SGUUB).

Proof: The proof can be easily completed by following the above design procedures from Step 1 to Step n. ■

4. SIMULATION STUDIES

To demonstrate the effectiveness of the above proposed control, the following continuous stirred tank reactors (CSTR) studied in (Ge and Zhang, 2003) is used for simulation here.

$$\begin{aligned}
\dot{x}_1 &= 1 - x_1 - c_1x_1 + c_2x_2^2 \\
\dot{x}_2 &= -x_2 + c_1x_1 - c_2x_2^2 - c_3x_2^2 + u \\
\dot{x}_3 &= -x_3 + c_3x_2^2 \\
y &= h(x) = x_1
\end{aligned}$$

It can be shown that the system has strong relative degree 2, with the normal form

$$\begin{aligned}
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= f_0(\xi_1, \xi_2) + g_0(\xi_1, \xi_2)u \\
\dot{\eta} &= -\eta + c_3 f_t \\
y &= \xi_1
\end{aligned}$$

where $f_0(\xi_1, \xi_2) = 2c_1c_2\sqrt{f_t}\xi_1 - (c_1+1)\xi_2 - 2c_2[1 + (c_2+c_3)\sqrt{f_t}]f_t$, $g_0(\xi_1, \xi_2) = 2c_2\sqrt{f_t}$, and $f_t = [(1 + c_1)\xi_1 + \xi_2 - 1]/c_2$. Assuming the parameters are chosen as follows: $c_1 = 20$, $c_2 = 0.1$ and $c_3 = 10$.

The control objective is to make y track the following reference model output y_d. For simplicity, the reference signal in this paper is zero. The proposed feedback controller (26), (27) and adaption law (29) are adopted here. Neural networks $\hat{W}^T S(Z)$ contains 256 nodes (i.e.,$l = 256$), with centers $\mu_l(l = 1, ..., l)$ evenly spaced in $[-4, 4] \times [-4, 4] \times [-4, 4] \times [-4, 4]$, and widths $\eta_l = 1.0(l = 1, ..., l_2)$. The design parameters of the above controller are $k_1 = 5.0$, $\Gamma = \text{diag}\{2.0\}$, $\sigma_1 = 0.1$. The initial weights $\hat{W}(0) = 0.0$. The initial conditions $[\xi_1(0), \xi_2(0)]^T = [1.0, 1.0]^T$.

The simulation results are shown in Figs 1 and 2. We can see that the tracking performance is good and the internal dynamics is stable. In addition, the boundedness of all other signals in the closed loop is also guaranteed.

5. CONCLUSION

In this paper, adaptive neural network (NN) control has been investigated for a class of general non-affine nonlinear system with unknown control direction, using backstepping method combining with the Mean Value Theorem and Nussbaum gain. The effectiveness of the proposed control has been proved with mathematic rigor and verified by the simulation results.

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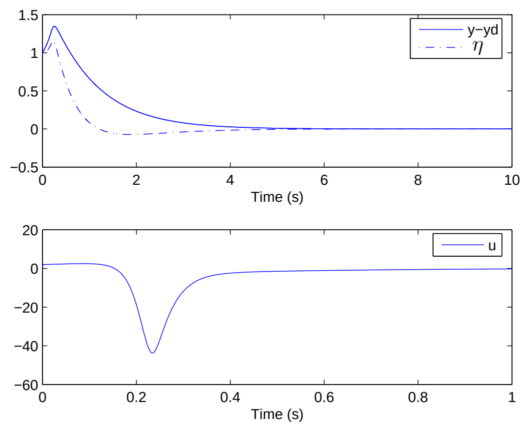

Fig. 1. Top: output tracking error $y - y_d$ and internal dynamics η; Bottom: control signal u.

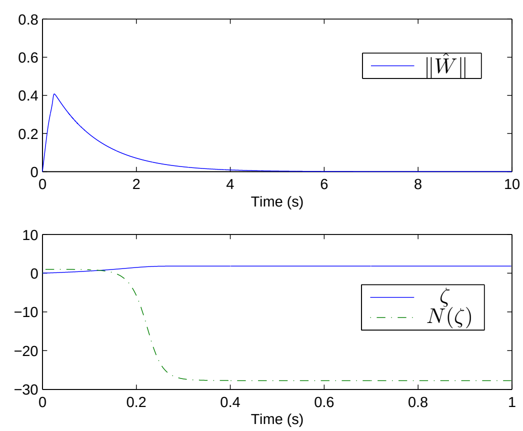

Fig. 2. Top: NN weight norm $\|\hat{W}\|$; Bottom: parameter ζ and Nussbaum function $N(\zeta)$.