Bounded Integral Control for Regulating Input-to-state Stable Non-linear Systems

George C. Konstantopoulos, Qing-Chang Zhong, Beibei Ren and Miroslav Krstic

***Abstract*— In this paper, a bounded integral controller (BIC) is proposed for regulating non-linear systems with guaranteed closed-loop system stability. It is proven that the BIC generates a bounded control output independently from the plant parameters and states and guarantees closed-loop system stability in the sense of boundedness with the only knowledge of the input-to-state (practical) stability (ISpS) of the plant. Furthermore, an analytic selection of the BIC parameters is presented to guarantee a given bound for the control output, suitably extending the BIC application to locally ISpS plant systems as well. Its performance is investigated using non-linear Lyapunov methods and it is proven that it can approximate the behavior of the traditional integral controller (IC) near steady state. Simulation results of a locally ISS system are provided to compare the BIC with the IC operation under a given bound of the control output.**

I. Introduction

During the last decades, integral control (IC) has been the core of control systems for achieving asymptotic regulation and disturbance rejection for systems with inherent parameter variations. However, even for linear systems, closed-loop system stability with an integral control action is only guaranteed under sufficiently small integral gain and under necessary and sufficient conditions of the plant [1], [2]. Particularly in [3], an analytic calculation of the maximum integral gain for guaranteeing closed-loop system stability of finite-dimensional linear systems has been presented.

IC has been suitably designed for non-linear systems with local closed-loop stability results [4], [5], [6]. Semi-global results were provided in [7], [8], [9] for minimum-phase systems using output feedback control and high-gain observers. The procedure was based on transforming the system into the normal form [10] and also using a saturating controller outside a compact set of interest. These results were further extended in [11] where a robust integral control was designed based on the relative-degree of the non-linear plant. Recently, conditional integrators were proposed [12], [13] which provide the integral action inside a boundary layer and act as a stable system outside of it. In many of the previous works, some of the assumptions mentioned for the plant are directly related to the input-to-state stability (ISS) property [14], [15] while in [11], the generalized small-gain theorem was used [16], [17], which represents a fundamental tool for robust stability. Similar approaches of IC in port-Hamiltonian systems for disturbance rejection can be also found in recent works [18], [19], where the port-Hamiltonian form is retained and closed-loop stability is guaranteed for systems with relative-degree higher than one.

In the previous works, the structure of the non-linear system (relative-degree, parameters etc.) directly affects the IC design which often results in a complicated control scheme. Furthermore, in several applications, the plant input should remain bounded due to physical limitations. In these cases, usually a saturation unit can be used at the controller output to achieve the desired bound, but this can cause undesired oscillations and damage the system. When actuator constraints are present, anti-windup techniques are often applied [20], [21], [22], [23] and can guarantee system stability, while depending on the system parameters in most of the cases. In the present paper, the plant input is initially assumed unconstrained and whenever bounded input requirements are present, they are related to stability properties, such as the locally ISS systems which have not been extensively exploited in the literature. Therefore, the existence of a generic integral controller that is independent from the system parameters and structure and can guarantee closed-loop system stability for either globally or locally ISS systems without the need of a saturation unit is investigated.

In this paper, a bounded integral controller (BIC) is proposed to guarantee non-linear closed-loop system stability for globally or locally input-to-state practically stable (ISpS) systems. Based on the widely used concept of the IC, the BIC consists of a non-linear second-order dynamic controller which approximates the IC near steady state and produces a bounded output. Using non-linear Lyapunov analysis and based only on the knowledge of the ISpS property of the plant [14], [15], it is proven that the proposed BIC guarantees closed-loop system stability in the sense of boundedness using the generalized small-gain theorem [16]. Although the plant input is considered unconstrained, often a given bound is introduced for stability reasons, such as for locally ISS systems. Therefore, an analytic selection of the controller parameters is presented to achieve a bounded controller output within a given range and approximate the IC near the steady state, thus extending the stability analysis to locally ISpS plant systems. Both a static and an adaptive choice

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of the controller gain are provided to investigate the BIC performance and also guarantee that no undesired oscillations can occur. Due to the generic structure of the BIC and the fact that it operates as the original IC near steady state, the proposed approach can be easily applied in combination with the existing IC techniques for non-linear systems. Simulation results of a locally ISS system are provided to verify the BIC method compared to the IC.

The paper is organized as follows: In Section II, a necessary theory is briefly presented and the traditional IC is revisited. In Section III, the main result is presented, where the proposed BIC is investigated and closed-loop system stability is proven. In Section IV, an analytic parameter selection is provided to result in the BIC with a given output bound. In Section V, simulation results are presented to compare the IC with the BIC operation and finally, in Section VI, some conclusions are drawn.

II. Preliminaries

A. Input-to-state (practical) stability

Consider the non-linear system

$$\dot{x} = f\left(x, u\right) \tag{1}$$

where $f : D \times D_u \to R^n$ is locally Lipschitz and D is an open neighborhood of the origin.

Definition 1: [4] The system (1) is said to be *locally input-to-state stable (LISS)* if there exist a class $\mathcal{KL}$ function β, a class $\mathcal{K}$ function γ and positive constants k_1 and k_2 such that for any initial state $x(0)$ with $\|x\left(0\right)\| < k_1$ and any input $u(t)$ with $\sup_{t\geq 0} \|u\left(\tau\right)\| < k_2$, the solution $x(t)$ exists and satisfies

$$\|x\left(t\right)\| \leq \beta\left(\|x\left(0\right)\|, t\right) + \gamma\left(\sup_{0\leq\tau\leq t} \|u\left(\tau\right)\|\right) \tag{2}$$

for all $t \geq 0$. It is said to be *input-to-state stable (ISS)* if $D = R^n$, $D_u = R^m$ and inequality (2) is satisfied for any initial state $x\left(0\right)$ and any bounded input $u(t)$.

Additionally, if there exists a non-negative constant d such that

$$\|x\left(t\right)\| \leq \beta\left(\|x\left(0\right)\|, t\right) + \gamma\left(\sup_{0\leq\tau\leq t} \|u\left(\tau\right)\|\right) + d \tag{3}$$

then the system (1) is said to be *input-to-state practically stable (ISpS)* if the conditions hold globally [16] (consequently LISpS in the local sense).

B. Small-gain theorem

Consider the composite feedback interconnection form Σ of two subsystems Σ_1 and Σ_2 shown in Fig. 1, where

$$\begin{aligned} \dot{x} &= f(x, z, u_1) && (4)\\ \dot{z} &= h(z, x, u_2). && (5) \end{aligned}$$

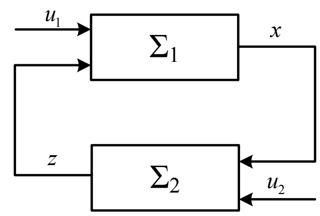

Fig. 1: Composite feedback interconnection

Small-gain theorem is used to guarantee stability of the interconnected system Σ [16], [24]. Although several versions of the small-gain theorem have been proposed in the literature [4], [16], [25], the theorem that follows introduces the generalized ISpS small-gain theorem [16] which proves ISpS of Σ by verifying the ISpS of Σ_1 and Σ_2 and checking a condition that relates their ISpS gains.

Theorem 1: Suppose that:

1) Σ_1 is ISpS (resp. ISS) with respect to $v_1 = (z, u_1)$, with gain γ_1 from z to x, i.e.,

$$\begin{aligned} \|x(t)\| \leq\; & \beta_1\left(\|x\left(0\right)\|, t\right) + \gamma_1\left(\sup_{0\leq\tau\leq t} \|z\left(\tau\right)\|\right) \\ & + \bar{\gamma}_1\left(\sup_{0\leq\tau\leq t} \|u_1\left(\tau\right)\|\right) + d_1 \end{aligned}$$

for some $\beta_1 \in \mathcal{KL}$, $\gamma_1, \bar{\gamma}_1 \in \mathcal{K}_\infty$ and $d_1 \geq 0$.

2) Σ_2 is ISpS (resp. ISS) with respect to $v_2 = (x, u_2)$, with gain γ_2 from x to z, i.e.,

$$\begin{aligned} \|z(t)\| \leq\; & \beta_2\left(\|z\left(0\right)\|, t\right) + \gamma_2\left(\sup_{0\leq\tau\leq t} \|x\left(\tau\right)\|\right) \\ & + \bar{\gamma}_2\left(\sup_{0\leq\tau\leq t} \|u_2\left(\tau\right)\|\right) + d_2 \end{aligned}$$

for some $\beta_2 \in \mathcal{KL}$, $\gamma_2, \bar{\gamma}_2 \in \mathcal{K}_\infty$ and $d_2 \geq 0$.

3) There exist two functions $\rho_1, \rho_2 \in \mathcal{K}_\infty$ and a non-negative real number s_l such that

$$\left.\begin{aligned} (Id + \rho_2) \circ \gamma_2 \circ (Id + \rho_1) \circ \gamma_1(s) \leq s \\ (Id + \rho_1) \circ \gamma_1 \circ (Id + \rho_2) \circ \gamma_2(s) \leq s \end{aligned}\right\} \; \forall s \geq s_l \tag{6}$$

where Id describes the identity function, i.e. $Id(s) = s$ and $\circ$ is the function composition.

Then Σ is ISpS (resp. ISS when $s_l = 0$) with respect to the input $u = (u_1, u_2)$.

C. Traditional integral control

For the plant system in the generalized non-linear form (1), assume that the control task is the regulation of a scalar function $g(x)$ to zero, which includes the common regulation scenario of a state variable x_i to a desired level x_i^{ref}, i.e. $g(x) = x_i^{ref} - x_i$. For simplicity consider a single-input system in the form of (1), but the results can be generalized for multiple-input systems.

It is well known that in many regulation problems, a dynamic controller is required. The most common and simple dynamic controller is the integral control, since it can guarantee a zero steady state error. Therefore, consider the simple IC, which is given in the following form:

$$u(t) = \int_0^t g\left(x(\tau)\right) d\tau \tag{7}$$

which introduces a dynamic controller that can be written as

$$u = w \tag{8}$$
$$\dot{w} = g(x). \tag{9}$$

Although the IC is widely used in control problems, the main drawback is that it cannot guarantee closed-loop system stability, even if the plant introduces useful properties such as the ISpS.

The purpose of this work is to propose a suitable generalized controller that acts in the same way as the IC, achieves accurate regulation $g(x) = 0$ and guarantees closed-loop system stability for a generic non-linear plant system with ISpS property. This problem is investigated in the following section.

III. Main result

A. Bounded integral controller (BIC)

The main task is to design a control scheme that achieves the desired regulation as the traditional IC (8)-(9) and can guarantee closed-loop stability. Such a controller is proposed below:

$$u = w \tag{10}$$
$$\begin{bmatrix} \dot{w} \\ \dot{w}_q \end{bmatrix} = A_c\left(w, w_q, g(x)\right)\begin{bmatrix} w \\ w_q \end{bmatrix} \tag{11}$$

with

$$A_c = \begin{bmatrix} -k\left(\frac{w^2}{u_{max}^2} + \frac{(w_q-b)^2}{\epsilon^2} - 1\right) & g(x)c \\ -\frac{\epsilon^2}{u_{max}^2}g(x)c & -k\left(\frac{w^2}{u_{max}^2} + \frac{(w_q-b)^2}{\epsilon^2} - 1\right) \end{bmatrix}$$

where w and w_q are the controller state variables, k and b are non-negative constants and ϵ, u_{max} and c are positive constants.

In order to investigate the proposed controller operation, consider the following Lyapunov function candidate

$$V = \frac{w^2}{u_{max}^2} + \frac{w_q^2}{\epsilon^2} \tag{12}$$

which has an ellipsoid form on $w - w_q$ plane with center the origin. Taking the time derivative of V, it yields

$$\begin{aligned} \dot{V} &= \frac{2w\dot{w}}{u_{max}^2} + \frac{2w_q\dot{w}_q}{\epsilon^2} \\ &= -2k\left(\frac{w^2}{u_{max}^2} + \frac{(w_q-b)^2}{\epsilon^2} - 1\right)\left(\frac{w^2}{u_{max}^2} + \frac{w_q^2}{\epsilon^2}\right) \\ &= -2k\left(\frac{w^2}{u_{max}^2} + \frac{(w_q-b)^2}{\epsilon^2} - 1\right)V. \end{aligned} \tag{13}$$

The sign of $\dot{V}$ is related to another ellipse at point $(0, b)$:

$$C = \left\{w, w_q \in R : \frac{w^2}{u_{max}^2} + \frac{(w_q-b)^2}{\epsilon^2} = 1\right\}. \tag{14}$$

Thus, $\dot{V}$ is negative outside of C and positive inside. Therefore, it becomes clear that for a given non-negative constant b, the controller states w and w_q are bounded signals independently from the values of $g(x)$ and since the controller introduces a double integrator structure, it forms a *bounded integral controller (BIC)*. This represents an extremely important property that can guarantee the stability of the closed-loop system as proven in the analysis that follows.

B. Closed-loop system stability

Consider the non-linear plant given in the form

$$\dot{x} = f(x, u, u_1) \tag{15}$$

where u describes the control input and u_1 is a vector of external uncontrolled inputs.

After closing the loop of the generic plant with the BIC, the resulting closed-loop system is described in Fig. 2, which is a composite feedback interconnection form as presented in Fig. 1. The plant is assumed to be ISpS and function $g(x)$ to be locally Lipschitz, which hold true in most control applications. Then, the following theorem can be formulated.

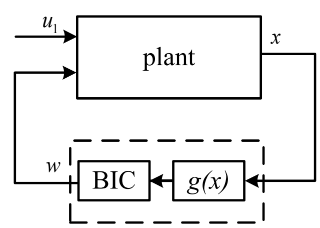

Fig. 2: Closed-loop system with BIC

Theorem 2: The feedback interconnection of plant system (15) with the proposed BIC (10)-(11) is ISpS with respect to input u_1, when the plant is ISpS with respect to (u, u_1).

Proof: As already described in subsection III-A, by selecting the Lyapunov function V from (12), it is proven that the derivative of V is negative outside C. Therefore, there exists a closed-set in the form of the Lyapunov function (12), outside of which $\dot{V} < 0$ holds true. This is represented by ellipse S in Fig. 3 which is in the form

$$\frac{w^2}{u_{max}^2} + \frac{w_q^2}{\epsilon^2} = \alpha^2 \tag{16}$$

for different values of α. Since S has the same form of C and they intersect at point $(0, b+\epsilon)$, then it holds true that $\alpha = \frac{b+\epsilon}{\epsilon}$. As a result

$$S = \left\{w, w_q \in R : \frac{w^2}{\left(\frac{(b+\epsilon)u_{max}}{\epsilon}\right)^2} + \frac{w_q^2}{(b+\epsilon)^2} = 1\right\} \tag{17}$$

simply implies that there exists an ultimate bound for the controller states w and w_q. Therefore it is proven that for any initial conditions $w(0)$ and $w_q(0)$, there exists a class $\mathcal{KL}$ function β and a future time instant $T \geq 0$ such that [4]

$$\left\| \begin{matrix} w \\ w_q \end{matrix} \right\| \leq \beta\left(\left\| \begin{matrix} w(0) \\ w_q(0) \end{matrix} \right\|, t\right) \quad \forall t \leq T \tag{18}$$

and

$$\left\| \begin{array}{c} w \\ w_q \end{array} \right\| \le \frac{(u_{max}+\epsilon)(b+\epsilon)}{\epsilon} \quad \forall t \ge T. \tag{19}$$

Inequality (19) results from the norm properties $\left(\|x\|_p \le \|x\|_1, \forall p \ge 1\right)$ and taking into account from (17) that for all $t \ge T$, i.e. after the time instant that w and w_q enter ellipse S, it holds true that $|w| \le \frac{(b+\epsilon)u_{max}}{\epsilon}$ and $|w_q| \le b+\epsilon$ which yield that $\left\| \begin{array}{c} w \\ w_q \end{array} \right\|_1 \le \frac{(u_{max}+\epsilon)(b+\epsilon)}{\epsilon}$.

Therefore, the control states solution can be written in the form:

$$\left\| \begin{array}{c} w \\ w_q \end{array} \right\| \le \beta\left(\left\| \begin{array}{c} w(0) \\ w_q(0) \end{array} \right\|, t\right) + d \tag{20}$$

where $d = \frac{(u_{max}+\epsilon)(b+\epsilon)}{\epsilon}$ is a positive constant. Since inequality (20) is satisfied independently from any bounded controller input $g(x)$, then the controller states can be written in the form of (3) with zero gain, i.e. $\gamma_{control} = 0$, regardless of the selection of the initial conditions w_0, w_{q0} and the parameters k, u_{max}, b and ϵ. This becomes clear from the fact that the function $g(x)$ does not affect the stability analysis of the controller dynamics, as presented in subsection III-A using the Lyapunov method.

As a result, since the closed-loop system, shown in Fig. 2, is given in the composite feedback interconnection form, then the small-gain theorem given from Theorem 1 can be applied. Particularly, since the controller gain is zero, then the condition (6) is obviously satisfied. Therefore, the closed-loop system is ISpS with respect to the external input u_1. ■

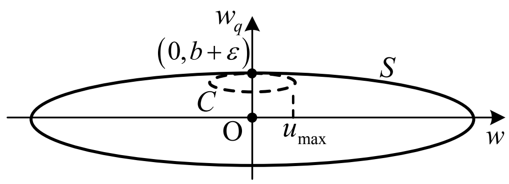

Fig. 3: Ellipse S and boundedness of states w and w_q

The special structure of the BIC guarantees the ISpS property for a wide class of non-linear systems. It is obvious that if no external input exists for the plant, i.e. $u_1 = 0$, the closed-loop system solution is globally bounded. It is also worth noting that Theorem 2 holds independently from the plant structure, the controller parameters k, $b \ge 0$ and u_{max}, $\epsilon > 0$ or the initial conditions of the BIC states and therefore the BIC provides a generic controller for non-linear ISpS systems.

IV. BIC WITH A GIVEN OUTPUT BOUND

A. Parameter selection

In the main result, the BIC output is proven to remain bounded. However, the actual bound is not known in general. In order for the control signal to remain inside a given bound $u \in [-u_{max}, u_{max}]$, where u_{max} denotes the maximum absolute value of the controller output, the BIC parameters can be selected as

$$b = 0,\ \epsilon = 1,\ c > 0,\ k \ge 0. \tag{21}$$

According to this selection, the BIC dynamics (11) become

$$\begin{bmatrix} \dot{w} \\ \dot{w}_q \end{bmatrix} = \begin{bmatrix} -k\left(\frac{w^2}{u_{max}^2} + w_q^2 - 1\right) & g(x)c \\ -\frac{1}{u_{max}^2} g(x)c & -k\left(\frac{w^2}{u_{max}^2} + w_q^2 - 1\right) \end{bmatrix} \begin{bmatrix} w \\ w_q \end{bmatrix} \tag{22}$$

where the initial conditions are chosen $w_0 = 0$ (initial condition of the IC, usually zero) and $w_{q0} = 1$. Now, considering the Lyapunov function candidate

$$W = \frac{w^2}{u_{max}^2} + w_q^2, \tag{23}$$

resulting from (12) with the proposed parameter selection, its derivative becomes

$$\dot{W} = -2k\left(\frac{w^2}{u_{max}^2} + w_q^2 - 1\right) W \tag{24}$$

which implies that the ellipse

$$C_0 = \left\{ w, w_q \in R : \frac{w^2}{u_{max}^2} + w_q^2 = 1 \right\} \tag{25}$$

as shown in Fig. 4 is an attractive ellipse and since the initial conditions are defined on C_0, then obviously

$$\dot{W} = 0 \Rightarrow W(t) = W(0) = 1,\ \forall t \ge 0 \tag{26}$$

proving that the BIC states will start and remain at all times on the ellipse C_0, i.e. the diagonal term $-k\left(\frac{w^2}{u_{max}^2} + w_q^2 - 1\right)$ will be zero and is only used to increase robustness with respect to external disturbances or calculation errors. By considering the following transformation

$$\begin{array}{c} w = u_{max}\sin\theta \\ w_q = \cos\theta, \end{array} \tag{27}$$

it yields from the BIC dynamics (22) that

$$\dot{\theta} = \frac{g(x)c}{u_{max}} \tag{28}$$

which proves that w and w_q will move on the ellipse C_0 with angular velocity $\dot{\theta}$ (Fig. 4). If it is assumed that there exists a desired equilibrium point for the plant with $u = u^*$, for which $g(x) = 0$, then w and w_q can stop at the desired equilibrium since $\dot{\theta} = 0$. Therefore the controller output $u = w$ will always remain in the range $[-u_{max}, u_{max}]$ and as soon as w and w_q approach the desired equilibrium, $\dot{\theta}$ will approach zero, making the controller states to slow down and allowing them to converge to the desired point $\left(u^*, w_q^*\right)$ on $w - w_q$ plane. This extends the BIC operation to guarantee stability for locally ISpS systems.

B. Design of gain c

From (28), it becomes clear that the gain $c > 0$ affects the angular velocity $\dot{\theta}$ and consequently the dynamic performance. From the first dynamic equation of (22) it yields

$$\dot{w} = g(x)cw_q. \tag{29}$$

It would be reasonable, near the steady state desired equilibrium, for the BIC to act similar as the IC. This offers the following possible choices.

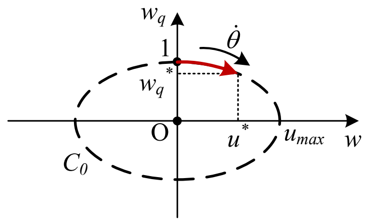

Fig. 4: BIC states on $w - w_q$ plane

Option1: Static choice

By directly comparing (29) and (9), a static choice of c can be

$$c = \frac{1}{w_q^*} = \frac{u_{max}}{\sqrt{u_{max}^2 - (u^*)^2}} \tag{30}$$

which means $cw_q^* \approx 1$ near the steady state and the BIC approximates the IC, if one compares (29) with (9). Note that the exact value of u^* is not required to be known exactly for the BIC operation. This is just to approximate the IC near steady state. Stability is always guaranteed independently from any $c > 0$ as proven in the main result.

Option2: Adaptive choice

Since parameter c does not affect the stability analysis, it doesn't have to be static. An adaptive choice of c can achieve a faster transient performance. Additionally, it is desirable for the BIC states to operate in the first 2 quadrants of $w - w_q$ plane where $u = w \in [-u_{max}, u_{max}]$, i.e. where $w_q > 0$ and avoid passing from the first to the fourth or from the second to the third quadrant (this might cause oscillations). For this reason, the angular velocity $\dot{\theta}$ should approach zero if the BIC states try to reach the horizontal axis, i.e., when $w_q \to 0$, independently from the function $g(x)$. To this end, parameter c can be tuned according to w_q as given below:

$$c = \frac{w_q}{\left(w_q^*\right)^2} = \frac{w_q u_{max}^2}{u_{max}^2 - (u^*)^2}. \tag{31}$$

Once again the accurate knowledge of u^* is irrelevant and since w_q starts from 1 and begins to decrease, initially cw_q is larger than 1 and the BIC obviously achieves a faster transient than the IC.

V. Simulation results

In order to verify the proposed BIC operation with respect to the traditional IC, the following non-linear system is considered

$$\dot{x} = -\frac{x}{1+x^2} + u. \tag{32}$$

It can be seen that the unforced system (32) ($u = 0$) is globally asymptotically stable at the origin, but for $|u| > 0.5$, the plant (32) is unstable [4]. Thus (32) forms a locally ISS system with $\sup_{t \geq 0} \|u(\tau)\| \leq u_{max} = 0.5$, which constitutes an interesting problem in terms of stability. Assume that the objective is to regulate the state x at the reference value $x^{ref} = 0.6$. This can be achieved using the traditional IC with $g(x) = k_I \left(x^{ref} - x\right)$ and $k_I = 0.02$. In the same frame, the BIC can be used with the same $g(x)$ and $u_{max} = 0.5$, $\epsilon = 1$, $b = 0$, $k = 10$ and c given from (31) with $u^* = 0.44$. For the simulated scenario, at time instant $t = 150s$, a continuously increasing error occurs at the sensor with rate -1%/sec (i.e. the measured state reduces by 1% of the actual state per second), in order to investigate the performance under a system failure which will shift the equilibrium outside the bounded range and force the plant input to increase.

As it is shown in Fig. 5a and 5b, before the error occurs to the sensor, the control input u stays below u_{max} and both the IC and the BIC achieve the desired regulation. Due to the adaptive choice of the gain c, the BIC achieves a faster transient. When the error occurs, the input u increases and exceeds u_{max} leading the closed-loop system with the IC to instability. On the other hand, the closed-loop system with the BIC slows down as $u \to u_{max}$ due to the adaptive choice of gain c. Therefore, even after the sensor failure, stability is always guaranteed and the BIC output is maintained in the given range. The BIC operation can be observed on the $w - w_q$ phase portrait in Fig. 5c, where the controller states move always on the ellipse C_0 and stay exclusively in the first quadrant.

VI. Conclusions

In this paper, a bounded integral control (BIC) was proposed to guarantee non-linear closed-loop system stability for globally or locally ISpS plants. With only knowledge the ISpS property of the plant, closed-loop system stability was proven using non-linear Lyapunov analysis and the generalized small-gain theorem. A suitable parameter selection was provided to produce a BIC output with a given bound and prove that the BIC approximates the performance of the traditional IC near steady state. A static and an adaptive choice of the controller gain were also presented, where it is proven that a faster transient can be achieved and no undesired oscillations will occur. Simulation results of a locally ISS system were provided to verify the proposed BIC approach compared to the traditional IC.

The BIC provides a generic solution for regulating ISpS plant systems. Since the closed-loop system stability of the main result is proven independently from the BIC parameters and initial conditions, a different parameter selection than the one proposed in the paper can be applied to guarantee a controller output with a given bound and achieve a different system response. Additionally, conditions for guaranteeing convergence to the desired equilibrium are also of great significance. These represent some interesting topics for future research.

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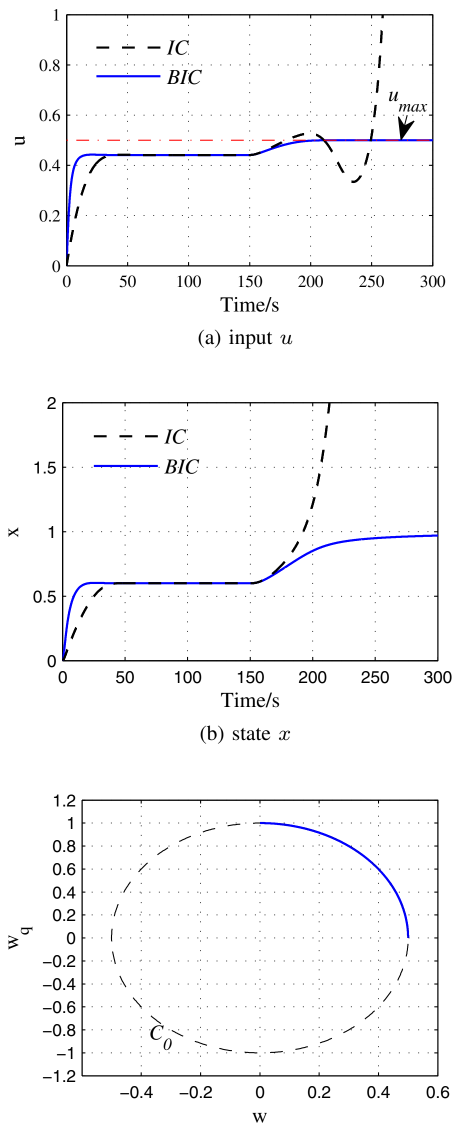

(a) input u

(b) state x

(c) $w - w_q$ plane (BIC)

Fig. 5: Simulation results using the IC and the BIC

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