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UDE-based Dynamic Surface Control for Strict-feedback Systems with Mismatched Disturbances

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Abstract—**This paper proposes a new method for the tracking control problem of strict-feedback systems with mismatched disturbances. The method, called the uncertainty disturbance estimator (UDE), is introduced into the conventional backstepping scheme to handle the mismatched disturbances due to its excellent performance in handling uncertainties and disturbances yet a simple control scheme. The dynamic surface control (DSC) is adopted to overcome the "explosion of complexity" problem in the backstepping scheme. Compared to the existing work in the literature, the backstepping deviation term between the synthetic virtual control and system state, is lumped into the "disturbance-like" term, together with the DSC filtering error and mismatched disturbances. The lumped term is then handled by the UDE method at each step, which simplifies the control design and analysis. In addition, the marriage of UDE with backstepping also relaxes the structural constraint of the UDE design for mismatched uncertainties. The uniformly ultimate boundedness of the closed-loop system is proved. Both numerical simulation and experimental validation are carried out to demonstrate the effectiveness of the proposed approach.**

I. Introduction

Most of modern control systems suffer various uncertainties like parameter perturbation, unmodeled dynamics and external disturbances. In order to attenuate the undesired influence of uncertainties, numerous adaptive and robust control strategies have been proposed by the control community, such as the sliding mode control and integrator backstepping control. Unlike the sliding mode control requires the matched conditions, the integrator backstepping control is an effective and popular strategy for a large class of nonlinear systems which are in the strict-feedback form with mismatched uncertainties [1], [2]. But an obvious problem comes from the "explosion of complexity", since it requires obtaining the derivatives of virtual controls. To overcome the above complexity, a methodology named dynamic surface control (DSC) was proposed in [3], [4] by introducing the first-order filtering of the synthetic controls at each step of the traditional backstepping approach.

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Since the DSC method avoids the calculation of derivatives of virtual control signals, it has drawn a lot of attention and has been applied to many applications in the recent decade. In [5], the DSC method was adopted into an adaptive neural network controller for a class of uncertain nonlinear systems in strict-feedback form. Furthermore, the DSC method has been adopted for handling different uncertainties and nonlinearities such as the unknown dead zone [6], stochastic systems [7], unknown time-delay [8], hysteresis systems [9], unknown direction of control gain and input saturation [10]. And in the practical engineering applications, the DSC method has been applied to the formation control and trajectory tracking of autonomous vehicles [11], [12], [13], flexible-joint robot control [14], permanent magnet synchronous motors control [15], etc. However, the common feature of above works is the utilization of neural networks (NNs) to approximate complex nonlinear functions and uncertainties. But it will cause another problem that the complexity of nonlinear functions approximated by NNs will grow dramatically as the order of the system increases. Same problems exist in fuzzy control based DSC control [6], [16], [17], etc.

Recently, a robust control methodology named uncertainty disturbance estimator (UDE) has demonstrated its excellent performance in handling uncertainties and disturbances and has been successfully applied in various systems [18], [19], [20], [21], [22]. As an easily implemented and real time scheme, the UDE method uses a strictly-proper stable filter to reconstruct the uncertainty and disturbance signals and it does not need any knowledge of these unknown signals [18], [23]. All the uncertainties including parameter perturbation, unmodeled dynamics, external disturbances and even the coupled states can be regarded as a "disturbance-like" term. Then the effects of the "disturbance-like" term on the system can be observed by the measured states and input signals and a compensator is constructed based on the filtering of this observation. However, the UDE method has a structural constraint which limits its application range [18], e.g., mismatched disturbances. Therefore, introducing the backstepping scheme into the conventional UDE is a promising solution to relax this constraint.

Motivated by the above results, this paper synthesizes both the UDE method and DSC method into the traditional backstepping approach to solve the tracking problem for strict-feedback systems with mismatched disturbances. Compared to the existing works in the literature, the backstepping deviation term between the synthetic virtual control and system state, is lumped into the "disturbance-like" term, together with the DSC filtering error and mismatched disturbances. The lumped term is then handled by the UDE method at each step, which simplifies the control design and analysis. In addition, the marriage of UDE with backstepping also relaxes the structural constraint in the conventional UDE design for mismatched uncertainties.

The remainder of this paper is organized as follows: In Section II, the UDE-based dynamic surface control via backstepping is developed for a third-order system, and a numerical example is provided for the validation. Then, Section III generalizes the strategy to the nth-order systems and gives a stability analysis. Finally, the experiment validation and concluding remarks are presented in Section IV and Section V, respectively.

II. An Illustrative Example

For the simplicity, consider the following third-order strict-feedback system with mismatched disturbances:

$$\begin{cases} \dot{x}_1 &= f_1(x_1) + x_2 + d_1 \\ \dot{x}_2 &= f_2(x_1, x_2) + x_3 + d_2 \\ \dot{x}_3 &= f_3(x_1, x_2, x_3) + u + d_3 \\ y &= x_1 \end{cases} \tag{1}$$

where x_1, x_2, x_3 are the system states, $f_1(\cdot), f_2(\cdot), f_3(\cdot)$ are known continuous and smooth functions, d_1, d_2, d_3 are bounded and smooth external disturbances and u is the control input. Without loss of generality, it is assumed that $f_i(0, ..., 0) = 0$. The control objective is to force the system output $y = x_1$ to track a smooth reference signal x_{1r}, i.e., $x_1 \to x_{1r}$ as $t \to \infty$.

In order to facilitate the control design, first define the variables z_1, z_2, z_3 as the system tracking errors, x_{2r}, x_{3r} as the virtual controls, ω_{2r}, ω_{3r} as the filtered virtual controls and y_2, y_3 as the virtual control errors. The UDE-based DSC control design is based on the following coordinates transformation [1], [4],

$$\begin{aligned} z_1 &= x_1 - x_{1r} \\ z_2 &= x_2 - \omega_{2r} = x_2 - y_2 - x_{2r} \\ z_3 &= x_3 - \omega_{3r} = x_3 - y_3 - x_{3r} \end{aligned} \tag{2}$$

where $y_i = \omega_{ir} - x_{ir}$, $i = 2, 3$ and the recursive design procedure contains three steps. The design procedure begins at the first equation which is progressively stabilized by virtual control that appears in the outer equations. The procedure terminates when the final external control is reached.

Step 1: Consider the first equation in (1). Since

$$\begin{aligned} z_1 &= x_1 - x_{1r} \\ \Longrightarrow \dot{z}_1 &= f_1 + x_2 + d_1 - \dot{x}_{1r} \end{aligned} \tag{3}$$

applying $z_2 = x_2 - y_2 - x_{2r}$ results in

$$\dot{z}_1 = f_1 + z_2 + y_2 + x_{2r} + d_1 - \dot{x}_{1r} \tag{4}$$

where the virtual control x_{2r} will be derived to stabilize the system (4). Following the UDE-based control design [18], the virtual control x_{2r} consists of the feedback linearization part and uncertainty compensation part as

$$x_{2r} = x_{2rl} + x_{2rd} \tag{5}$$

where x_{2rl} is for the feedback linearization and x_{2rd} is for the uncertainty compensation. Since z_2 and y_2 are unknown for system (4), let $u_{d1} = z_2 + y_2 + d_1$ as the lumped "disturbance-like" term in (4). This is different from the conventional backstepping method where z_2 remains in this step and will be eliminated in the next step. Select

$$x_{2rl} = -k_1 z_1 - f_1 + \dot{x}_{1r} \tag{6}$$

where $k_1 > 0$ and

$$x_{2rd} = -(z_2 + y_2 + d_1) = -u_{d1} \tag{7}$$

the closed-loop system becomes

$$\dot{z}_1 = -k_1 z_1 \tag{8}$$

where $k_1 > 0$. Therefore, the system state z_1 is exponentially stable as $z_1 = e^{-k_1 t} z_1(0) \to 0$, with $t \to \infty$. However, u_{d1} is unknown, and can not be used directly. The disturbance compensation should be redesigned. Substituting (5), (6) into the system (4) yields

$$\dot{z}_1 = -k_1 z_1 + x_{2rd} + u_{d1}. \tag{9}$$

Solving u_{d1} there is

$$u_{d1} = \dot{z}_1 + k_1 z_1 - x_{2rd} \tag{10}$$

which indicates that the uncertainty term can be observed by the known signals z_1, x_{2rd}. Using a proper low-pass filter g_{f1} which has the unity gain and zero phase shift in the spectrum of u_{d1}, the uncertainty term can be estimated by

$$\hat{u}_{d1} = g_{f1} * (\dot{z}_1 + k_1 z_1 - x_{2rd}) \tag{11}$$

where $*$ is the convolution operator and the disturbance compensation is written as

$$\begin{aligned} x_{2rd} &= -\hat{u}_{d1} \\ &= -g_{f1} * (\dot{z}_1 + k_1 z_1 - x_{2rd}). \end{aligned} \tag{12}$$

Solving x_{2rd} yields

$$x_{2rd} = -\mathcal{L}^{-1}\left(\frac{G_{f1}}{1 - G_{f1}}\right) * (\dot{z}_1 + k_1 z_1) \tag{13}$$

where G_{f1} is the Laplace transfer of g_{f1}, $\mathcal{L}^{-1}$ is the inverse Laplace operator. Hence, the virtual control (5) for the system becomes

$$\begin{aligned} x_{2r} = & \left[-k_1 z_1 - f_1 + \dot{x}_{1r}\right] \\ & -\mathcal{L}^{-1}\left(\frac{G_{f1}}{1 - G_{f1}}\right) * (\dot{z}_1 + k_1 z_1) \end{aligned} \tag{14}$$

To obtained a filtered virtual control, x_{2r} is passed through a first-order filter

$$\tau_2 \dot{\omega}_{2r} + \omega_{2r} = x_{2r} \quad \omega_{2r}(0) = x_{2r}(0) \tag{15}$$

with $\tau_2 > 0$ is a time constant. Consequently, substituting (5), (6), (11) and (12) into (4) results in the closed-loop system

$$\dot{z}_1 = -k_1 z_1 + (u_{d1} - \hat{u}_{d1}) \tag{16}$$

Step 2 Consider the second equation in (1)

$$\begin{aligned} z_2 &= x_2 - \omega_{2r} \\ \Rightarrow \dot{z}_2 &= f_2 + x_3 + d_2 - \dot{\omega}_{2r} \end{aligned} \tag{17}$$

Applying $z_3 = x_3 - y_3 - x_{3r}$ results in

$$\dot{z}_2 = f_2 + z_3 + y_3 + x_{3r} + d_2 - \dot{\omega}_{2r} \tag{18}$$

Similarly, as z_3 and y_3 are unknown for system (18), let $u_{d2} = z_3 + y_3 + d_2$ as the lumped "disturbance-like" term and follow the same procedure with Step 1. The virtual control x_{3r} is obtained as

$$x_{3r} = x_{3rl} + x_{3rd} \tag{19}$$

where the feedback linearization part is

$$x_{3rl} = -k_2 z_2 - f_2 + \dot{\omega}_{2r} \tag{20}$$

with $k_2 > 0$ and the uncertainty compensation part is

$$x_{3rd} = -\mathcal{L}^{-1}\left(\frac{G_{f2}}{1 - G_{f2}}\right) * (\dot{z}_2 + k_2 z_2). \tag{21}$$

Hence, the virtual control is

$$\begin{aligned} x_{3r} &= [-k_2 z_2 - f_2 + \dot{\omega}_{2r}] \\ &\quad -\mathcal{L}^{-1}\left(\frac{G_{f2}}{1 - G_{f2}}\right) * (\dot{z}_2 + k_2 z_2) && (22) \\ &= \left[-k_2 z_2 - f_2 + \frac{x_{2r} - \omega_{2r}}{\tau_2}\right] \\ &\quad -\mathcal{L}^{-1}\left(\frac{G_{f2}}{1 - G_{f2}}\right) * (\dot{z}_2 + k_2 z_2) && (23) \end{aligned}$$

Then, we pass x_{3r} though a first-order filter to obtain the filtered virtual control

$$\tau_3 \dot{\omega}_{3r} + \omega_{3r} = x_{3r} \quad \omega_{3r}(0) = x_{3r}(0) \tag{24}$$

with $\tau_3 > 0$ is a time constant and the resulting closed-loop system is

$$\dot{z}_2 = -k_2 z_2 + (u_{d2} - \hat{u}_{d2}) \tag{25}$$

Step 3 Consider the last equation in (1),

$$\begin{aligned} z_3 &= x_3 - \omega_{3r} \\ \Rightarrow \dot{z}_3 &= f_3 + u + d_3 - \dot{\omega}_{3r} \end{aligned} \tag{26}$$

Let the lumped "disturbance-like" term be $u_{d3} = d_3$ and follow the same procedure with Steps 1 and 2, the control input is obtained as

$$\begin{aligned} u &= [-k_3 z_3 - f_3 + \dot{\omega}_{3r}] \\ &\quad -\mathcal{L}^{-1}\left(\frac{G_{f3}}{1 - G_{f3}}\right) * (\dot{z}_3 + k_3 z_3) && (27) \\ &= \left[-k_3 z_3 - f_3 + \frac{x_{3r} - \omega_{3r}}{\tau_3}\right] \\ &\quad -\mathcal{L}^{-1}\left(\frac{G_{f3}}{1 - G_{f3}}\right) * (\dot{z}_3 + k_3 z_3) && (28) \end{aligned}$$

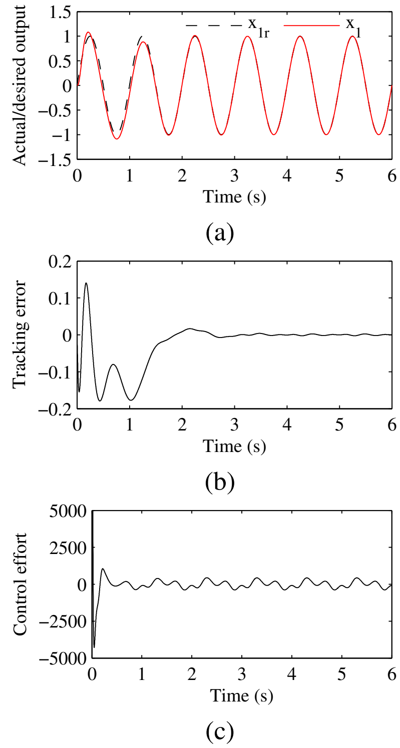

Figure 1. Simulation results of Example 1

where $k_3 > 0$, $\dot{x}_{3r}$ is obtained from the previous step. Then the resulting closed-loop system is

$$\dot{z}_3 = -k_3 z_3 + (u_{d3} - \hat{u}_{d3}) \tag{29}$$

Example 1. Consider the following third-order system

$$\begin{aligned} \dot{x}_1 &= x_1^2 + x_2 + d_1 && (30) \\ \dot{x}_2 &= -x_1 x_2^2 + x_3 + d_2 && (31) \\ \dot{x}_3 &= u + d_3 && (32) \end{aligned}$$

The disturbances are $d_1 = 0.5 \sin 2\pi t$, $d_2 = 0.15 \sin 2\pi t$, $d_3 = 0.5 \cos 2\pi t$ and the initial value of the system is chosen as $x_{10} = x_{20} = x_{30} = 0$. The design parameters of the controller are taken as $k_1 = 3$, $k_2 = k_3 = 25$, and the first-order filters are $G_{f1} = G_{f2} = G_{f3} = \frac{2\pi s}{s^2 + 2\pi s + 2\pi}$. The time constants for DSC are set as $\tau_2 = \tau_3 = 0.01$s. The output signal x_1 is required to track the reference signal $x_{1r} = \sin 2\pi t$.

Fig. 1 shows the performance of the proposed method. The system output can successfully track the reference signal and the tracking error is remained below 0.5%.

III. Control Design and Stability Analysis for a nth-Order System

More generally, the results in Section II can be generalized to the nth-order case:

$$\begin{cases} \dot{x}_1 & = f_1(x_1) + x_2 + d_1 \\ \dot{x}_2 & = f_2(x_1, x_2) + x_3 + d_2 \\ & ... \\ \dot{x}_{n-1} & = f_{n-1}(x_1, ..., x_{n-1}) + x_n + d_{n-1} \\ \dot{x}_n & = f_n(x_1, ..., x_n) + u + d_n \\ y & = x_1 \end{cases} \tag{33}$$

where x_i , $i = 1, 2, ..., n$, are the system states, $f_i(\cdot)$ are known continuous and smooth functions, d_i are bounded and smooth external disturbances, y is the system output, and u is the control input. Using the following coordinate transformation

$$z_1 = x_1 - x_{1r} \tag{34}$$

$$z_i = x_i - \omega_{ir} = x_i - y_i - x_{ir} \tag{35}$$

$$y_i = \omega_{ir} - x_{ir} \tag{36}$$

and

$$\tau_i \dot{\omega}_{ir} + \omega_{ir} = x_{ir}, \quad \omega_{ir}(0) = x_{ir}(0) \tag{37}$$

where $i = 2, ..., n$, $\tau_i > 0$ are time constants, x_{ir} are virtual controls, ω_{ir} are filtered virtual controls, and y_i are deviations of virtual controls.

The UDE-based DSC algorithm is summarized as the following Algorithm 1.

Algorithm 1 UDE-based DSC for strict-feedback systems (33)

$z_1 = x_1 - x_{1r}$
$x_{2r} = [-k_1 z_1 - f_1 + \dot{x}_{1r}] - \mathcal{L}^{-1}(\frac{G_{f1}}{1-G_{f1}}) * (\dot{z}_1 + k_1 z_1)$
$\tau_2 \dot{\omega}_{2r} + \omega_{2r} = x_{2r}$, $\omega_{2r}(0) = x_{2r}(0)$
$z_2 = x_2 - \omega_{2r}$
for $i = 3$ to n **do**
  $x_{ir} = [-k_{i-1} z_{i-1} - f_{i-1} + \frac{x_{(i-1)r} - \omega_{(i-1)r}}{\tau_{i-1}}] - \mathcal{L}^{-1}(\frac{G_{fi}}{1-G_{fi}}) * (\dot{z}_{i-1} + k_{i-1} z_{i-1})$
  $\tau_i \dot{\omega}_{ir} + \omega_{ir} = x_{ir}$, $\omega_{ir}(0) = x_{ir}(0)$
  $z_i = x_i - \omega_{ir}$
  $i = i + 1$
end for
$u = [-k_n z_n - f_n + \frac{x_{nr} - \omega_{nr}}{\tau_n}] - \mathcal{L}^{-1}(\frac{G_{fn}}{1-G_{fn}}) * (\dot{z}_n + k_n z_n)$

The following theorem shows the stability and control performance of the closed-loop system.

Theorem 2 (Stability)**.** *Consider the nth-order strict-feedback system (33) and the Algorithm 1, with a series of virtual controls $x_{2r}, x_{3r}, ..., x_{nr}$ and the control input u. Assume d_i, $i = 1, 2..., n$ are bounded and the control parameters satisfy that $k_i > \frac{1}{2}, k_n > \frac{1}{2}, \tau_{i+1} < 1$, $i = 2, ..., n-1$, then the closed-loop system is stable, i.e., the system tracking error $|x_1 - x_{1r}|$ is bounded and the bound is adjustable by control parameters.*

Proof: By using the Algorithm 1, the resulting closed-loop system is as follows

$$\dot{z}_i = -k_i z_i + (u_{di} - \hat{u}_{di}) \tag{38}$$

$$\dot{z}_n = -k_n z_n + (u_{dn} - \hat{u}_{dn}) \tag{39}$$

where k_i, $k_n > 0$, $u_{di} = z_{i+1} + y_{i+1} + d_i$, $i = 1, 2, ..., n-1$ and $u_{dn} = d_n$. The filtered errors $y_i = \omega_{ir} - x_{ir}$ from (37) will result in the following filtered error dynamics

$$\dot{y}_{i+1} = -\frac{1}{\tau_{i+1}} y_{i+1} - \dot{x}_{(i+1)r} \quad i = 1, ..., n-1 \tag{40}$$

By differentiating the virtual controls (40), there is

$$\left| \dot{y}_2 + \frac{1}{\tau_2} y_2 \right| \le \xi_2 \tag{41}$$

Furthermore,

$$\left| \dot{y}_{i+1} + \frac{1}{\tau_{i+1}} y_{i+1} \right| \le \xi_{i+1} \tag{42}$$

where $\xi_2(z_1, y_2, x_{1r}, \dot{x}_{1r}, \ddot{x}_{1r}) = -k_1 \dot{z}_1 - \frac{\partial f_1}{\partial x_1} \dot{x}_1 + \ddot{x}_{1r} - \frac{\partial}{\partial t} \mathcal{L}^{-1}\left(\frac{G_{fi}}{1-G_{fi}}\right) * (\dot{z}_1 + k_1 z_1)$ and $\xi_{i+1}(z_1, ..., z_{i+1}, y_2, ..., y_{i+1}, x_{1r}, \dot{x}_{1r}, \ddot{x}_{1r}) = -k_i \dot{z}_i - \sum_{k=1}^{i} \frac{\partial f_i}{\partial x_k} \dot{x}_k + \frac{1}{\tau_i}(\dot{x}_{ir} - \dot{\omega}_{ir}) - \frac{\partial}{\partial t} \mathcal{L}^{-1}\left(\frac{G_{fi}}{1-G_{fi}}\right) * (\dot{z}_i + k_i z_i)$ $i = 1, ..., n-1$ are continuous functions. Therefore,

$$\begin{aligned} \dot{y}_{i+1} y_{i+1} &\le -\frac{y_{i+1}^2}{\tau_{i+1}} + |y_{i+1}| \xi_{i+1} \\ &\le -\frac{y_{i+1}^2}{\tau_{i+1}} + |y_{i+1}|^2 + \frac{1}{4} \xi_{i+1}^2 \end{aligned} \tag{43}$$

Consider the Lyapunov function candidates

$$V_{zi} = \frac{1}{2} z_i^2 \quad i = 1, ..., n \tag{44}$$

and

$$V_{y(i+1)} = \frac{1}{2} y_{i+1}^2 \quad i = 1, ..., n-1 \tag{45}$$

Using the Young's inequality and (43) results in

$$\begin{aligned} \dot{V}_{zi} &= z_i \dot{z}_i \\ &= -k_i z_i^2 + z_i (u_{di} - \hat{u}_{di}) \\ &\le -k_i z_i^2 + \frac{z_i^2}{2} + \frac{(u_{di} - \hat{u}_{di})^2}{2} \end{aligned} \tag{46}$$

and

$$\dot{V}_{y(i+1)} = y_{i+1} \dot{y}_{i+1} \le -\frac{y_{i+1}^2}{\tau_{i+1}} + |y_{i+1}|^2 + \frac{1}{4} \xi_{i+1}^2 \tag{47}$$

Define the composite Lyapunov function candidate

$$V = \sum_{i=1}^{n} V_{zi} + \sum_{i=1}^{n-1} V_{y(i+1)} \tag{48}$$

By differentiating V respect to t results in

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{n}\dot{V}_{zi}+\sum_{i=1}^{n-1}\dot{V}_{y(i+1)} \\
&\leq \sum_{i=1}^{n}\left[-k_i z_i^2+\frac{z_i^2}{2}+\frac{(u_{di}-\hat{u}_{di})^2}{2}\right] \\
&\quad +\sum_{i=1}^{n-1}\left(-\frac{y_{i+1}^2}{\tau_{i+1}}+y_{i+1}^2+\frac{1}{4}\xi_{i+1}^2\right) \\
&\leq \sum_{i=1}^{n}\left(-k_i+\frac{1}{2}\right)z_i^2+\sum_{i=1}^{n-1}\left(-\frac{1}{\tau_{i+1}}+1\right)y_{i+1}^2 \\
&\quad +\sum_{i=1}^{n}\frac{((1-g_f)*u_{di})^2}{2}+\sum_{i=1}^{n-1}\frac{1}{4}\xi_{i+1}^2 \qquad (49)
\end{aligned}$$

Consider the compact sets

$$\Omega_r=\left\{(x_{1r},\dot{x}_{1r},\ddot{x}_{1r})\in\mathbb{R}^3\right\}$$

and

$$\Omega_{pi} = \left\{(z_1,...,z_i,y_2,...,y_i)\in\mathbb{R}^{2i-1}| \sum_{k=1}^{i}z_k^2+\sum_{k=1}^{i-1}y_{k+1}^2\leq 2p\right\}$$

where $p>0$.

Therefore, $\xi_{i+1}(z_1,...,z_{i+1},y_2,...,y_{i+1},x_{1r},\dot{x}_{1r},\ddot{x}_{1r})$ has a maximum, say M_{i+1} on $\Omega_r\times\Omega_{p(i+1)}$, $i=1,...,n-1$. Furthermore, since the external disturbances, d_i , are assumed to be bounded and $u_{di}=z_{i+1}+y_{i+1}+d_i$, $i=1,2,...,n-1$ and $u_{dn}=d_n$. Thus, for each $(1-g_f)*u_{di}$ there exists a bound say N_i, $i=1,2,...,n$. If the control parameters are fixed to satisfy $k_i>\frac{1}{2}$, $k_n>\frac{1}{2}$ and $\tau_{i+1}<1$, $i=1,...,n-1$,

$$\begin{aligned}
\dot{V} &\leq -\sum_{i=1}^{n}\left(k_i-\frac{1}{2}\right)z_i^2-\sum_{i=1}^{n-1}\left(\frac{1}{\tau_{i+1}}-1\right)y_{i+1}^2 \\
&\quad +\frac{1}{2}\sum_{i=1}^{n}N_i^2+\frac{1}{4}\sum_{i=1}^{n-1}M_{i+1}^2 \\
&\leq -\rho V+c \qquad (50)
\end{aligned}$$

where $\rho=\min\left\{k_i-\frac{1}{2},k_n-\frac{1}{2},\frac{1}{\tau_{i+1}}-1,i=1,...,n-1\right\}>0$ and $c=\frac{1}{2}\sum_{i=1}^{n}N_i^2+\frac{1}{4}\sum_{i=1}^{n-1}M_{i+1}^2>0$. Let $\rho>\frac{c}{p}$, then $\dot{V}<0$ on $V=p$. Thus the set $\Omega_p=\left\{(z_1,...,z_n,y_2,...,y_n)\in\mathbb{R}^{2n-1}|V\leq p\right\}$ is an invariant set, i.e., if $V(0)+\frac{c}{\rho}\leq p$ then $V(t)\leq p$ for all $t\geq 0$. Solving the inequality (50) gives

$$0\leq V(t)\leq V(0)e^{-\rho t}+\frac{c}{\rho}\left(1-e^{-\rho t}\right)\quad t\geq 0 \qquad (51)$$

thus, $V(t)$ is bounded by $\frac{c}{\rho}$ while $t\to\infty$ and all signals of the closed-loop system are bounded, i.e., $z_1,...,z_n,y_2,...,y_n$ are uniformly ultimately bounded. By increasing parameters $k_1,...,k_n$, or decreasing $\tau_2,...,\tau_n$, the quantity $\frac{c}{\rho}$ can be adjusted to be arbitrarily small. Then the tracking error $z_1=x_1-x_{1r}$ can be made arbitrarily small. This completes the proof. ■

IV. Experimental Validation

In order to illustrate the performance of the proposed method, an experimental study is carried out on a rotary servo system, as shown in Fig. 2. The servo system can be modeled

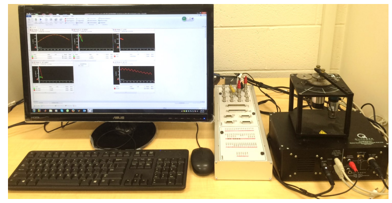

Figure 2. DC servo motor

by the following second-order system

$$\dot{x}_1 = x_2+d_1 \qquad (52)$$
$$\dot{x}_2 = -30x_2+80u+d_2 \qquad (53)$$

where x_1, x_2 represent the position and velocity, respectively; u is the control input and d_1, d_2 are unmodeled dynamics or disturbances appearing in the system. It is reasonable to assume these uncertainties to be bounded in the real physical system. The control objective is to make the position x_1 track a step change.

The set-point is chosen as 40 degree, which is associated to the sensed voltage 1.14V. To overcome the jump point at $t=0$, a reference model is selected as $\dot{x}_{1r}=-5x_{1r}+5\times 1.14$, which makes the reference signal x_{1r} become a smooth and continuous function. To facilitate the UDE design, the time constant in the first-order $G_{f1}(s)=G_{f2}(s)=\frac{1}{\tau s+1}$ is chosen as $\tau=0.05$s and the DSC time constant is chosen as $\tau_2=0.01$s. In addition, the control parameters are chosen as $k_1=75$ and $k_2=50$. The performance is shown in Fig. 3 (a) - (c). It can be seen that the servo arm successfully tracks the set point after 1 second and the steady state error is achieved at 0.5%.

V. Conclusion

The UDE-based dynamic surface control has been developed for strict-feedback systems with mismatched external disturbances. Taking advantages of the backstepping approach (which could handle the mismatched disturbances), and the DSC method (which could avoid the calculation of derivatives of the virtual controls), this paper introduced the UDE method for the uncertainty compensation. Inheriting from the conventional UDE method, the developed control approach could achieve the very good tracking performance in real-time and is easily to be implemented. The stability analysis of the closed-loop system was also presented. Finally, the performance of the proposed method was validated by the numerical simulation and experimental studies.

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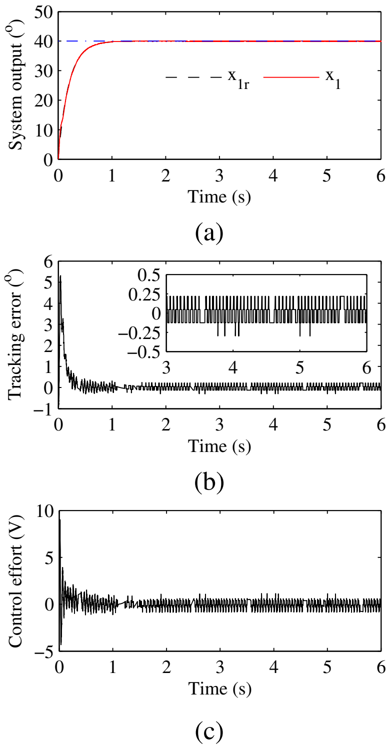

Figure 3. Experimental results for set-point reference tracking

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