Robust Control for Parallel Operated L-Inverters with Uncertainty and Disturbance Estimator

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Abstract—In conventional droop control, accurate proportional load sharing could not be achieved among parallel operated inverters due to the mismatch of output impedance and system disturbances. In this paper, an uncertainty and disturbance estimator (UDE)-based robust droop control strategy is proposed for accurate proportional load sharing, particularly reactive power sharing, among parallel operated L-inverters. The reactive power dynamics is derived from power delivery equation passed by a low-pass filter, while the reactive power reference is fed back to generate the reactive power reference, power flow control of grid-connected inverters [29] etc. In recent years, the UDE-based control demonstrates excellent performance in handling uncertainties and disturbances, e.g., large or fast load change [10], [11], time varying of output impedance [15], [16].

One effective way to achieve accurate power sharing is through communication among parallel operated inverters [9], [10], [11], [14], such as hierarchical control [6], [15], [17], [18], [19]. However, hierarchical droop control also has its own drawbacks, such as cost of communication [11], reliability problem, location problem and slow responses [8], [11], [13]. The virtual impedance is another common application to improve power sharing performance among parallel operated inverters [9], [20], [21], for example, three nested control loops with adjustable output impedance value are designed in [8] to achieve equal sharing of linear and nonlinear loads, also with good harmonic sharing. Adding a virtual inductor and estimating the effect of the line impedance are proposed in [12] to improve the situation via changing the droop coefficients. However, the sharing performance of virtual impedance based method will be affected by time varying output impedance [15], [16]. Other methods for proportional load sharing without communication are made with the re-design of voltage droop strategy in [13], [22], Q-V dot droop control method in [23] and virtual flux droop method in [24].

In this paper, a robust control strategy based on the uncertainty and disturbance estimator (UDE) [25] method without communication is developed for parallel operated inverters to achieve accurate proportional load sharing for L-inverters, which are a kind of inverters with inductive output impedances [1]. The UDE control algorithm, which is proposed in [25], is based on the assumption that the uncertainty and disturbance can be estimated by using a filter with the appropriate bandwidth. In recent years, the UDE-based control demonstrates excellent performance in handling uncertainties and disturbances in different systems, and is successfully applied to robust trajectory tracking [26], a class of non-affine nonlinear systems [27], variable-speed wind turbine control [28], and power flow control of grid-connected inverters [29] etc. In this paper, the reactive power dynamics is developed from power delivery equation passed by a low-pass filter. The load voltage is fed back to generate the reactive power reference,
and the UDE-based method is applied for accurate reactive power control to achieve reactive power sharing. With adoption of UDE method, the model nonlinearity and uncertainty (e.g., power angle) and system operation disturbances (e.g., time varying of output impedance, and load change) can be estimated and compensated with accurate reactive power sharing. The real power sharing is achieved followed the design of conventional droop method [1], [12], [15]. The effectiveness of the proposed UDE-based robust droop control method is demonstrated through experimental studies through two parallel operated Texas Instruments (TI) L-inverters. The similar results can be extended to R-inverters or C-inverters.

The rest of this paper is organized as follows. Section II provides an overview of the conventional droop control. In Section III, the UDE-based robust droop control is proposed for parallel operated L-inverters. Effectiveness of the proposed approach is demonstrated through experimental studies in Section IV, before the concluding remarks are made in Section V.

II. OVERVIEW OF CONVENTIONAL DROOP CONTROL

A single L-inverter \( E \angle \delta \) delivering power to the grid \( V_o \angle 0^\circ \) through an inductive impedance \( Z_o \angle \theta \) with \( \theta = 90^\circ \) can be modeled as shown in Fig. 1. The real power \( P \) and the reactive power \( Q \) received by the grid \( V_o \angle 0^\circ \) are shown in [1] as

\[
P = \frac{E V_o}{Z_o} \sin \delta, \quad (1)
\]

\[
Q = \frac{E V_o}{Z_o} \cos \delta - \frac{V_o^2}{Z_o}. \quad (2)
\]

where \( \delta \) is the phase difference between the inverter and the grid, often called the power angle.

Based on the assumption of a small power angle, the conventional droop controller for \( i \)th parallel operated L-inverter can be expressed as

\[
E_i = E^* - n_i Q_i, \quad (3)
\]

\[
\omega_i = \omega^* - m_i P_i, \quad (4)
\]

where \( E_i \) is the voltage set-point, \( \omega_i \) is the frequency set-point, \( E^* \) is the rated voltage and \( \omega^* \) is the rated frequency. \( n_i \) and \( m_i \) are the droop coefficients, which are related to the power rating of inverters, and usually defined by the requirement of customers. The control scheme of conventional droop control is shown in Fig. 2(a).

The limitations of conventional droop control with mismatch of output impedance have been pointed out in [1], and the output impedance of inverters always drift in different conditions [15], [16], such as inductance change with magnetic saturation by high current, and resistance change by high temperature. The system operation disturbances, e.g., large or fast load change [10], [11], time variations of output impedance [15], [16], will affect the power sharing performance in conventional droop control.

III. UDE-BASED ROBUST DROOP CONTROL

In this paper, a robust droop control scheme is proposed based on the UDE (uncertainty and disturbance estimator) [25] method for L-inverters to deal with mismatched output impedance and system operation disturbances. Its structure is shown in Fig. 2(b). Both reactive power sharing and real power sharing are achieved through different control channels.

The reactive power sharing control method is designed first. In practice, in order to filter the switching harmonics and other high order noises, low-pass filters are commonly adopted for the real power and reactive power calculations in (1) and (2) as shown in Fig. 2(b). With the choice of a first-order low-pass filter \( G_{qi}(s) = \frac{1}{1 + \tau_{qi}s} \) with the time constant \( \tau_{qi} \), the reactive power in (2) for the \( i \)th inverter can be re-written as

\[
Q_i = \frac{1}{1 + \tau_{qi}s} \left( \frac{E_i V_o}{Z_{oi}} \cos \delta_i - \frac{V_o^2}{Z_{oi}} \right), \quad (5)
\]

or, in the time-domain,

\[
Q_i = \frac{E_i V_o}{\tau_{qi} Z_{oi}} \cos \delta_i - \frac{V_o^2}{\tau_{qi} Z_{oi}} - Q_i. \quad (6)
\]
Since the power angle $\delta_i$ depends on the load and the inverter in parallel operation, which is quite uncertain, the reactive power dynamics (6) can be re-written as

$$\dot{Q}_i = \frac{V_o}{\tau_{qi} Z_{oi}} E_i - \frac{V_o^2}{\tau_{qi}^2 Z_{oi}} \frac{Q_i}{\tau_{qi}} + \Delta_{qi}, \quad (7)$$

where

$$\Delta_{qi} = \frac{E_i V_o}{\tau_{qi} Z_{oi}} (\cos \delta_i - 1) \quad (8)$$

represents the uncertainty term, including the nonlinearity and uncertainty of the power angle.

The set-point voltage droop in (3) can be re-designed with load voltage for the reactive power reference as shown in Fig. 2(b)

$$Q_{ri} = \frac{E^* - V_o}{n_i} \quad (9)$$

With the feedback of load voltage $V_o$, all parallel operated inverters can achieve accurate reactive power sharing, as $n_i Q_{ri}$ are equal for all the units, with the same $E^*$ and $V_o$.

With the reactive power dynamics developed in (7) and the reactive power reference designed in (9), the reactive power control is developed to design a control law such that the reactive power in (7) asymptotically tracks the reactive power reference in (9). Then, the reactive power sharing can be achieved. In particular, the tracking error

$$e_{qi} = Q_{ri} - Q_i \quad (10)$$

satisfies the desired dynamic equation

$$\dot{e}_{qi} = -K_{qi} e_{qi}, \quad (11)$$

where $K_{qi} > 0$ is a constant error feedback gain.

Combining (7) and (9) - (11), then

$$\dot{Q}_{ri} = \frac{V_o}{\tau_{qi} Z_{oi}} E_i + \frac{V_o^2}{\tau_{qi}^2 Z_{oi}} \frac{Q_i}{\tau_{qi}} + \Delta_{qi} = -K_{qi} e_{qi}. \quad (12)$$

So, $E_i$ needs to satisfy

$$E_i = \frac{\tau_{qi} Z_{oi}}{V_o} \left( \dot{Q}_{ri} + \frac{V_o^2}{\tau_{qi}^2 Z_{oi}} \frac{Q_i}{\tau_{qi}} + K_{qi} e_{qi} - \Delta_{qi} \right). \quad (13)$$

Following the procedures provided in [25], $\Delta_{qi}$ can be estimated from (7) with

$$\dot{\Delta}_{qi} = \Delta_{qi} * g_{fi} = (\dot{Q}_i - \frac{V_o}{\tau_{qi} Z_{oi}} E_i + \frac{V_o^2}{\tau_{qi}^2 Z_{oi}} \frac{Q_i}{\tau_{qi}}) * g_{fi}, \quad (14)$$

where $g_{fi}(t)$ is the impulse response of the strictly proper stable filter $G_{fi}(s)$ with the appropriate frequency characteristics. Replacing $\Delta_{qi}$ with $\Delta_{qi}$ in (13) results in

$$E_i = \frac{\tau_{qi} Z_{oi}}{V_o} \left( \dot{Q}_{ri} + \frac{V_o^2}{\tau_{qi}^2 Z_{oi}} \frac{Q_i}{\tau_{qi}} + K_{qi} e_{qi} \right.$$

$$\left. - (\dot{Q}_i - \frac{V_o}{\tau_{qi} Z_{oi}} E_i + \frac{V_o^2}{\tau_{qi}^2 Z_{oi}} \frac{Q_i}{\tau_{qi}}) * g_{fi} \right). \quad (15)$$

Then, the UDE-based robust droop control for reactive power sharing can be formulated as

$$E_i = \frac{\tau_{qi} Z_{oi}}{V_o} \left[ \dot{Q}_{ri} + \frac{V_o^2}{\tau_{qi}^2 Z_{oi}} \frac{Q_i}{\tau_{qi}} + L^{-1} \left\{ \frac{1}{1 - G_{fi}(s)} \right\} * (Q_{ri} + K_{qi} e_{qi}) \right.$$\n
$$\left. - L^{-1} \left\{ \frac{sG_{fi}(s)}{1 - G_{fi}(s)} \right\} * Q_i \right]. \quad (16)$$

When the reactive power tracks the reactive power reference with the UDE-based control (16), the reactive power sharing can be achieved for all parallel operated L-inverters.

For real power sharing, as shown in Fig. 2 (b), the conventional droop control (4) with the addition of a first order low-pass filter is adopted, as it can achieve accurate real power proportional sharing for L-inverters [1], [12], [15] with same frequency for all parallel units.

IV. EXPERIMENTAL RESULTS

A. Experimental Setup

Fig. 3(a) shows a test rig with two TI L-inverters in parallel operation to verify the effectiveness of the UDE-based robust droop control. The corresponding circuit diagram is shown in Fig. 3(b), where the load consists of a resistor $R_L = 40 \ \Omega$ in parallel with two 45 $\mu F$ capacitors $C_{1,1}$ and $C_{1,2}$. The capacitor $C_{1,2}$ is initially bypassed by a switch $CB2$. The ON/OFF status of switch $CB2$ is used for the load change. The parameters of the inverters are given in Table I. The capacity of Inverter 1 is assumed as 500 VA, and Inverter 2 is 250 VA. And the droop coefficients are chosen as $n_1 = 0.022$ and $n_2 = 0.044$; $m_1 = 0.0004\pi$ and $m_2 = 0.0008\pi$. Hence, it is expected that $P_1 = 2P_2$ and $Q_1 = 2Q_2$. Though the same model of inverters with same output impedance are used, a different sharing proportion (2:1) is set in the experiment.
B. Control Parameters

The filter in the UDE algorithm should cover the spectrum of disturbances with the unity gain and zero phase shift. Here, the UDE filter $G_i(s)$ is chosen as the following first-order low-pass filter

$$G_i(s) = \frac{1}{1 + \tau_i s}.$$  (17)

with the time constant $\tau_i$ such that the bandwidth is wide enough to cover the spectrum of $\Delta q_i$. Then, the UDE-based robust droop control for reactive power sharing (16) with the voltage set-point $E_i$ is derived as

$$E_i = V_o + \frac{Q_i Z_{oi}}{V_o} + \frac{\tau_i Z_{oi}}{V_o} \left[ Q_{ri} + (K_{q1} + \frac{1}{\tau_i} r_{q1} + \frac{K_{q2}}{\tau_i} \int_0^t e_{q1} dt) \right].$$

(18)

The control parameters for the final control law (18) are shown in Table II.

### C. System Performance

In this experimental studies, two disturbances conditions are considered: the output impedance change and load change. Initially, the load is connected to the Inverter I only, with switch $CB1$ ON and switch $CB2$ OFF. Inverter II is connected to the loads at $t = 2$ s by turning switch $CB1$ ON. At $t = 6$ s, two disturbances conditions are injected into the system, separately. At $t = 10$ s, Inverter II is disconnected.

1) Case 1) Change of output impedance: For output impedance change disturbance, a virtual output impedance $R_{v1} = 3 \Omega$ with feedback current is added in Inverter I at $t = 6$ s. This virtual output impedance mimics the time varying disturbance of output impedance. The experimental results are shown in the left column of Fig. 4. After $t = 6$ s, there is a negative spike in real power of Inverter I, as the increase of output impedance reduces power output of inverter I, the real power of inverter II has a positive spike correspondingly, as shown Fig. 4(a). The frequency responses are shown in Fig. 4(b). Both the real power and frequency converge to stable status with 2:1 sharing quickly within 0.5 s. The reactive powers of both inverters only have vary small spike and still keep sharing ratio very well as shown in 4(c), the output voltage has small drop and goes back quickly shown in Fig. 4(d). The UDE-based robust droop control can effectively reject the disturbance of the output impedance change.

2) Case 2) Change of load: For load change, at $t = 6$ s, switch $CB2$ is turned OFF to change capacitive load from 45 $\mu$F to 22.5 $\mu$F. The system responses are shown in the right column of Fig. 4. After $t = 6$ s, the real power and frequency almost remain unchanged as shown in Fig.4(a) and (b), as the resistive load keeps the same. The reactive power converges to a new state in a very short time (within 0.5 s), and the sharing ratio can still keep 2:1 shown in Fig. 4(d). In Fig. 4(c), the output voltage goes down with lower reactive power output. At $t = 10$ s, Inverter II is disconnected, the real power and frequency of Inverter I are almost back to initial state, while the reactive power is about half of initial state due to the half capacitive load. The voltage is lower than initial state due to the lower reactive power output. The experimental results indicate that this method has the good robustness against the load change.

### V. CONCLUSION

In this paper, an UDE-based robust droop control has been proposed for accurate proportional load sharing among parallel operated L-inverters, and its effectiveness has been validated on a test rig with two parallel operated TI L-inverters. The new droop control strategy, including the derivation of the reactive power dynamics, the design of reactive power reference, and UDE-based reactive power control, has been developed to achieve accurate power sharing, particularly reactive power sharing, for parallel operated L-inverters. And the similar results can be extended to R-inverters or C-inverters to achieve accurate proportional load sharing. The nonlinearity, uncertainty, and system operation disturbances have been well handled by the proposed approach with good robustness to achieve accurate proportional power sharing.

### ACKNOWLEDGMENT

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### REFERENCES


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**Table I**

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### Table 1

**Inverter parameters.**

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### Table II

**Control parameters final control law (18).**

**REFERENCES**

Figure 4. Experimental results: adding virtual impedance (left column), and load change (right column).


