UDE-based Robust Control for Nonlinear Systems with Mismatched Uncertainties and Input Saturation

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Abstract: In this paper, the uncertainty and disturbance estimator (UDE)-based control is adopted for a class of nonlinear systems with mismatched uncertainties and input saturation. By integrating with the backstepping approach, the robustness for the mismatched uncertainties is guaranteed by the recursive structure of backstepping. The "disturbance-like" terms which consist of the unknown dynamics, disturbances and the system intermediate tracking errors are estimated by low-pass filters with the knowledge of spectrum information. An auxiliary system is constructed to deal with the input saturation while only the system order information is known. Stability analysis of the closed-loop system shows that the uniformly ultimate boundedness is guaranteed. A numerical example is provided to illustrate the effectiveness of the proposed method.

Key Words: Uncertainty and disturbance estimator, Mismatched uncertainties, Input saturation

1 Introduction

With the increasing demand of precise control in industry, alleviating the effect of uncertainties in the nonlinear systems has drawn a lot of attention in both theoretic and practical aspects among control community. A number of techniques to handle the uncertainties have been proposed in the past decades [1, 2]. Adaptive control which adjusts the control parameters is capable of handling the parametric uncertainties [1]. With the basis of approximation theorem in a compact set, non-parametric uncertainties like the unknown dynamics, can be approximated by universal approximators e.g, neural networks (NNs) or fuzzy logic (FL) [2]. By using the high gain feedback to force the system to slide along a sliding mode surface, sliding mode control is another effective way to handle the uncertainties [3]. Besides above methods, it is well known that once the estimation of uncertainties is available, the uncertainties can be compensated directly. This idea results in a large class of uncertainties compensation methods, e.g, the active disturbance rejection control (ADRC) [4], the disturbance observer (DOB) [5], the equivalent input disturbance (EID) [6], the extended state observer (ESO) [7], the uncertainty and disturbance estimator (UDE)-based control [8]. Both unknown dynamics and external disturbances are regarded as the uncertainties in the system. If the uncertainties are estimated in real time then they can be compensated effectively. Hence, the focal point is how the uncertainties can be estimated.

In the framework of ADRC and ESO, the uncertainties are formulated as an augmented state and estimated by a state observer. DOB is another way which is based on the inverse of the nominal transfer function of the plant. EID regards the uncertainties as an unknown disturbance in the input, then a state observer is used to estimate it. Based on the observation of uncertainties, UDE-based control adopts a low-pass filter to reconstruct an estimation of the uncertainties, then the control action is modified by this estimation. However, most above methods require the uncertainties satisfy the so-called matching condition which implies that the uncertainties act via the same channel as the control input, as discussed in [9, 10] for ESO and DOB. In addition, the UDE-based control requires a structure constraint [8]. Unfortunately, in many applications, the uncertainties will affect the states directly rather than through the input channel. Therefore, solving the mismatched uncertainties has more general and wide applicability.

To tackle with the mismatched uncertainties, some extension results for above methods have been presented. In [9], a generalized version of ESO is proposed for general systems with mismatched uncertainties by appropriately developing a disturbance compensation gain. The similar technique is also utilized with DOB to handle the mismatched uncertainties in [10]. In [11], DOB combining with sliding mode control solves the mismatched uncertainties. Another effective technique to handle the mismatched uncertainties is the adaptive backstepping approach [1]. With the recursive structure, adaptive backstepping is capable for both matched and mismatched uncertainties. Though UDE-based control is very suitable for handling uncertainties and has received more and more attention [12–14], all the existing UDE related works only handle the matched uncertainties due to the structural constraint [8]. This paper will utilize the recursive structure of backstepping approach to extend the UDE-based control to systems with mismatched uncertainties.

Another issue that should be considered in the real application is the input saturation since actuators are usually limited by the physical characteristics or under some safety considerations [15]. When a controller is developed without considering the input saturation, it will degrade the performance of the system and even result in the instability [16]. The anti-windup control is a common technique to deal with the input saturation for linear systems [17]. A saturation compensator is designed to remove the effect of input saturation by using the difference between the actual input and controller output. For nonlinear systems, auxiliary system [18, 19], command filter [20, 21], approximation-based methods [22, 23] are proposed to deal with the input saturation. To cope with the input saturation as well as the uncertainties, ADRC and anti-windup are combined in [24]; DOB and NNs are integrated in [23]; ESO and sliding mode are merged in [25]. To the best of our knowledge, there are few works related to the UDE-based control considering the input saturation. This paper will integrate an auxiliary system with the UDE to tackle the input saturation.

The objective of this paper is to develop the UDE-based control for systems with mismatched uncertainties and input saturation. The main contributions of this paper are summarized as follows: 1) The backstepping technique is employed to extend the UDE-based method to cope with the mismatched uncertainties; 2) The reference model is utilized in the design process which relaxes the derivation of virtual controls and avoids the complexity explosion in the conventional backstepping approach; 3) The "disturbance-like" terms, which consist of the unknown dynamics, disturbances and the system intermediate tracking errors, are estimated and compensated by the UDE-based control only with the knowledge of their spectrum information; 4) An auxiliary system is constructed to handle the input saturation and only the system order is required to be known.

The remainder of this paper is organized as follows: In Section 2, the problem formulation is presented. In Section 3, the UDE-based backstepping design with an auxiliary system is developed. Then the stability of the closed-loop system is investigated in Section 4. Finally, a numerical example and some concluding remarks are provided in Section 5 and Section 6, respectively.

2 Problem Formulation and Preliminaries

Consider the following nth-order single-input-single-output (SISO) nonlinear system

$$\begin{aligned} \dot{x}_i &= f_i(\bar{x}_i) + x_{i+1} + d_i, \quad i = 1, ..., n-1 \\ \dot{x}_n &= f_n(\bar{x}_n) + sat(u) + d_n \\ y &= x_1, \quad \bar{x}_n(0) = x_0 \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}$, $i = 1, ..., n$, $\bar{x}_i = [x_1, ..., x_i]^T$ are the system states and state vectors, $y \in \mathbb{R}$ is the output, $u \in \mathbb{R}$ is the controller output and $sat(u) \in \mathbb{R}$ is the actual input, $f_i(\bar{x}_i)$ are the system dynamics which are continuous and $d_i \in \mathbb{R}$ are the external disturbances. x_0 is the initial value and $sat(\cdot)$ is the saturation function,

$$sat(u) = \begin{cases} u, & \rho_{min} \le u \le \rho_{max} \\ \rho_{max}, & u > \rho_{max} \\ \rho_{min}, & u < \rho_{min} \end{cases} \tag{2}$$

where ρ_{max} and ρ_{min} are upper and lower bounds of u. The control objective is to make the system output y to asymptotically track the desired trajectory y_d in the presence of unknown dynamics, disturbance and input saturation. Define $e = y - y_d$ be the tracking error then the objective is $\lim_{t\to\infty} e \to 0$.

Assumption 1. [21] The system (1) is input-to-state stable (ISS).

Remark 2. [21] The Assumption 1 is reasonable, because an unstable system can not be stabilized in the presence of input saturation.

Assumption 3. The system dynamics $f_i(), i = 1, ..., n$ are continuous and can be completely or partially unknown. In this paper, they are all regarded as unknown dynamics.

Assumption 4. The external disturbances $d_i, i = 1, ..., n$ are bounded, but their bounds are not necessary to be known.

3 Controller Design

The basic idea is integrating the UDE-based control with the recursive structure of backstepping method to handle the system uncertainties and disturbances at each step. In order to suppress the effect of the input saturation, the following auxiliary system will be applied into the controller design [19]

$$\begin{aligned} \dot{\lambda}_i &= -c_i\lambda_i + \lambda_{i+1}, \ i = 1, ..., n-1 \\ \dot{\lambda}_n &= -c_n\lambda_n + \Delta u \\ \lambda_i(0) &= 0 \end{aligned} \tag{3}$$

where $\lambda_i, i = 1, ..., n$ are auxiliary states and constants $c_i, i = 1, ..., n$ are chosen to make the polynomial $c_1 s^{n-1} + c_2 s^{n-2} + ... + c_{n-1}s + c_n$ be Hurwitz. Let $\Delta u = sat(u) - u$ denotes the difference between the actual input and the controller output. Then the auxiliary signals λ_i can be represented as filtered versions of Δu and remain zero while no saturation occurs, and nonzero while saturation occurs.

The following change of coordinates are defined,

$$\begin{aligned} z_1 &= x_1 - y_d - \lambda_1 \\ z_i &= x_i - \beta_i - \lambda_i, \ i = 2, ..., n \\ &= x_i - y_i - \alpha_i - \lambda_i \end{aligned} \tag{4}$$

where $z_1, ..., z_n$ are intermediate system errors and $\beta_2, ..., \beta_n$ are reference signals generated by the following first-order reference models through the virtual controls $\alpha_2, ..., \alpha_n$,

$$\begin{aligned} \dot{\beta}_i &= -a_i\beta_i + b_i\alpha_i, \ i = 2, ..., n \\ \beta_i(0) &= \alpha_i(0) \end{aligned} \tag{5}$$

where a_i, b_i are design parameters. Furthermore, β_i can be regarded as a filtered version of the virtual control α_i and y_i are defined as $y_i = \beta_i - \alpha_i, i = 2, ..., n$. Usually, taking the derivative of the virtual controls in the backstepping approach will result in the "complexity explosion", but with the help of the reference models inherited from UDE, the "complexity explosion" can be avoided. The reference dynamics (5) are also in the same spirit of dynamic surface control [26]. The control design consists of n steps:

Step *1* Taking the derivative of the first equation in (4) results in

$$\begin{aligned} \dot{z}_1 &= \dot{x}_1 - \dot{y}_d - \dot{\lambda}_1 \\ &= f_1 + d_1 + x_2 - \dot{y}_d + c_1\lambda_1 - \lambda_2 \\ &= f_1 + d_1 + z_2 + \beta_2 + \lambda_2 - \dot{y}_d + c_1\lambda_1 - \lambda_2 \\ &= f_1 + d_1 + z_2 + y_2 + \alpha_2 - \dot{y}_d + c_1\lambda_1 \\ &= \Delta_1 + \alpha_2 - \dot{y}_d + c_1\lambda_1 \end{aligned} \tag{6}$$

where $\Delta_1 = f_1 + d_1 + z_2 + y_2$ is regarded as the lumped "disturbance-like" term. If the virtual control is chosen as $\alpha_2 = -\kappa_1 z_1 + \dot{y}_d - c_1\lambda_1 - \Delta_1$, $\kappa_1 > 0$ then $\dot{z}_1 = -\kappa_1 z_1$ which is asymptotically stable. However, Δ_1 is totally unknown and should be estimated. According to the idea of the UDE-based method [8], the virtual control consists of two parts,

$$\alpha_2 = \alpha_{2l} + \alpha_{2d} \tag{7}$$

where the uncertainty compensation term α_{2d} will be designed later and the linear feedback term α_{2l} is chosen as

$$\alpha_{2l} = -\kappa_1 z_1 + \dot{y}_d - c_1\lambda_1 \tag{8}$$

such that

$$\dot{z}_1 = -\kappa_1 z_1 + \alpha_{2d} + \Delta_1 \tag{9}$$

From (9), it has

$$\Delta_1 = \dot{z}_1 + \kappa_1 z_1 - \alpha_{2d} \tag{10}$$

The information of the "disturbance-like" term Δ_1 can be estimated by using a proper stable low-pass filtering with a suitable bandwidth,

$$\hat{\Delta}_1 = g_{f1} * [\dot{z}_1 + \kappa_1 z_1 - \alpha_{2d}] \tag{11}$$

where g_f is a low-pass filter with the unity gain and zero phase shift in the spectrum of Δ_1 and $*$ is the convolution operator. Hence, the uncertainty compensation term α_{2d} can be designed as

$$\alpha_{2d} = -\hat{\Delta}_1 \tag{12}$$

Substituting (11) into (12), solving for α_{2d} yields

$$\alpha_{2d} = -\mathcal{L}^{-1}\left\{\frac{G_{f1}}{1-G_{f1}}\right\} * [\dot{z}_1 + \kappa_1 z_1] \tag{13}$$

where G_{f1} is the Laplace transform of g_{f1} and $\mathcal{L}^{-1}$ is the inverse Laplace operator. Hence, the virtual control has the form of

$$\begin{aligned} \alpha_2 &= -\kappa_1 z_1 + \dot{y}_d - c_1\lambda_1 \\ &\quad -\mathcal{L}^{-1}\left\{\frac{G_{f1}}{1-G_{f1}}\right\} * [\dot{z}_1 + \kappa_1 z_1] \end{aligned} \tag{14}$$

Substituting (7), (8), (12) into (6), the error dynamics is obtained as

$$\dot{z}_1 = -\kappa_1 z_1 + \tilde{\Delta}_1$$

where $\tilde{\Delta}_1 = \Delta_1 - \hat{\Delta}_1$.

Step *i* ($i = 2, ..., n-1$) Taking the derivative of the ith equation in (4) and considering (3), (5),

$$\begin{aligned} \dot{z}_i &= \dot{x}_i - \dot{\beta}_i - \dot{\lambda}_i \\ &= f_i + d_i + x_{i+1} - \dot{\beta}_i + c_i\lambda_i + \lambda_{i+1} \\ &= f_i + d_i + z_{i+1} + \beta_{i+1} + \lambda_{i+1} - \dot{\beta}_i + c_i\lambda_i - \lambda_{i+1} \\ &= f_i + d_i + z_{i+1} + y_{i+1} + \alpha_{i+1} - \dot{\beta}_i + c_i\lambda_i \\ &= \Delta_i + \alpha_{i+1} + a_i\beta_i - b_i\alpha_i + c_i\lambda_i \end{aligned} \tag{15}$$

where $\Delta_i = f_i + d_i + z_{i+1} + y_{i+1}$ is regarded as the lumped "disturbance-like" term. The virtual control consists of $\alpha_{i+1} = \alpha_{(i+1)l} + \alpha_{(i+1)d}$ with the linear feedback term

$$\alpha_{(i+1)l} = -\kappa_i z_i - a_i\beta_i + b_i\alpha_i - c_i\lambda_i. \tag{16}$$

Similar to Step 1, the uncertainty compensation term is designed as

$$\begin{aligned} \alpha_{(i+1)d} &= -\hat{\Delta}_i \\ &= -\mathcal{L}^{-1}\left\{\frac{G_{fi}}{1-G_{fi}}\right\} * [\dot{z}_i + \kappa_i z_i] \end{aligned}$$

Then the virtual control is obtained as

$$\begin{aligned} \alpha_{i+1} &= -\kappa_i z_i - a_i\beta_i + b_i\alpha_i - c_i\lambda_i \\ &\quad -\mathcal{L}^{-1}\left\{\frac{G_{fi}}{1-G_{fi}}\right\} * [\dot{z}_i + \kappa_i z_i] \end{aligned} \tag{17}$$

The error dynamics can be obtained as

$$\dot{z}_i = -\kappa_i z_i + \tilde{\Delta}_i \tag{18}$$

where $\tilde{\Delta}_i = \Delta_i - \hat{\Delta}_i$.

Step *n* Taking the derivative of the nth equation in (4) and considering (3), (5), there is

$$\begin{aligned} \dot{z}_n &= \dot{x}_n - \dot{\beta}_n - \dot{\lambda}_n \\ &= \dot{x}_n - \dot{\beta}_n + c_n\lambda_n - (sat(u) - u) \\ &= f_n + d_n + sat(u) - \dot{\beta}_n + c_n\lambda_n - (sat(u) - u) \\ &= \Delta_n + u + a_n\beta_n - b_n\alpha_n + c_n\lambda_n \end{aligned} \tag{19}$$

where $\Delta_n = f_n + d_n$ is regarded as the lumped "disturbance-like" term. Thus the control input consists of $u = u_l + u_d$ with the linear feedback term

$$u_l = -\kappa_n z_n - a_n\beta_n + b_n\alpha_n - c_n\lambda_n \tag{20}$$

and the uncertainty compensation term

$$\begin{aligned} u_d &= -\hat{\Delta}_n \\ &= -\mathcal{L}^{-1}\left\{\frac{G_{fn}}{1-G_{fn}}\right\} * [\dot{z}_n + \kappa_n z_n] \end{aligned} \tag{21}$$

Then the final control input is obtained as

$$\begin{aligned} u &= -\kappa_n z_n - a_n\beta_n + b_n\alpha_n - c_n\lambda_n \\ &\quad -\mathcal{L}^{-1}\left\{\frac{G_{fn}}{1-G_{fn}}\right\} * [\dot{z}_n + \kappa_n z_n] \end{aligned} \tag{22}$$

Furthermore, the error dynamics is obtained as

$$\dot{z}_n = -\kappa_n z_n + \tilde{\Delta}_n \tag{23}$$

where $\tilde{\Delta}_n = \Delta_n - \hat{\Delta}_n$.

4 Stability Analysis

The proposed control structure is shown in Fig. 1. The following theorem shows the stability and control performance of the closed-loop system.

Theorem 5. *Consider the closed-loop system which consists of the nth-order system (1), the auxiliary system (3), the reference dynamics (5), the virtual controls (14) (17) and the control input (22). Define the compact sets as* $\bar{x} = (x_1, ..., x_n) \in \mathbb{R}^n$, $\Omega_x = \{\bar{x} : \|\bar{x}\|_2 \le p_1\}$, $\bar{z} = (z_1, ..., z_n) \in \mathbb{R}^n$, $\Omega_z := \{\bar{z} : \|\bar{z}\|_2 \le 2p_2\}$ *and* $\bar{y} = (y_2, ..., y_n) \in \mathbb{R}^{n-1}$, $\Omega_y := \{\bar{y} : \|\bar{y}\|_2 \le 2p_3\}$, $\bar{\lambda} = (\lambda_1, ..., \lambda_n) \in \mathbb{R}^n$, $\Omega_\lambda = \{\bar{\lambda} : \|\bar{\lambda}\|_2 \le 2p_4\}$, p_1, p_2, p_3, p_4 *are pre-defined arbitrary positive constants. If the following conditions are satisfied,*

C1) $(y_d, \dot{y}_d) \in \mathcal{N}_y$, $\mathcal{N}_y$ *is a compact set in* $\mathbb{R}^2$.

C2) $(d_1, ..., d_n) \in \mathcal{N}_d$, $\mathcal{N}_d$ *is a compact set in* $\mathbb{R}^n$.

C3) $c_1 > \frac{1}{2}$, $c_i > 1, i = 2, ..., n-1$, *and* $c_n > \frac{3}{4}$.

C4) $\kappa_i > \frac{1}{4}$, $a_i > 1, i = 2, ..., n$.

then

i) All signals in the closed-loop system are bounded.

ii) The system tracking error z_1 can be adjusted within an arbitrary small compact set by changing the control parameters $\kappa_1, ..., \kappa_n, c_1, ..., c_n, a_2, b_2, ..., a_n, b_n$ *appropriately.*

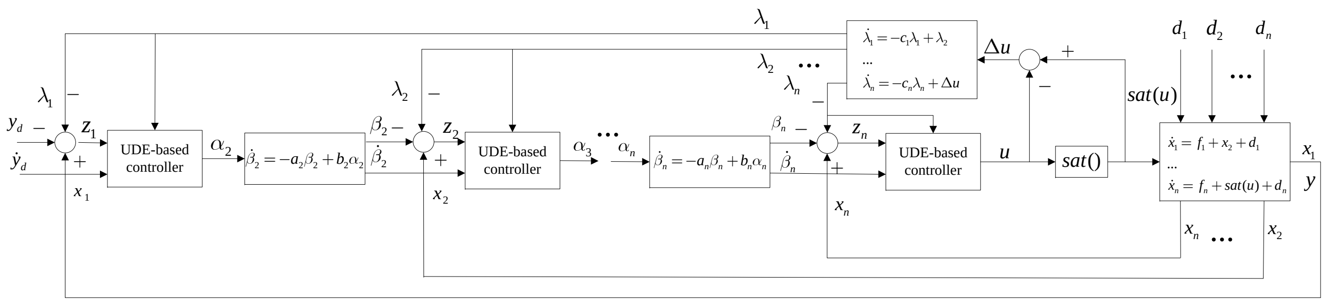

Fig. 1: The proposed control structure.

Proof. By substituting the virtual controls (14), (17) and the control input (22) into the original system (1) results in the following closed-loop system

$$\begin{aligned}\dot{z}_i &= -\kappa_i z_i + \tilde{\Delta}_i, i = 1, ..., n-1 \\ \dot{z}_n &= -\kappa_n z_n + \tilde{\Delta}_n\end{aligned} \tag{24}$$

where $\tilde{\Delta}_i = \Delta_i - \hat{\Delta}_i$ and $\Delta_i = f_i + d_i + z_{i+1} + y_{i+1}, i = 1, ..., n-1$, $\Delta_n = f_n + d_n$. Using the Young's inequality, there is

$$\begin{aligned}z_i \dot{z}_i &= -\kappa_i z_i^2 + z_i \tilde{\Delta}_i \\ &\leq -\kappa_i z_i^2 + \frac{z_i^2}{4} + \tilde{\Delta}_i^2 \\ &= -\bar{\kappa}_i z_i^2 + \tilde{\Delta}_i^2\end{aligned} \tag{25}$$

where $\bar{\kappa}_i = \kappa_i - \frac{1}{4}$, and condition $C4$ guarantees that $\bar{\kappa}_i > 0$. Furthermore, consider the following Lyapunov function candidate

$$V_z = \frac{1}{2}\sum_{i=1}^{n} z_i^2. \tag{26}$$

Taking the derivative of both sides of the above equation leads to

$$\begin{aligned}\dot{V}_z &= \sum_{i=1}^{n} z_i \dot{z}_i \\ &\leq -\sum_{n=1}^{n} \bar{\kappa}_i z_i^2 + \sum_{i=1}^{n} \tilde{\Delta}_i^2 \\ &\leq -\bar{c}_z V_z + \sum_{i=1}^{n} \tilde{\Delta}_i^2\end{aligned} \tag{27}$$

where $\bar{c}_z = \min\{\bar{\kappa}_i, i = 1, ..., n\}$, and the functions $\tilde{\Delta}_i = (\delta - g_f) * \Delta_i = (\delta - g_f) * (f_i + d_i + z_{i+1} + y_{i+1})$ are continuous functions of $x_1, ..., x_i, z_1, ..., z_i, y_2, ..., y_i, d_1, ..., d_i$, where δ is the Dirac delta function. According to (5), there is

$$\dot{y}_i = -a_i y_i + (b_i - a_i)\alpha_i - \dot{\alpha}_i \tag{28}$$

thus

$$|\dot{y}_i + a_i y_i| \leq \xi_i(y_d, \dot{y}_d, \ddot{y}_d, z_1, ...z_i, y_2, ..., y_i) \tag{29}$$

where $\xi_i(y_d, \dot{y}_d, \ddot{y}_d, z_1, ...z_i, y_2, ..., y_i, \lambda_1, ..., \lambda_{i-1}) = (b_i - a_i)\alpha_i - \dot{\alpha}_i$ is a continuous function of variables $y_d, \dot{y}_d, \ddot{y}_d, z_1, ..., z_i, y_2, ..., y_i, \lambda_1, ..., \lambda_{i-1}$. Furthermore,

$$\begin{aligned}\dot{y}_i y_i &\leq -a_i y_i^2 + |y_i| \xi_i \\ &\leq -a_i y_i^2 + y_i^2 + \frac{\xi_i^2}{4} \\ &= -\bar{a}_i y_i^2 + \frac{\xi_i^2}{4}\end{aligned} \tag{30}$$

where $\bar{a}_i = a_i - 1$, and condition $C4$ guarantees that $\bar{a}_i > 0$. Consider the Lyapunov function candidate

$$V_y = \frac{1}{2}\sum_{i=2}^{n} y_i^2 \tag{31}$$

thus,

$$\begin{aligned}\dot{V}_y &= \sum_{i=2}^{n} y_i \dot{y}_i \\ &\leq -\sum_{i=2}^{n} \bar{a}_i y_i^2 + \frac{1}{4}\sum_{i=2}^{n} \xi_i^2 \\ &\leq -\bar{c}_y V_y + \frac{1}{4}\sum_{i=2}^{n} \xi_i^2\end{aligned} \tag{32}$$

where $\bar{c}_y = \min\{\bar{a}_i, i = 2, ..., n\}$. From (3), while $i = 1, ..., n-1$, there is

$$\begin{aligned}\lambda_i \dot{\lambda}_i &= -c_i \lambda_i^2 + \lambda_i \lambda_{i+1} \\ &\leq -c_i \lambda_i^2 + \frac{\lambda_i^2}{2} + \frac{\lambda_{i+1}^2}{2}\end{aligned} \tag{33}$$

While $i = n$,

$$\begin{aligned}\lambda_n \dot{\lambda}_n &= -c_n \lambda_n^2 + \lambda_n \Delta u \\ &\leq -c_n \lambda_n^2 + \frac{\lambda_n^2}{4} + \Delta u^2\end{aligned} \tag{34}$$

Consider the Lyapunov function candidate

$$V_\lambda = \frac{1}{2}\sum_{i=1}^{n} \lambda_i^2 \tag{35}$$

Taking its derivative results in

$$\begin{aligned} \dot{V}_\lambda &= -\sum_{i=1}^{n} c_i\lambda_i^2 + \sum_{i=1}^{n-1} \lambda_i\lambda_{i+1} + \lambda_n \Delta u \\ &\leq -\sum_{i=1}^{n} c_i\lambda_i^2 + \frac{\lambda_1^2}{2} + \sum_{i=2}^{n-1} \lambda_i^2 + \frac{3}{4}\lambda_n^2 + \Delta u^2 \\ &\leq -\sum_{i=1}^{n} \bar{c}_i\lambda_i^2 + \Delta u^2 \\ &\leq -\bar{c}_\lambda V_\lambda + \Delta u^2 \end{aligned} \tag{36}$$

where $\bar{c}_1 = c_1 - \frac{1}{2}$, $\bar{c}_i = c_i - 1, i = 2, ..., n-1$, $\bar{c}_n = c_n - \frac{3}{4}$ and $\bar{c}_\lambda = \min\{\bar{c}_i, i = 1, ..., n\}$. The condition $C3$ guarantees that $\bar{c}_i > 0$.

Let $V = V_z + V_y + V_\lambda$ be the composite Lyapunov function candidate, thus

$$\begin{aligned} \dot{V} &= \dot{V}_z + \dot{V}_y + \dot{V}_\lambda \\ &\leq -\bar{c}_z V_z - \bar{c}_y V_y - \bar{c}_\lambda V_\lambda + \\ & \quad \sum_{i=1}^{n} \tilde{\Delta}_i^2 + \frac{1}{4}\sum_{i=2}^{n} \xi_i^2 + \Delta u^2 \end{aligned} \tag{37}$$

Under the conditions $C1$ and $C2$, firstly, consider the compact set $\Omega_1 = \mathcal{N}_d \times \Omega_x \times \Omega_z \times \Omega_y$. The functions $\tilde{\Delta}_i$ are continuous functions inside Ω_1, thus there exists a maximum value for $\sum_{i=1}^{n} \tilde{\Delta}_i^2$, say M_1. Next, consider the compact set $\Omega_2 = \mathcal{N}_y \times \Omega_z \times \Omega_y \times \Omega_\lambda$. Since the functions ξ_i are continuous functions inside Ω_2, there exists a maximum value for $\frac{1}{4}\sum_{i=2}^{n} \xi_i^2$, say M_2. Similarly, the function Δu is also a continuous function in Ω_2, and it must have a maximum value say M_3. Therefore, (37) can be rewritten as

$$\dot{V} \leq -\mu V + \zeta \tag{38}$$

where $\mu = \min\{\bar{c}_z, \bar{c}_y, , \bar{c}_\lambda\} > 0$ and $\zeta = M_1 + M_2 + M_2 > 0$. Solving (38) gives

$$0 \leq V(t) \leq V(0)e^{-\mu t} + \frac{\zeta}{\mu}\left(1 - e^{-\mu t}\right) \tag{39}$$

Let $p = p_2 + p_3 + p_4$, if the control parameters, $\kappa_1, ..., \kappa_n, c_1, ..., c_n, a_2, b_2, ..., a_n, b_n$, are chosen to satisfy that $p > \frac{\zeta}{\mu}$ and from (38) there is $\dot{V} < 0$ on $V = p$. Furthermore, if the initial condition satisfies that $V(0) \leq p - \frac{\zeta}{\mu}$, there is $V(t) \leq p$ for all $t \geq 0$. As shown in Fig. 2, the compact set $\Omega = \Omega_x \times \Omega_z \times \Omega_y$ is an invariant set. It can be concluded

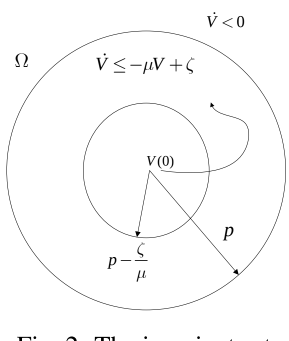

Fig. 2: The invariant set

that: (i) the signals $x_1, ..., x_n, z_1, .., z_n, y_2, ..., y_n, \lambda_1, ..., \lambda_n$ are bounded. As $t \to \infty$, $V(t) < \frac{\zeta}{\mu}$, then $|z_1| = |x_1 - y_d - \lambda_1| < \sqrt{\zeta/\mu}$; (ii) by adjusting the control parameters $\kappa_1, ..., \kappa_n, c_1, ..., c_n, a_2, b_2, ..., a_n, b_n$, we can increase μ and decrease ζ to make $\zeta/\mu \to 0$, such that $z_1 \to 0$ can be achieved and z_1 can be bounded in an arbitrarily small compact set. Furthermore, from (36), it is obvious that $\lambda_1 \to 0$ can be guaranteed by tuning the control parameters appropriately. Thus $\lim_{t\to\infty} e = \lim_{t\to\infty}(x_1 - y_d) \to 0$ can be achieved. □

5 Simulation Results

To illustrate the proposed method, the following example is considered,

$$\begin{aligned} \dot{x}_1 &= x_2 - \frac{x_1}{1 + x_1^4} + d_1 \\ \dot{x}_2 &= sat(u) - x_2 e^{-x_1^2} + d_2 \\ y &= x_1 \end{aligned} \tag{40}$$

where $d_1 = 0.1\sin(x_1)$ and $d_2 = 0.1\cos(x_2)$. The reference signal is given as $y_d = \sin t$. The input saturation is described by

$$sat(u) = \begin{cases} 0.8 & , u > 0.8 \\ u & , -1 \leq u \leq 0.8 \\ -1 & , u < -1 \end{cases} \tag{41}$$

The sampling time in the simulation is $0.001s$ and the initial conditions of the states are chosen as $x_1(0) = 0.05, x_2(0) = 0.2$. The low-pass filters $G_{f1} = G_{f2} = \frac{1}{0.05s+1}$ are used in the UDE-based method. Other parameters are chosen as $c_1 = c_2 = 5$ and $a_2 = b_2 = 100$.

The results are shown in Fig. 3. As expected, the tracking performance is shown in Figs. 3 (a) and (d). It can be seen that the output signal can successfully track the reference signal very well. Figs. 3 (b) and (e) demonstrate the controller output u and the actual input $sat(u)$ respectively, where the actual input $sat(u)$ is saturated when the controller output u exceeds the limitation. Figs. 3 (c) and (f) illustrate that the uncertainties Δ_1, Δ_2 are estimated accurately which results in the good performance of the UDE-based control in handling the mismatched uncertainties as well as the input saturation.

6 Conclusions

In this paper, an UDE-based backstepping control method was developed for nonlinear systems with mismatched uncertainties and input saturation. The backstepping approach was integrated with the UDE to deal with the mismatched uncertainties. An auxiliary system was constructed to handle the input saturation. In the development of the UDE-based control, only the spectrum of uncertainties and the system order information are required. Stability analysis has shown that all signals in the closed-loop system are bounded. Furthermore, a simulation example verified the effectiveness of the proposed method.

References

[1] M. Krstic, P. V. Kokotovic, and I. Kanellakopoulos, *Nonlinear and adaptive control design*. John Wiley & Sons, Inc., 1995.

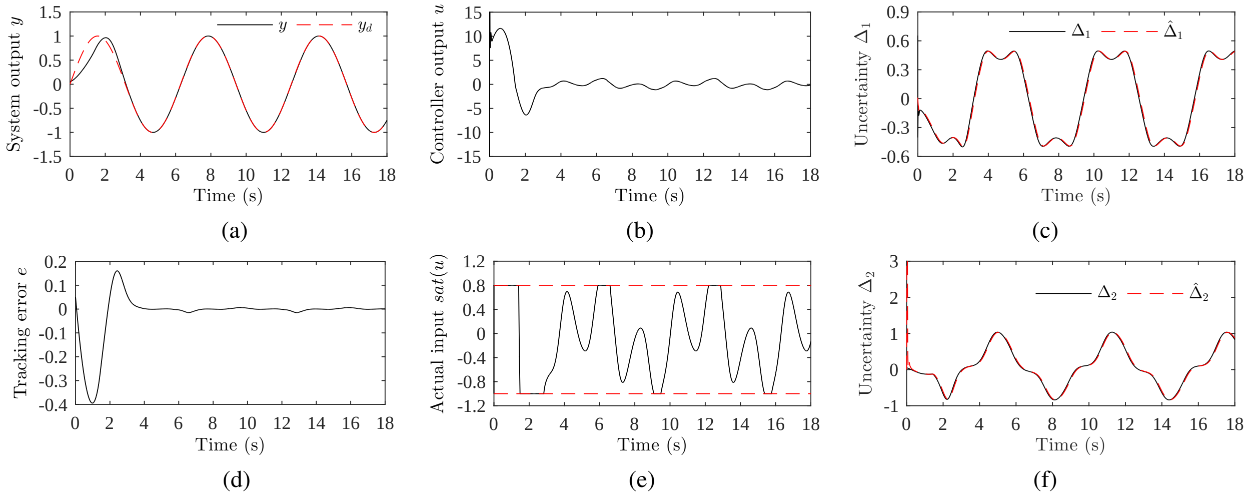

Fig. 3: Simulation results

[2] S. S. Ge, C. C. Hang, T. H. Lee, and T. Zhang, *Stable adaptive neural network control*. Springer Science & Business Media, 2013, vol. 13.

[3] A. Levant, "Sliding order and sliding accuracy in sliding mode control," *International Journal of Control*, vol. 58, no. 6, pp. 1247–1263, 1993.

[4] J. Han, "From PID to active disturbance rejection control," *IEEE Tran. on Industrial Electronics*, vol. 56, no. 3, pp. 900–906, 2009.

[5] K. Ohishi, M. Nakao, K. Ohnishi, and K. Miyachi, "Microprocessor-controlled DC motor for load-insensitive position servo system," *IEEE Tran. on Industrial Electronics*, vol. 1, no. IE-34, pp. 44–49, 1987.

[6] J.-H. She, X. Xin, and Y. Pan, "Equivalent-input-disturbance approach — analysis and application to disturbance rejection in dual-stage feed drive control system," *IEEE/ASME Tran. on Mechatronics*, vol. 16, no. 2, pp. 330–340, 2011.

[7] B.-Z. Guo and Z.-l. Zhao, "On the convergence of an extended state observer for nonlinear systems with uncertainty," *Systems & Control Letters*, vol. 60, no. 6, pp. 420–430, 2011.

[8] Q.-C. Zhong and D. Rees, "Control of uncertain LTI systems based on an uncertainty and disturbance estimator," *Journal of Dynamic Systems, Measurement, and Control*, vol. 126, no. 4, pp. 905–910, 2004.

[9] S. Li, J. Yang, W.-H. Chen, and X. Chen, "Generalized extended state observer based control for systems with mismatched uncertainties," *IEEE Tran. on Industrial Electronics*, vol. 59, no. 12, pp. 4792–4802, 2012.

[10] J. Yang, W.-H. Chen, and S. Li, "Non-linear disturbance observer-based robust control for systems with mismatched disturbances/uncertainties," *IET Control Theory & Applications*, vol. 5, no. 18, pp. 2053–2062, 2011.

[11] J. Yang, S. Li, and X. Yu, "Sliding-mode control for systems with mismatched uncertainties via a disturbance observer," *IEEE Tran. on Industrial Electronics*, vol. 60, no. 1, pp. 160–169, 2013.

[12] A. Kuperman and Q.-C. Zhong, "Robust control of uncertain nonlinear systems with state delays based on an uncertainty and disturbance estimator," *International Journal of Robust and Nonlinear Control*, vol. 21, no. 1, pp. 79–92, Mar. 2010.

[13] B. Ren, Q.-C. Zhong, and J. Chen, "Robust control for a class of non-affine nonlinear systems based on the uncertainty and disturbance estimator," *IEEE Tran. on Industrial Electronics*, vol. 62, no. 9, pp. 5881–5888, 2015.

[14] B. Ren, Y. Wang, and Q.-C. Zhong, "UDE-based control of variable-speed wind turbine systems," *International Journal of Control*, 2015, DOI 10.1080/00207179.2015.1126678. [Online]. Available: http://dx.doi.org/10.1080/00207179.2015.1126678

[15] V. Kapila and K. Grigoriadis, *Actuator saturation control*. CRC Press, 2002.

[16] N. O. Pérez-Arancibia, T.-C. Tsao, and J. S. Gibson, "Saturation-induced instability and its avoidance in adaptive control of hard disk drives," *IEEE Tran. on Control Systems Technology*, vol. 18, no. 2, pp. 368–382, 2010.

[17] S. Tarbouriech and M. Turner, "Anti-windup design: an overview of some recent advances and open problems," *IET Control Theory & Applications*, vol. 3, no. 1, pp. 1–19, 2009.

[18] K. Esfandiari, F. Abdollahi, and H. A. Talebi, "Adaptive control of uncertain nonaffine nonlinear systems with input saturation using neural networks," *IEEE Tran. on Neural Networks and Learning Sysytems*, vol. 26, no. 10, pp. 2311–2322, 2015.

[19] J. Zhou and C. Wen, *Adaptive backstepping control of uncertain systems: Nonsmooth nonlinearities, interactions or time-variations*. Springer, 2008.

[20] M. Chen, S. S. Ge, and B. Ren, "Adaptive tracking control of uncertain mimo nonlinear systems with input constraints," *Automatica*, vol. 47, no. 3, pp. 452–465, 2011.

[21] C. Wen, J. Zhou, Z. Liu, and H. Su, "Robust adaptive control of uncertain nonlinear systems in the presence of input saturation and external disturbance," *IEEE Tran. on Automatic Control*, vol. 56, no. 7, pp. 1672–1678, 2011.

[22] J. Ma, Z. Zheng, and P. Li, "Adaptive dynamic surface control of a class of nonlinear systems with unknown direction control gains and input saturation," *IEEE Transactions on Cybernetics*, vol. 45, no. 4, pp. 728–741, 2015.

[23] Z. Yan-long and C. Mou, "Disturbance-observer-based fault tolerant control for near space vehicles with input saturation," in *Proceedings of 33rd Chinese Control Conference (CCC)*. IEEE, 2014, pp. 2101–2105.

[24] X. Wu, J. Pang, and T. Yang, "Anti-windup design for active disturbance rejection control," in *Proceedings of 33rd Chinese Control Conference (CCC)*. IEEE, 2014, pp. 2389–2395.

[25] H. Li, T. Zhang, and Z. Guo, "Adaptive control for a class of uncertain chaotic systems with saturation nonlinear input," in *Proceeedings of 9th International Conference on Electronic Measurement & Instruments*. IEEE, 2009, pp. 3–583.

[26] D. Swaroop, J. K. Hedrick, P. P. Yip, and J. C. Gerdes, "Dynamic surface control for a class of nonlinear systems," *IEEE Tran. on Automatic control*, vol. 45, no. 10, pp. 1893–1899, Oct. 2000.