HIGH BANDWIDTH CONTROL OF A PIEZOELECTRIC STAGE IN THE PRESENCE OF RATE-DEPENDENT HYSTERESIS

INTRODUCTION

Piezoelectric nanpositioning actuators are widely employed in many applications, including the hard disk drives [1–3], gyroscopes [4] and the scanning probe microscopes [5]. Besides the precise position control, high-bandwidth tracking control is crucial for the applications of piezoelectric actuators, since, for example, it will allow hard disk drives to meet the demand for higher data storage capacities with lower access times [6]. However, the existence of vibration dynamics, the coupled hysteresis and other uncertainty (e.g. creep) greatly limit the positioning and tracking performance of the piezoelectric actuators over a broad bandwidth. As a solution to tackle this problem and achieve precise control over a broad frequency range, stiffer piezoelectric actuators that have higher resonant frequencies can be employed. In this way, the scan frequency at which the vibration is significant becomes higher, while, as a major disadvantage, the range of motion is reduced [7]. This highly motivates the development of control techniques that enable high-bandwidth nanpositioning of piezoelectric-actuated systems.

In order to perform the high-bandwidth high-precision tracking on the piezoelectric-actuated systems, numerous controllers have been developed based on a cascade model of the rate-independent hysteresis and vibration dynamics [5,8]. Different from these work, the input-output relation of the piezoelectric stage can be also considered as a rate-dependent hysteresis model [9]. In this paper, to achieve the high-bandwidth control on a piezoelectric stage, the rate-dependent hysteresis is reconstructed as a linear function plus an unknown nonlinear term. Then, in order to estimate and compensate such an unknown nonlinear term together with other uncertainties, the uncertainty-and-disturbance-estimator (UDE)-based controller will be further developed based on the internal model principle [10]. Experimental results will be provided to show that the proposed UDE-based controller can well achieve high-bandwidth precise control.

EXPERIMENTAL SETUP AND SYSTEM CHARACTERIZATION

The experimental setup is shown in Fig. 1. The detailed specification of the piezoelectric nanpositioning system can be referred to http://www.pi.ws. The sampling frequency is chosen as 40kHz.

In order to show that the hysteresis in piezoelectric stage is indeed rate-dependent, three sinusoidal signals in the form of \( v(t) = 3(\sin(2\pi ft) + 1) \) with the frequencies \( f = 1\text{Hz}, 50\text{Hz}, 100\text{Hz} \) are used as the excitation to obtain a series of major hysteresis loops, which are shown in Fig. 2(a). It can

FIGURE 1. Experimental setup of P-753.31c nanpositioning system.
be clearly seen that the hysteresis between the input voltage \( v(t) \) and the output displacement \( y(t) = H[v(t)] + d(t) \) exhibits a rate-dependent effect with the increase of the frequency of the input, where \( d(t) \) includes some unknown bounded disturbances and uncertainties (e.g., creep effect and modeling error). To compensate the rate-dependent hysteresis \( H[v(t)] \), it will firstly be constructed as a linear function plus the unknown nonlinear term \( d_H(t) = H[v(t)] - kv(t) \) in this paper, i.e.,

\[
H[v(t)] = kv(t) + d_H(t),
\]

where \( k \) should be chosen as the slope of the asymptotes of the hysteresis loops to ensure the boundedness of \( d_H(t) \) as illustrated in Fig. 2(a). The resulted nonlinear term \( d_H(t) \) when \( k = 4.33 \) is shown in Fig. 2(b), where the horizontal axis is scaled by the periods of the signals.

**CONTROLLER DESIGN**

To facilitate the controller design, a filter will be added on the output \( y(t) \) to generate the filtered output \( x(t) \). For simplification, a first-order low-pass filter is adopted, which can be also replaced by other higher-order low-pass filters, then

\[
x(t) = \mathcal{L}^{-1} \left\{ \frac{\rho}{s + \rho} \right\} \ast (H[v(t)] + d(t)),
\]

where “*” is the convolution operator, \( \mathcal{L}^{-1} \{ \cdot \} \) is the operator of inverse Laplace transform. By choosing a proper value of \( \rho > 0 \), it holds that \( x(t) \approx y(t) \). Based on (1) and (2), the dynamics of the piezoelectric stage is rewritten as

\[
\dot{x}(t) = -\rho x(t) + \rho H[v(t)] + \rho d(t)
\]

\[
= -\rho x(t) + \rho kv(t) + u_d(t),
\]

with \( u_d(t) \triangleq \rho d_H(t) + \rho d(t) \).

For the required specifications of the closed-loop system, the following stable reference model \( (A_m > 0) \) can be chosen with the piecewise continuous and uniformly bounded command \( c(t) \)

\[
\dot{x}_m(t) = -A_m x_m(t) + B_m c(t).
\]

The control objective is to design a control signal \( v(t) \) such that, for any desired continuous trajectory \( x_m(t) \), the tracking error defined as \( e(t) = x_m(t) - x(t) \) asymptotically converges to zero. The desired error dynamics governed by \( \dot{e}(t) = -K e(t) \), where \( K > 0 \) is the error feedback gain. From (3), there is \( u_d(t) = \dot{x}(t) + \rho x(t) - \rho kv(t) \). By using the UDE [8], \( u_d(t) \) can be estimated as

\[
\dot{u}_d(t) = g_f(t) \ast u_d(t) = g_f(t) \ast (\dot{x}(t) + \rho x(t) - \rho kv(t))
\]

where \( g_f(t) \) is the impulse response of a low-pass filter \( G_f(s) \), which should be stable and strictly proper, and have the unity gain and zero phase shift over the spectrum of \( u_d(t) \) and zero gain elsewhere. According to the filter design based on the internal model principle [10], the filter \( G_f(s) \) is chosen as follows

\[
G_f(s) = 1 - \frac{s(s^2 + \omega^2)}{(s + \alpha)(s^2 + \alpha_1 s + \beta_1)},
\]

where \( \omega = 2\pi f \), and \( \alpha, \alpha_1 \) and \( \beta_1 \) are design parameters. By combining (3), (4) and the desired error dynamics, there is

\[
-K e(t) = -A_m x_m + B_m c + \rho x(t) - \rho kv(t) - u_d(t).
\]

By replacing \( u_d(t) \) in (7) with \( \dot{u}_d(t) \) in (5), the UDE-based controller can be obtained as

\[
v(t) = \frac{1}{\rho k} \left[ \rho x(t) - \mathcal{L}^{-1} \left\{ \frac{s G_f(s)}{1 - s G_f(s)} \right\} \ast x(t) + \mathcal{L}^{-1} \left\{ \frac{1}{1 - G_f(s)} \right\} \ast (-A_m x_m + B_m c + K e(t)) \right].
\]

**EXPERIMENTAL RESULTS**

To verify the effectiveness of the UDE-based controller (8) with the filter \( G_f(s) \) in (6) is implemented on the experimental setup illustrated in Fig. 1 to track the desired trajectory \( x_m(t) \) in (4) with \( c(t) = A \sin(2\pi f t - \pi / 2) + 1 \). The parameters are chosen in the following manner. First, \( A_m \) and \( B_m \) are set as 80000.
controller that is developed based on the internal model principle. Experimental results had been provided to show that the desired trajectory in a wide range of frequency can be accurately tracked by the proposed UDE-based controller.

REFERENCES


CONCLUSION

In this paper, the rate-dependent hysteresis in a piezoelectric stage is firstly reconstructed as a linear term plus an unknown nonlinear term. The unknown term together with other uncertainties are then estimated and compensated by the UDE-based