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Robust Tracking for a Class of Uncertain Switched Linear Systems Based on the Uncertainty and Disturbance Estimator

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Abstract—**The robust tracking problem of a class of linear switched systems with both model uncertainty and external disturbance is investigated. Firstly, some new stability results for the linear switched systems with external disturbance are obtained in the following two cases: one is under arbitrary switching paths, the other is under switching paths with average dwell time. Secondly, a robust switched control strategy based on the existing uncertainty and disturbance estimator (UDE)-based control method is developed for a class of linear switched system with both model uncertainty and external disturbance. The asymptotic stability of the closed-loop linear switched system is established under the above two mentioned switching paths. Finally, the important features and good performance of the proposed method are demonstrated through numerical simulation studies.**

I. INTRODUCTION

In the past decades, switched systems and their applications have attracted considerable attention, and many important results have been published for the synthesis and analysis of those switched systems, e.g., [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12] and the references therein. Many efforts have been devoted to the linear switched systems described by linear difference or equations in the following aspects: stability [1], [2], [3], observability [4], controllability [5], stabilization [7], [6], [8], and so forth. There are two important methods to deal with the switched system: one is the common Lyapunov function (CLF) method [9], [10]; the other is the multiple Lyapunov functions (MLF) method [11]. According to the CLF method, for a given switched system if there exists a CLF, this switched system is globally asymptotically stable (GAS) under arbitrary switching laws if all subsystems are GAS. The MLF method is another powerful tool to investigate the stability problem of such switched systems in which all subsystems cannot share a CLF. For a given switched system whose subsystems cannot share a CLF, it is well known that such a system is GAS for any switching path if the dwell time, in which all the subsystems are active, is sufficiently large, and this method is called the dwell time (DT) method. Later, the average dwell time (ADT) method is proposed, and becomes a powerful tool to study the synthesis and analysis of those switched systems whose subsystems cannot share a CLF. Recently, an modified ADT method [12] is presented to improve the existing ADT method in the following two aspects: 1) The application conditions of the existing ADT method is simplified; 2) The conservativeness in estimating the minimum lower bound of ADT is relaxed.

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It should be noticed that among the above mentioned switched systems, system uncertainty and external disturbance are not considered. Unfortunately, this is not the case in practice. As a matter of fact, model uncertainty and external disturbance appear frequently in practical applications, and may deteriorate the performance (e.g., system stability, robustness of tracking). For the switched systems, a lot of methods have been proposed to investigate the robust stability and stabilization problems under both arbitrary switching and some specifically designed switching paths [13], [14], [15], [16], [17], [18], [19], [20], [21]. For example, model reference robust adaptive control (MRRAC) [13], adaptive robust control [14], sliding-mode control [15], [16], the disturbance-observer-based control (DOBC) [17], robust fault detection observer (RFDO) [18], H_∞ control [19], [20], [21], and so on. However, the assumption that the disturbance is bounded in some forms is required in most of above methods. For example, $d(t)$ in [17] belongs to $L_2^n[0,+\infty)$, or $d(t)$ in [18] is assumed to be L_2-norm bounded, etc. More importantly, the robust control and disturbance rejection problems in the above mentioned works are mainly solved by the linear matrix inequalities (LMIs). However, the stability conditions, which are derived by the LMIs, are only sufficient conditions and often result in the conservativeness of the conclusions. The robust control problem of linear switched systems with both model uncertainty and external disturbance has remained a very challenging problem.

The UDE-based control method [22] was proposed to replace the time-delay control (TDC) [23], and it has the following two advantages: 1) the system model or a disturbance model is not necessarily known completely; 2) both structured (or unstructured) uncertainties and external disturbances can be well handled. Being an effective robust control strategy, UDE-based control has found widespread applications in various non-switched systems, [24], [25], [26], [27]. Naturally, it is of interest to apply the UDE-based control to switched systems with both model uncertainty and external disturbance. To the best of the anthors' knowledge, this problem has not been addressed in the existing literature. Therefore, the main goal of this paper is to develop a UDE-based switching control for the linear switched systems.

Motivated by the aforementioned discussions, the robust tracking problem will be studied thoroughly in this paper. Based on the CLF method, MLF method, and the obtained new stability results, the UDE-based switching control will be developed.

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Then, sufficient conditions to guarantee the asymptotic stability of the closed-loop linear switched system and the asymptotic tracking will be derived. The main contributions of this paper are summarized as follows: 1) some new stability results of the linear switched systems with disturbance are obtained in two cases: one is under arbitrary switching paths, the other is under switching paths with the restricted average dwell time; 2) the UDE-based control is used to study the robust tracking problem of the linear switched systems with both system uncertainty and external disturbance; 3) the UDE-based controllers are designed for each subsystem, and the whole closed-loop linear switched system is GASfor the aforementioned two cases.

II. Preliminaries and Problem formulation

A. Overview of the stability analysis of the linear switched time-invariant system

Consider the following linear switched time-invariant (LTI) system:

$$\dot{x}(t) = A_{\sigma(t)}x(t), \tag{1}$$

where $x(t) \in R^n$ is the state vector, $\sigma(t) : [0,\infty) \to \Lambda = \{1, 2, \cdots, N\}$ is called the switching path, e.g., $\sigma(t) = i$ means that the i-th subsystem of the system (1) is active, A_i is the known matrix, $i \in \Lambda$, $\{t_m\}_{m=0}^{+\infty}$ is called the switching time sequence which is assumed to satisfy $0 \le t_0 < t_1 < \cdots < t_n < \cdots < +\infty$. Let $\tau_k = t_k - t_{k-1}$, which is called the dwell time, where $k \ge 1$. In addition, it is assumed that the state of the linear switched system (1) is continuous, and only a finite number of switches can take place in any finite time interval.

Consider the linear switched system (1) in which all subsystems are stable. Some stability results are introduced.

Lemma 1: [1] Consider the linear switched system (1). If A_i shares a CLF $V(x)$, where $i \in \Lambda$, then the system (1) is GAS under arbitrary switching paths.

Lemma 2: [12] Consider the linear switched system (1). If $V_i(x) = x^T P_i x$ is a Lyapunov function candidate of the i-th subsystem, where $P_i > 0$, $i \in \Lambda$, then the system (1) is GAS under any switching path with the following restricted ADT:

$$\tau_a \ge \tau_a^* = \frac{a}{\lambda_0}, \tag{2}$$

where

$$a = \ln\mu, \mu = \max_{i\in\Lambda}\frac{\lambda_{\max}(P_i)}{\lambda_{\min}(P_i)}, \lambda_0 = \min_{i\in\Lambda}\lambda_i, \tag{3}$$

with $\lambda_i > 0$ given as

$$\lambda_i = \frac{\lambda_{\min}(Q_i)}{\lambda_{\max}(P_i)},$$

and $Q_i > 0$, $i \in \Lambda$, which satisfies the following condition:

$$P_i^T A_i + P_i A_i = -Q_i, i \in \Lambda. \tag{4}$$

B. Problem Formulation

Consider the following switched system in which both model uncertainty and external disturbance exist:

$$\dot{x}(t) = (A_{\sigma(t)} + F_{\sigma(t)})x(t) + B_{\sigma(t)}u_{\sigma(t)}(t) + d_{\sigma(t)}(t), \tag{5}$$

where $x(t) \in R^n$ is the state vector, A_i is the known state matrix, F_i is a matrix which is not known in advance and is called the model uncertainty, B_i is the control matrix which is assumed to have full column rank, (A_i, B_i) is a controllable pair, $u_i(t) = (u_{i1}(t), u_{i2}(t), \cdots, u_{ir}(t))^T \in R^r$ is the control input vector, and $d_i(t)$ is the unpredictable external disturbance vector, of the i-th subsystem, respectively, $i \in \Lambda$.

The objective of this paper is to investigate the robust tracking problem of the switched system (5) by the UDE-based control method in the following two cases: one is under arbitrary switching paths, the other is under switching paths with the restricted average dwell time. In this paper, the tracked signal $c(t) \in R^r$ is a given command, which is assumed to be a piecewise smooth (continuous) and uniformly bounded, for example, trigonometric functions.

III. Main Results

A. Some new stability results of the linear switched systems with disturbance

Consider the following switched system with disturbance:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + \tilde{u}_{d\sigma(t)}(t), \tag{6}$$

where $x(t) \in R^n$ is the state vector, A_i is the known matrix, and $\tilde{u}_{di}(t)$ is the external disturbance, $i \in \Lambda$.

Based on Lemma 1 and Lemma 2, some new results of the linear switched system (6) are obtained in the following.

Theorem 1: For the linear switched system (6), if the following conditions:

C1: A_i, $i \in \Lambda$, are Hurwitz matrices, which implies that there exist $L_1 > 0, L_2 > 0$ and $\lambda > 0$ satisfying the condition: $L_1 e^{-\lambda t} \le \|e^{A_i t}\| \le L_2 e^{-\lambda t}$, $i \in \Lambda$;

C2: $A_iA_j = A_jA_i, i, j \in \Lambda$;

C3: $\|\tilde{u}_{di}(t)\| \le \alpha(t)$, $i \in \Lambda$, where $\alpha(t)$ satisfies $\int_0^{+\infty} e^{\lambda t}\alpha(t)dt < \infty$;

are satisfied, then the system (6) is GAS under arbitrary switching paths.

Proof: Let $\sigma(t) = i_k$ for $t \in [t_{k-1}, t_k), k \ge 1$. From (6), it results in

$$\dot{x}(t) = A_{i_k}x(t) + \tilde{u}_{di_k}(t), t \in [t_{k-1}, t_k),$$

which implies that

$$x(t) = e^{A_{i_k}(t-t_{k-1})}x(t_{k-1}) + \int_{t_{k-1}}^{t} e^{A_{i_k}(t-s)}\tilde{u}_{di_k}(s)ds.$$

By induction, for $t \in [t_{k-1}, t_k)$, it results in

$$\begin{aligned} x(t) \;=\; & e^{A_{i_k}(t-t_{k-1})}e^{A_{i_{k-1}}(t_{k-1}-t_{k-2})}\cdots e^{A_{i_1}(t_1-t_0)}x(t_0) \\ & + \textstyle\int_{t_0}^{t_1} e^{A_{i_k}(t-t_{k-1})}e^{A_{i_{k-1}}(t_{k-1}-t_{k-2})} \\ & \cdots e^{A_{i_1}(t_1-s)}\tilde{u}_{di_1}(s)ds \\ & + \textstyle\int_{t_1}^{t_2} e^{A_{i_k}(t-t_{k-1})}e^{A_{i_{k-1}}(t_{k-1}-t_{k-2})} \\ & \cdots e^{A_{i_2}(t_2-s)}\tilde{u}_{di_2}(s)ds \\ & + \cdots \textstyle\int_{t_{k-1}}^{t} e^{A_{i_k}(t-s)}\tilde{u}_{di_k}(s)ds. \end{aligned}$$

Let $\tau_i(0, t)$ be the total time of the i-th subsystem of the linear switched system (6) active during the time interval $[0, t)$. From condition C2, it follows that

$$x(t) = e^{A_1\tau_1(0,t)}e^{A_2\tau_2(0,t)}\cdots e^{A_n\tau_n(0,t)}x(t_0) + \int_{t_0}^{t_1} e^{A_1\tau_1(s,t)}e^{A_2\tau_2(s,t)}\cdots e^{A_n\tau_n(s,t)}\tilde{u}_{di_1}(s)ds + \int_{t_1}^{t_2} e^{A_1\tau_1(s,t)}e^{A_2\tau_2(s,t)}\cdots e^{A_n\tau_n(s,t)}\tilde{u}_{di_2}(s)ds + \cdots \int_{t_{k-1}}^{t} e^{A_1\tau_1(s,t)}e^{A_2\tau_2(s,t)}\cdots e^{A_n\tau_n(s,t)}\tilde{u}_{di_k}(s)ds.$$

Using conditions C1 and C3, it results in

$$\begin{aligned} \|x(t)\| &\leq L_2^N e^{-\lambda t}\|x(t_0)\| + L_2^N \int_{t_0}^{t_1} e^{-\lambda(t-s)}\|\tilde{u}_{di_1}(s)\|ds \\ &\quad + L_2^N \int_{t_1}^{t_2} e^{-\lambda(t-s)}\|\tilde{u}_{di_2}(s)\|ds \\ &\quad + L_2^N \int_{t_2}^{t_3} e^{-\lambda(t-s)}\|\tilde{u}_{di_3}(s)\|ds \\ &\quad + \cdots L_2^N \int_{t_{k-1}}^{t} e^{-\lambda(t-s)}\|\tilde{u}_{di_k}(s)\|ds \\ &\leq L_2^N\left(e^{-\lambda t}\|x(t_0)\| + \int_{t_0}^{t} e^{-\lambda(t-s)}\alpha(s)ds\right) \\ &= e^{-\lambda t}L_2^N\left(\|x(t_0)\| + \int_{t_0}^{t} e^{\lambda s}\alpha(s)ds\right). \end{aligned}$$

Thus, $\lim_{t\to\infty}\|x(t)\| = 0$. This completes the proof. ■

Remark 1: The condition: $\int_0^{+\infty} e^{\lambda t}\alpha(t)dt < \infty$, implies that $\alpha(t) = o(e^{-\lambda t})$. Moreover, according to the equation (6), there exists a constant $L_3 > 0$ such that $L_3\|e^{A_i t}\| \leq \|x(t)\|$, $i \in \Lambda$. Thus, an appropriate $\alpha(t)$ and a positive constant ε are chosen to satisfy the condition: $\alpha(t) \leq \varepsilon\|x\|$. For example,

$$\alpha(t) = \varepsilon L_3 L_1 e^{-(\lambda+1)t} \leq \varepsilon L_3 L_1 e^{-\lambda t} \leq \varepsilon L_3\|e^{A_i t}\| \leq \varepsilon\|x(t)\|.$$

Next, the condition C2 is relaxed to the following condition:

C4: There exist a group of matrices $Q_i > 0$, $P > 0$, such that

$$P^T A_i + A_i P = -Q_i, i \in \Lambda, \tag{7}$$

which implies that $V(x) = x^T P x$ is a CLF for the linear switched system: $\dot{x}(t) = A_{\sigma(t)}x(t)$.

In this case, one possible choice for $\alpha(t)$ is

$$\alpha(t) = \varepsilon_1 L_3 L_1 e^{-(\lambda+\epsilon)t}, \tag{8}$$

which implies that $\alpha(t) \leq \varepsilon_1\|x(t)\|$, where ϵ is an arbitrary positive number, and ε_1 is determined by

$$\varepsilon_1 = \frac{\lambda_{\min}}{2\lambda_{\max}(P)}, \lambda_{\min} = \min_{i\in\Lambda}\{\lambda_{\min}(Q_i)\}. \tag{9}$$

For the linear switched system (6), a conclusion is obtained as follows.

Theorem 2: Consider the linear switched system (6). If the conditions C1, C3 and C4 are satisfied, where $\alpha(t)$ is given in (8), then the system (6) is GAS under arbitrary switching paths.

Proof: Let $V(x) = x^T P x$, using the conditions C1, C4 and $\alpha(t)$, then it results in

$$\begin{aligned} \dot{V}(x(t)) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \\ &= x^T(t)\left(A_i^T P + PA_i\right)x(t) + \tilde{u}_{di}^T(t)Px(t) \\ &\quad + x^T(t)P\tilde{u}_{di}(t) \\ &= -x^T(t)Q_i x(t) + 2x^T(t)P\tilde{u}_{di}(t) \\ &\leq -\lambda_{\min}\|x(t)\|^2 + 2\lambda_{\max}(P)\|x(t)\|\|\tilde{u}_{di}(t)\| \\ &\leq -\lambda_{\min}\|x(t)\|^2 + 2\lambda_{\max}(P)\|x(t)\|\alpha(t) \\ &\leq -\lambda_{\min}\|x(t)\|^2 + 2\varepsilon_1\lambda_{\max}(P)\|x(t)\|^2 \\ &= -(\lambda_{\min} - \lambda_{\min})\|x(t)\|^2 \\ &= 0. \end{aligned} \tag{10}$$

Thus, the system (6) is GAS under arbitrary switching paths. ■

As for the linear switched system (6), if only the conditions C1 and C3 are satisfied, one possible choice for $\alpha(t)$ is

$$\alpha(t) = \varepsilon_2 L_3 L_1 e^{-(\lambda+\epsilon)t}, \tag{11}$$

which implies that $\alpha(t) \leq \varepsilon_2\|x(t)\|$, where ϵ is an arbitrary positive number, and ε_2 is determined by

$$\varepsilon_2 = \max_{i\in\Lambda}\left\{\frac{\lambda_{\min}(Q_i) - l_i\lambda_{\max}(P_i)}{2\lambda_{\max}(P)}\right\}, \tag{12}$$

with $l_i > 0$, which is chosen in advance to meet the following condition:

$$\lambda_{\min}(Q_i) - l_i\lambda_{\max}(P_i) > 0, i \in \Lambda, \tag{13}$$

and $P_i > 0$, $Q_i > 0$, are given in (4), $i \in \Lambda$.

Theorem 3: Consider the linear switched system (6). If the conditions C1 and C3 are satisfied, where $\alpha(t)$ is given in (11), then the linear switched system (6) is GAS under any switching path with ADT $\tau_a \geq \tau_a^*$, where τ_a^* is designed as

$$\tau_a^* = \frac{a}{l_0}, \tag{14}$$

with

$$a = \ln\mu, \mu = \max_{i\in\Lambda}\left\{\frac{\lambda_{\max}(P_i)}{\lambda_{\min}(P_i)}\right\}, l_0 = \min_{i\in\Lambda}\{l_i\}, \tag{15}$$

and P_i, Q_i given in (4), l_i is given in (12), $i \in \Lambda$.

Proof: Let $V_i(x) = x^T P_i x$, using the condition C1, the equation (4) and $\alpha(t)$, it follows

$$\begin{aligned} \dot{V}_i(x(t) &= \dot{x}^T(t)P_i x(t) + x^T(t)P_i\dot{x}(t) \\ &= x^T(t)\left(A_i^T P_i + P_i A_i\right)x(t) + \tilde{u}_{di}^T(t)P_i x(t) \\ &\quad + x^T(t)P_i\tilde{u}_{di}(t) \\ &= -x^T(t)Q_i x(t) + 2x^T(t)P\tilde{u}_{di}(t) \\ &\leq -\lambda_{\min}(Q_i)\|x(t)\|^2 + 2\lambda_{\max}(P_i)\|x(t)\|\|\tilde{u}_{di}(t)\| \\ &\leq -\lambda_{\min}(Q_i)\|x(t)\|^2 + 2\lambda_{\max}(P_i)\|x(t)\|\alpha(t) \\ &\leq -\lambda_{\min}(Q_i)\|x(t)\|^2 + 2\varepsilon_2\lambda_{\max}(P_i)\|x(t)\|^2 \\ &= -\lambda_{\min}(Q_i)\|x(t)\|^2 \\ &\quad + 2\lambda_{\max}(P_i)\frac{\lambda_{\min}(Q_i) - l_i\lambda_{\max}(P_i)}{2\lambda_{\max}(P)}\|x(t)\|^2 \\ &\leq -l_i\lambda_{\max}(P_i)\|x(t)\|^2 \\ &\leq -l_i V_i(x(t)). \end{aligned} \tag{16}$$

According to the proof of Lemma 2, the system (6) is GAS under any switching path with ADT $\tau_a \geq \tau_a^*$. This completes the proof. ■

B. Robust tracking problem of linear switched system with both model uncertainty and external disturbance by the UDE-based control method

In this subsection, the robust tracking problem of the switched system (5) is investigated by the UDE-based control method in refs [24], [25], [26], [27]. The control design involves two steps. The first step is to design a switched reference model which is given as

$$\begin{aligned} \dot{x}_m(t) &= (A_{\sigma(t)} + B_{\sigma(t)}K_{\sigma(t)})x_m(t) + B_{m\sigma(t)}c_{\sigma(t)}(t) \\ &\quad + D_{m\sigma(t)}\dot{c}(t), \end{aligned} \tag{17}$$

where $x_m(t) \in R^n$ is the reference state vector, $\sigma(t)$ is the switching path, $c(t) = (c_1(t), c_2(t), \cdots, c_r(t))^T \in R^r$ is a given command, which is piecewise smooth and uniformly bounded to the i-th subsystem, K_i is the feedback gain matrix of the i-th subsystem with appropriate dimension, which is chosen in advance to make the matrix $A_i + B_iK_i$ be Hurwitz, and $B_{mi} = -M_{i1}$, where M_{i1}, M_{i2} are the blocks of matrix $A_i + B_iK_i$, i.e.,

$$A_i + B_iK_i = (M_{i1}, M_{i2}), D_{mi} = \begin{pmatrix} I_r \\ 0 \end{pmatrix} \in R^{n\times r}, i \in \Lambda.$$

Then, the desired switched error system is described as follows:

$$\dot{e}(t) = (A_{\sigma(t)} + B_{\sigma(t)}K_{\sigma(t)})e(t). \quad (18)$$

The second step is to design controllers $u_i, i \in \Lambda$. According to the results in refs [24], [25], [26], [27], the UDE-based control law for the i-th subsystem of the linear switched system (5) can be obtained as

$$\begin{aligned} u_i(t) \;=\; & B_i^+\mathcal{L}^{-1}\left[\tfrac{1}{1-G_{fi}(s)}\right] * B_iK_ix(t) \\ & +B_i^+\mathcal{L}^{-1}\left[\tfrac{1}{1-G_{fi}(s)}\right] * [B_{mi}c(t) + D_{mi}\dot{c}(t)] \\ & -B_i^+\left\{\mathcal{L}^{-1}\left[\tfrac{sG_{fi}(s)}{1-G_{fi}(s)}\right] * x(t)\right\} \\ & +B_i^+\left\{\mathcal{L}^{-1}\left[\tfrac{G_{fi}(s)}{1-G_{fi}(s)}\right] * (A_ix(t))\right\}, \end{aligned} \quad (19)$$

where $B_i^+ = (B_i^TB_i)^{-1}B_i^T$ is the pseudo-inverse of B_i, $\mathcal{L}^{-1}\{.\}$ is the inverse Laplace transform operator, and $G_{fi}(s)$ is a strictly proper stable filter with the appropriate frequency characteristics for the i-th subsystem of the linear switched system (5) , $i \in \Lambda$.

According to Theorem 1, the following conclusion is obtained.

Theorem 4: Consider the system (5) and reference model (17). If $B_iK_i = S_i - A_i$, where S_i, $i \in \Lambda$, are Hurwitz and pairwise commutative, i.e., $S_iS_j = S_jS_i, i, j \in \Lambda$, and the filter $G_{fi}(s)$ is designed to make $\tilde{u}_{di}$ satisfy the condition C3, $i \in \Lambda$, then the state $x(t)$ of the switched system (5) asymptotically tracks the state $x_m(t)$ of the switched reference model (17) under arbitrary switching paths.

Proof: If $B_iK_i = S_i - A_i$, i.e., $A_i + B_iK_i = S_i$, $i \in \Lambda$, then the switched reference model is given as

$$\dot{x}_m(t) = S_{\sigma(t)}x_m(t) + B_{m\sigma(t)}c(t) + D_{m\sigma(t)}\dot{c}(t). \quad (20)$$

Substituting the controller $u(t)$ given in (19) into the system (5), then the switched error dynamics becomes

$$\dot{e}(t) = S_ie(t) - \tilde{u}_{di}(t), \quad (21)$$

where $\tilde{u}_{di}$ is the estimated error (the difference between the uncertainty and disturbance term and their estimated value) of the i-th subsystem, respectively, $i \in \Lambda$. Note that $\tilde{u}_{di}$ satisfies the condition C3, $i \in \Lambda$, thus the switched error dynamics in (21) is GAS under arbitrary switching paths according to Theorem 1. This completes the proof. ∎

Based on Theorem 2, a conclusion is obtained as follows.

Theorem 5: Consider the system (5) and the reference model (17). If $B_iK_i = S_i - A_i$, where S_i is Hurwitz, $i \in \Lambda$, and the conditions: C3 and C4 are satisfied, where $\alpha(t)$ is given in (8), then the state $x(t)$ of the switched system (5) asymptotically tracks the state $x_m(t)$ of the switched reference model (17) under arbitrary switching paths.

Proof: Similarly, the obtained switched error dynamics is

$$\dot{e}(t) = S_ie(t) - \tilde{u}_{di}(t), \quad (22)$$

where $\tilde{u}_{di}$ is the estimated error the i-th subsystem, respectively, $i \in \Lambda$. Note that R_i is Hurwitz, $i \in \Lambda$, and the conditions: C3 and C4 are satisfied, where $\alpha(t)$ is given in (8), thus the switched error dynamics in (22) is GAS under arbitrary switching paths according to Theorem 2, which completes the proof. ∎

For the system (5), if $A_i + B_iK_i = S_i$, where S_i is Hurwitz, $i \in \Lambda$, but the condition: C4 is not satisfied, the following result is obtained according to Theorem 3.

Theorem 6: Consider the system (5) and the reference model (17). If $B_iK_i = S_i - A_i$, where S_i is Hurwitz, $i \in \Lambda$, and the condition: C3 is satisfied, where $\alpha(t)$ is given in (11), then the state $x(t)$ of the switched system (5) asymptotically tracks the state $x_m(t)$ of the switched reference model (17) under any switching signal with ADT τ_a, where τ_a^* is determined by the equations (3) and (4).

Proof: Similarly, the obtained switched error dynamics is

$$\dot{e}(t) = S_ie(t) - \tilde{u}_{di}(t), \quad (23)$$

where $\tilde{u}_{di}$ is the estimated error of the i-th subsystem, respectively, $i \in \Lambda$. Note that the matrices S_i, $i \in \Lambda$, are Hurwitz, and the condition: C3 is satisfied, where $\alpha(t)$ is given in (11), thus the switched system in (23) is GAS under any signal with the restricted ADT τ_a according to Theorem 2. ∎

It should be noticed that the filters $G_{fi}(s)$, $i \in \Lambda$, which guarantee $\tilde{u}_{di}$ satisfy the condition C3, especially for $\alpha(t)$ given in (8) and (11), are designed according to the results in [27]. The good performance will be shown by the simulation studies in Section IV.

IV. Simulation Studies

In this section, the numerical validation is carried out for an example in which the matrices of the closed loop linear switched systems cannot share a CLF.

Consider the following linear switched system with both model uncertainty and external disturbance:

$$\dot{x} = (A_{\sigma(t)} + F_{\sigma(t)})x + B_{\sigma(t)}u_{\sigma(t)}(t) + d_{\sigma(t)}(t), \quad (24)$$

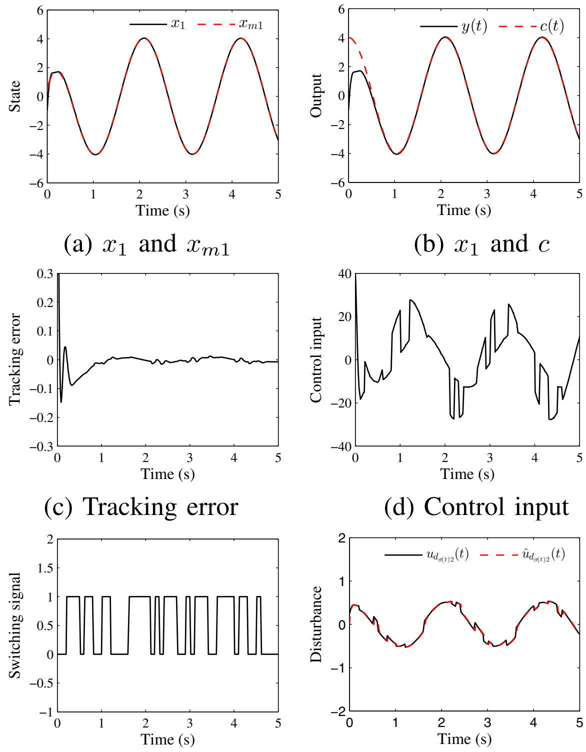
where $\Lambda = \{1, 2\}$,

$$A_1 = \begin{pmatrix} 0 & 1 \\ 12 & -11 \end{pmatrix}, A_2 = \begin{pmatrix} -2 & 13 \\ 10 & 1 \end{pmatrix},$$

$$B_1 = B_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

$$F_1 = \begin{pmatrix} 0.02 & 0 \\ 0 & 0.12 \end{pmatrix}, F_2 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.11 \end{pmatrix},$$

and



(a) x_1 and x_{m1} (b) x_1 and c

(c) Tracking error (d) Control input

(e) Switching signal (f) $d(t)$ and $\hat{d}(t)$

Figure 1. The results of the UDE-based robust tracking performance of switched system (24) where the matrices of the closed loop switched systems cannot share CLF.

$$d_1(t) = \begin{pmatrix} 0.1\sin(0.1t) \\ 0.02\cos(0.5t) \end{pmatrix}, d_2(t) = \begin{pmatrix} 0.02\cos(0.1t) \\ 0.1\sin(t) \end{pmatrix}.$$

The objective is to design the controller $u_i(t)$ such that $x_1(t)$ asymptotically tracks $c(t) = \cos(5t)$.

The first step is to design a linear switched reference system which is described as follows:

$$\dot{x}_m = (A_{\sigma(t)} + K_{\sigma(t)})x_m + B_{m\sigma(t)}c(t) + D_{m\sigma(t)}\dot{c}(t). \quad (25)$$

For the feedback gains K_i , $i \in \Lambda$, one feasible choice is

$$K_1 = \begin{pmatrix} -11 & -11 \end{pmatrix}, K_2 = \begin{pmatrix} -20 & -12 \end{pmatrix},$$

and then

$$A_1 + B_1K_1 = \begin{pmatrix} -11 & -10 \\ 1 & -22 \end{pmatrix},$$
$$A_2 + B_2K_2 = \begin{pmatrix} -22 & 1 \\ -10 & -11 \end{pmatrix},$$

$$B_{m1} = B_{m2} = \begin{pmatrix} 21 \\ 21 \end{pmatrix}, D_{m1} = D_{m2} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

The second step is to design the filters $G_{fi}(s)$, $i \in \Lambda$. According to [27], the filters can be designed as

$$G_{f1}(s) = G_{f2}(s) = \frac{a_1 s + (a_2 - \omega_0^2)}{s^2 + a_1 s + a_2},$$

where $\omega_0 = 0.5$, $a_1 = 10\omega_0$ and $a_2 = 100\omega_0^2$.

According to the results in [3], the lower bound of ADT $\tau_a^* = \frac{1}{2}\log(10) = 1.1513$. According to Theorem 6, the following performance:

$$\lim_{t\to\infty} x(t) = x_m(t), \lim_{t\to\infty} x_1(t) = c(t)$$

is achieved under any switching signal with ADT $\tau_a \geq 1.1513$.

With the initial condition: $x(0) = (1, -2)^T$ and the following switching path:

$$\sigma(t) = \begin{cases} 2, & t \in [t_{2k}, t_{2k+1}), t_{2k+1} - t_{2k} = 1.1513 + rand \\ 1, & t \in [t_{2k+1}, t_{2k+2}), t_{2k+2} - t_{2k+1} = 1.1513 \end{cases},$$

where $k = 0, 1, 2\cdots$, and $rand \in (0, 1)$ is a random number, the simulation results are shown in Fig. 1.

From Fig. 1, it can be observed that the state $x(t)$ of the system (24) asymptotically tracks the state $x_m(t)$ of the reference model (25) quickly and the state $x_1(t)$ also asymptotically tracks the command $c(t)$ quickly. The simulation results show that the proposed control is very effective in achieving the robust tracking of a class of linear switched systems with model uncertainty and external disturbance.

V. Conclusions

In this paper, the robust tracking problem of a class of linear switched systems with both system uncertainty and external disturbance has been investigated. Firstly, some new stability results about linear switched systems with disturbance have been presented in two cases: one is under arbitrary switching paths, the other is under switching paths restricted with average dwell time. Then, based on the obtained stability results and the existing UDE-based control method, the asymptotic stability of the linear switched system has been established in the above mentioned two cases, which guaranteed that the states of the linear switched systems can asymptotically track the command in the presence of both uncertainty and disturbance. Finally, a numerical example is provided to verify effectiveness of the proposed method.

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