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Bounded Integral Controller for Nonlinear MIMO Systems with Inputs Constraint

Yeqin Wang, Beibei Ren, and Qing-Chang Zhong

Abstract—**In this paper, a bounded integral controller (BIC) is developed for nonlinear multiple-input and multiple-output (MIMO) systems to handle the sum of squares constraint of control inputs. The BIC guarantees the inputs constraint independently from the plant parameters, and keeps the properties of traditional integral controller (IC) with eliminating tracking errors and achieving disturbances rejection. The closed-loop stability is analyzed with the input-to-state (practical) stability (ISpS). Two simulation cases with practical applications are provided to demonstrate the effectiveness of the BIC design.**

I. Introduction

The inputs constraint is a common problem in control system design, which relates to the stability of systems with bounded control inputs [1]–[3]. Also, the inputs constraint is subject to the actuator physical limitations [4], [5], e.g. limited control input force for spacecraft rendezvous [6], [7], and voltage limitation for electric machines [8]–[10].

For single-input single-output (SISO) systems, a lot of research has been done with input constraint, e.g., saturation unit with anti-windup designs for different linear or nonlinear systems [11], or constrained adaptive control [12]. Especially, a bounded integral controller (BIC) is proposed in [1], [13] to keep both bounded control input and closed-loop system stability. However, inputs constraint problem becomes more challenging and complex in multiple-input and multiple-output (MIMO) systems, as MIMO systems have multiple channels, coupling effects, etc. For some MIMO systems, it is enough to individually regulate the constraint of each input channel with similar methods for SISO system. It is more difficult to consider the constraint of all inputs together, e.g., the norm constraint of all input channels [14]–[16], i.e. $\|u\| \leq M$ problem, where $u \in \mathbb{R}^n$ is a vector representing the multiple control inputs, $\|\cdot\|$ denotes the Euclidean norm, and M is a constant.

In some previous works, the norm constraint of the control inputs is considered in linear systems [4], [17], [18], where the saturation function, adopted in SISO systems, similarly can be used for bounded inputs design. A popular solution for $\|u\| \leq M$ problem is converting the inputs constraint into the optimization problem and solving linear matrix inequality (LMI) for nonlinear MIMO systems. In [19], the nonlinear system is represented into a Takagi-Sugeno (TS) fuzzy model with linear input-output relations, where the constraint problem of control inputs is transferred into the optimization problem and handled by the LMI method. The similar idea with LMI optimization is extended in [2], [20]–[23] for inputs constraint. The inputs constraint, H_∞ control, and poles constraint are considered in spacecraft rendezvous [6], [7], which is also cast into a convex optimization problem subject to LMI constraints. A robust non-fragile piecewise-affine controller with inputs constraint is studied in [24], which is cast as an optimization problem subject to a set of LMIs. The LMI method is used to guarantee the inputs constraint for spacecraft trajectory tracking control [25]. The $\|u\| \leq M$ problem is formulated as optimization problem with LMIs in [26], [27], etc. However, the LMI is usually very complex, and could not be used in real-time control systems with fast response requirement [10]. Apart from LMI optimization, a bounded Lyapunov-based control is proposed in [3] for linear systems to keep inputs constraint. It is further extended into nonlinear system for inputs constraint with an invariant subset [5], [28], [29]. An improved integral control (IC) is provided in [9] to regulate the duty-ratio of voltage source inverter in the permitted range for vector control of induction motors.

Uncertainties and disturbances are common in many engineering systems, which can be handled through the robustness designs. The integral property is very common in robustness designs to eliminate tracking errors and achieve disturbances rejection. The optimization methods with LMI are not easy to be applied to robustness designs with inputs constraint of $\|u\| \leq M$ problem, due to the effects of the unknown disturbances and the fast response requirement [10]. Though the saturation functions can be used for the inputs constraint, the complex anti-windup designs are usually required for the integral property in robustness designs. In this paper, inspired by the BIC for SISO system in [1], [13], the BIC is extended to nonlinear MIMO systems to handle the sum of squares constraint of control inputs. Compared with the inputs constraint of $\|u\| \leq M$ problem, additional weights for control inputs are considered to represent a more general case. The BIC design can be treated as the modification and extension of traditional IC, as it keeps the properties of traditional IC with eliminating tracking errors and achieving disturbances rejection. Also, the BIC fulfills desired inputs constraint independently from the plant parameters and system states with a simple structure, without any complex anti-windup designs. Compared with the method in [9], the BIC is developed to deal with the general sum of squares constraint of control inputs, and additional terms are

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provided in the BIC to eliminate numerical errors or parameter drifts for inputs constraint. The closed-loop stability is investigated with small-gain theorem for the plant with input-to-state (practical) stability (ISpS). Two simulation cases with practical applications are studied to validate the BIC design.

II. Problem formulation

Consider a class of MIMO systems

$$\begin{aligned}\dot{x}_1 &= f_1(x_1, x_2, \cdots, x_n, u_1)\\ \dot{x}_2 &= f_2(x_1, x_2, \cdots, x_n, u_2)\\ &\vdots\\ \dot{x}_n &= f_n(x_1, x_2, \cdots, x_n, u_n)\end{aligned} \tag{1}$$

where $x_i = [x_{i1}, x_{i2}, ..., x_{im}] \in \mathbb{R}^m$, $i = 1, 2, \ldots, n$, are system state vectors, m is the dimension of the states x_i, and $u_i \in \mathbb{R}$ are the control inputs. Each subsystem is assumed as a mth-order SISO system. f_i are system dynamics which are assumed to be ISpS.

In this paper, the sum of squares constraint of control inputs is considered. Without loss of generality, the control inputs are constrained in the sense of

$$\sum_{i=1}^{n} c_i u_i^2 \leq \beta \tag{2}$$

where $\beta > 0$ is a constant, which usually represents the limitation of total actuators, e.g. force limitation, voltage limitation, or power limitation. c_i are bounded weights for u_i, and considered as positive constants in this paper. For simplicity, the index i from 1 to n is omitted for the summation notation $\sum$ in this paper.

Some existing solutions proposed for $\|u\| \leq M$ problem also can be applied to the inputs constraint of condition (2). However, most of those solutions are converting the $\|u\| \leq M$ problem into the optimization problem and solving LMI equations. If the system includes some unknown disturbances, those optimization technologies might not be applicable. And the complexity of LMI makes it difficult to be applied when both the system and the controller need fast responses [10]. Though the robustness designs, e.g. IC, or other robust controllers, can handle unknown disturbances and uncertainties, the saturation functions with complex anti-windup designs are usually considered to deal with inputs constraint. Is it possible to develop a control strategy to keep same performances of robustness designs, e.g. IC, with achieving asymptotic regulation and disturbances rejection, and to handle the inputs constraint of condition (2) simultaneously?

III. BIC design for MIMO systems

In [1], [13], a BIC is proposed for SISO systems with ISpS stability, where the system input is regulated inside a given bound, and the closed-loop stability is guaranteed. In this paper, inspired by the BIC in [1], [13], a BIC is extended to nonlinear MIMO systems with the sum of squares constraint of control inputs, i.e., $\sum c_i u_i^2 \leq \beta$, and ISpS stability of the closed-loop system is investigated.

A. Controller design

In this paper, the control aims of the MIMO systems (1) are considered as the tracking control for the first element of the state vectors x_i, i.e., $x_{i1} \in \mathbb{R}$. The state tracking errors are defined as $e_i(t) = x_{i1}^{ref} - x_{i1}$, where $x_{i1}^{ref} \in \mathbb{R}$ are desired state references. The dynamics controller of the BIC is designed as

$$\begin{aligned}\dot{u}_1 &= -k\left(\frac{\sum c_i u_i^2}{\beta} + u_0^2 - 1\right) u_1 + k_1 e_1 u_0^2\\ \dot{u}_2 &= -k\left(\frac{\sum c_i u_i^2}{\beta} + u_0^2 - 1\right) u_2 + k_2 e_2 u_0^2\\ &\vdots\\ \dot{u}_n &= -k\left(\frac{\sum c_i u_i^2}{\beta} + u_0^2 - 1\right) u_n + k_n e_n u_0^2\\ \dot{u}_0 &= -k\left(\frac{\sum c_i u_i^2}{\beta} + u_0^2 - 1\right) u_0 - \frac{c_1 u_1}{\beta} k_1 e_1 u_0\\ &\quad - \frac{c_2 u_2}{\beta} k_2 e_2 u_0 - \cdots - \frac{c_n u_n}{\beta} k_n e_n u_0\end{aligned} \tag{3}$$

where u_i and u_0 are controller states, k_i are integral gains, and k is a positive constant. It is worth noting that the BIC design is the modification of traditional IC, it keeps same properties of traditional IC with $k_i e_i$ terms.

Consider the following Lyapunov function candidate

$$V(t) = \frac{\sum c_i u_i^2}{\beta} + u_0^2.$$

Take the time derivative of $V(t)$ along with (3), it yields

$$\begin{aligned}\dot{V} &= \frac{\sum 2 c_i u_i \dot{u}_i}{\beta} + 2 u_0 \dot{u}_0\\ &= -\frac{2kc_1u_1^2}{\beta}\left(\frac{\sum c_i u_i^2}{\beta} + u_0^2 - 1\right) + \frac{2c_1u_1}{\beta} k_1 e_1 u_0^2\\ &\quad -\frac{2kc_2u_2^2}{\beta}\left(\frac{\sum c_i u_i^2}{\beta} + u_0^2 - 1\right) + \frac{2c_2u_2}{\beta} k_2 e_2 u_0^2\\ &\quad \vdots\\ &\quad -\frac{2kc_nu_n^2}{\beta}\left(\frac{\sum c_i u_i^2}{\beta} + u_0^2 - 1\right) + \frac{2c_nu_n}{\beta} k_2 e_n u_0^2\\ &\quad -2ku_0^2\left(\frac{\sum c_i u_i^2}{\beta} + u_0^2 - 1\right) - \frac{2c_1u_1}{\beta} k_1 e_1 u_0^2\\ &\quad -\frac{2c_2u_2}{\beta} k_2 e_2 u_0^2 - \cdots - \frac{2c_nu_n}{\beta} k_n e_n u_0^2\\ &= -2k\left(\frac{\sum c_i u_i^2}{\beta} + u_0^2\right)\left(\frac{\sum c_i u_i^2}{\beta} + u_0^2 - 1\right)\\ &= -2kV^2(t) + 2kV(t).\end{aligned} \tag{4}$$

Then, solve (4), it gives

$$\begin{aligned}V(t) &= \frac{e^{2kt}V(0)}{e^{2kt}V(0) - V(0) + 1}\\ &= \frac{1}{1 - e^{-2kt}(1 - \frac{1}{V(0)})}.\end{aligned} \tag{5}$$

Through the initial design with $u_0(0) = 1$, and $u_i(0) = 0$, i.e., $V(0) = 1$, then

$$V(0) = 1 \Rightarrow V(t) = 1, \forall t \geq 0.$$

It is worth noting that if there are any numerical errors or parameter drifts in $V(t)$, $V(t)$ still will converge to 1 with $t \to \infty$, according to equation (5), and the rate of convergence can be adjusted by parameter k. The BIC introduces additional terms $-k\left(\frac{\sum c_i u_i^2}{\beta} + u_0^2 - 1\right) u_i$ to handle numerical errors or parameter drifts, which is different from the method proposed in [9]. In this way, it always holds that $\frac{\sum c_i u_i^2}{\beta} \leq 1$, such that the inputs constraint is fulfilled with $\sum c_i u_i^2 \leq \beta$. In the BIC design, an additional controller state u_0 is introduced so that the controller states always move on the controller sphere

$$\frac{\sum c_i u_i^2}{\beta} + u_0^2 = 1.$$

An illustration of this controller sphere is shown in Fig. 1 with a spherical surface S_r, when two inputs u_1, u_2 and u_0 are considered. When the initial condition is chosen at the apex O_t, the BIC will always regulate the controller states on the upper half of the spherical surface S_r between the apex O_t and the circle C_o. As long as u_0 does not converge to 0, the proposed BIC (3) will perform as the n integral controllers with dynamic integral gains $k_i u_0^2$ to eliminate the tracking errors. In the steady-state, $\dot{u}_i$, $\dot{u}_0$ and e_i will be regulated to 0, and the controller states u_1, u_2, and u_0 will converge to the equilibrium point, e.g., point E.

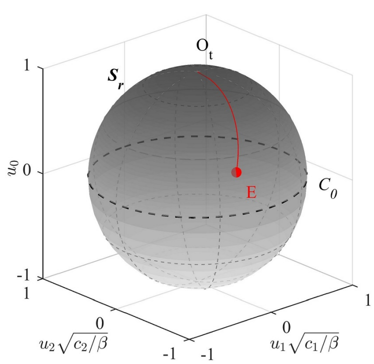


Fig. 1. An illustration of control states when two inputs u_1, u_2 and u_0 are considered with the BIC (3).

B. Closed-loop system stability

In this section, the small-gain theorem [13], [30] is used for the stability analysis of the closed-loop system. The plant (1) is assumed with ISpS, as many engineering systems are bounded-input bounded-output (BIBO) stable due to their inherent dissipative structure.

After applying the BIC (3) into plant (1), the closed-loop system is described as a composite interconnection form with two nonlinear subsystems, Σ_1 and Σ_2, as shown in Fig. 2, where $x = [x_{11}, x_{21}, ..., x_{n1}]$, $u = [u_1, u_2, ..., u_n]$, d and r are external input vectors.

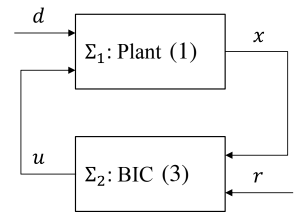


Fig. 2. The closed-loop system with the composite interconnection form.

For the subsystem Σ_1 with ISpS, there exists a class $\mathcal{KL}$ function β_1, class $\mathcal{K}_\infty$ functions γ_u and γ_d, and positive constants K_1, K_2 and K_3, such that for any state $x(0)$ with $\|x(0)\| < K_1$, any input $u(t)$ with $\sup_{t\geq 0} \|u(t)\| < K_2$, and any external input $d(t)$ with $\sup_{t\geq 0} \|d(t)\| < K_3$, the $x(t)$ exists and satisfies

$$\begin{aligned} \|x(t)\| \quad \leq \quad & \beta_1(\|x(0)\|, t) + \gamma_u\left(\sup_{0\leq\ \tau\ \leq t} \|u(\tau)\|\right) \\ & + \gamma_d\left(\sup_{0\leq\ \tau\ \leq t} \|d(\tau)\|\right) + \alpha_1 \end{aligned}$$

for all $t \geq 0$, where α_1 is an arbitrary non-negative constant.

With the boundedness design of (3), the output of Σ_2, u_i, satisfy

$$\sum c_i u_i^2 \leq \beta$$

then

$$\|u(t)\| \leq \frac{\beta}{c_{min}}$$

where $c_{min} = \min\{c_i\} > 0$.

For the subsystem Σ_2, there exists a class $\mathcal{KL}$ function β_2, class $\mathcal{K}_\infty$ functions γ_x and γ_r, and positive constants K_4, K_5 and K_6, such that for any state $u(0)$ with $\|u(0)\| < K_4$, any input $x(t)$ with $\sup_{t\geq 0} \|x(t)\| < K_4$, and any external input $r(t)$ with $\sup_{t\geq 0} \|r(t)\| < K_6$, the $u(t)$ exists and satisfies

$$\begin{aligned} \|u(t)\| \quad & \leq \quad \frac{\beta}{c_{min}} \\ & \leq \quad \beta_2(\|u(0)\|, t) + \gamma_x\left(\sup_{0\leq\ \tau\ \leq t} \|x(\tau)\|\right) \\ & \quad\quad + \gamma_r\left(\sup_{0\leq\ \tau\ \leq t} \|r(\tau)\|\right) + \alpha_2 \end{aligned}$$

for all $t \geq 0$, where $\alpha_2 = \frac{\beta}{c_{min}}$ is a positive constant, and functions β_2, γ_r and γ_r can be arbitrary small.

Then, two $\mathcal{K}_\infty$ functions ρ_1 and ρ_2 and a non-negative real number s_l can be found to achieve

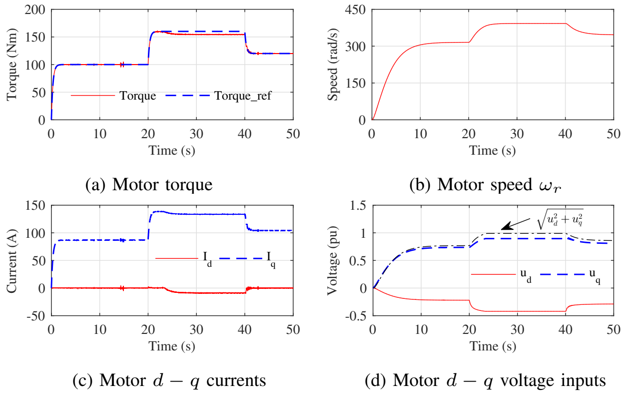
$$\left.\begin{array}{lcl} (I_d + \rho_2) \circ \gamma_x \circ (I_d + \rho_1) \circ \gamma_u\,(s) & \leq & s \\ (I_d + \rho_1) \circ \gamma_u \circ (I_d + \rho_2) \circ \gamma_x\,(s) & \leq & s \end{array}\right\} \forall s \geq s_l$$

where I_d is the identity function, and $\circ$ is the function composition, when γ_x can be arbitrary small.

As a result, since the closed-loop system, shown in Fig. 2, is given in the composite interconnection form, by applying the small-gain theorem, the closed-loop system is ISpS with respect to the external inputs d and r.

TABLE I
SYSTEM PARAMETERS FOR CASE 1

Parameters	Values	Parameters	Values
L_d, L_q	0.635 mH	V_{dc}	580 V
ψ_f	0.192 V·s	J_m	1.5 kg·m^2
p	4	λ_1	0.001 N·m·s/rad
R_s	0.05 Ω	λ_2	0.001 N·m·s^2/rad^2



(a) Motor torque (b) Motor speed ω_r

(c) Motor $d-q$ currents (d) Motor $d-q$ voltage inputs

Fig. 3. Vector control results with the BIC for the PMSM.

IV. SIMULATION STUDIES

In this section, two application cases are considered to verify the BIC design. One is the vector control of the permanent-magnet synchronous motor (PMSM) with voltage inputs constraint; the other is the multilevel heating system with power constraint.

A. Case 1: Vector control of the PMSM with voltage inputs constraint

In the vector control of the PMSM, the model of the PMSM in $d-q$ reference frame is given as [31]

$$\begin{aligned} U_d &= L_d \dot{i}_d + R_s i_d - \omega_e L_q i_q \\ U_q &= L_q \dot{i}_q + R_s i_q + \omega_e L_d i_d + \omega_e \psi_f \end{aligned}$$

where U_d and U_q are stator voltages, i_d and i_q are stator currents, L_d and L_q are stator winding inductances, R_s is stator winding resistance, ψ_f is the core magnetic flux, and ω_e is the electrical angular velocity. The torque control is usually adopted for PMSMs in many applications, e.g. electric vehicles, wind turbines. For a surface-mounted magnet type PMSM, the electromagnetic torque of the PMSM is given as

$$T_e = \frac{3}{2} p \psi_f i_q \tag{6}$$

where p is the number of pole pairs, L_d and L_q are equal, and $i_d = 0$ control is usually adopted.

For the PMSM control, the control inputs U_d and U_q are limited by the DC-bus voltage and the design of pulse width modulation (PWM) method. Usually the normalized values of U_d and U_q can be expressed as

$$\sqrt{u_d^2 + u_q^2} \leq 1.$$

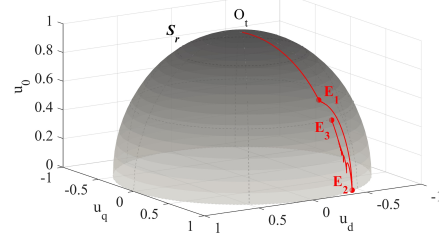


Fig. 4. The operation of controller states on the controller sphere S_r with the BIC for the PMSM.

A PMSM control system with the BIC (3) is built in Matlab/Simulink/SimPowerSystems. The current control is used to achieve the torque command of PMSM, where the i_d setting is 0, and the i_q setting can follow equation (6) with the desired torque reference. The motor load is chosen as a general mechanical load with $T_l = \lambda_1 \omega_r + \lambda_2 \omega_r^2$, where $\omega_r = \omega_e / p$ is the mechanical angular velocity. The system parameters are shown in Table I, where J_m is the total moment of inertia of the whole system. The integral gains for currents are selected as $k_d = 225$ and $k_q = 450$, k is chosen with $k = 1000$, and the initial condition is chosen with $u_d(0) = 0$, $u_q(0) = 0$, and $u_0(0) = 1$ for the BIC (3).

The system responses are shown in Fig. 3. Initially, the torque reference is set to 100 N · m by passing through a first-order low-pass filter. The output torque of PMSM can track the reference well, as shown in Fig. 3(a), and the motor speed converges to its steady-state within about 12 s, as shown in Fig. 3(b). The motor currents are shown in Fig. 3(c), wherein the i_d is well regulated to 0, i_q is regulated to a positive number to generate a positive torque value. The motor voltages are shown in Fig. 3(d) with negative u_d and positive u_q. At $t = 20$ s, the torque reference is changed to 160 N · m, both motor torque and motor currents are well controlled while $\sqrt{u_d^2 + u_q^2}$ does not reach the maximum value. At about $t = 22$ s, $\sqrt{u_d^2 + u_q^2}$ reaches the maximum value when the motor speed increases, the BIC still regulates $\sqrt{u_d^2 + u_q^2}$ within maximum value, as shown in Fig. 3(d). There are some deviations in the regulations of both torque and currents, because the voltages u_d and u_q already reach the maximum value and cannot support desired torque and currents. The system still keeps stable operation, though $\sqrt{u_d^2 + u_q^2}$ reaches the maximum value. When the torque reference is set to 120 N·m at $t = 40$s, both motor torque and motor currents are well regulated again due to the decrease of $\sqrt{u_d^2 + u_q^2}$ from the maximum value. And the motor speed decreases with lower torque. The BIC can always keep $\sqrt{u_d^2 + u_q^2} \leq 1$ in different stages of torque control of the PMSM, and always keep the system stable in both steady and transient states. The operations of controller states u_d, u_q, and u_0 are illustrated in Fig. 4, where it clearly shows that the controller states remain on the controller sphere S_r with the BIC design. The equilibrium points E_1, E_2, and E_3 represent the three steady states of the system, at 12 s $\sim$ 20 s, 25 s $\sim$ 40 s, and 45 s $\sim$ 50 s respectively.

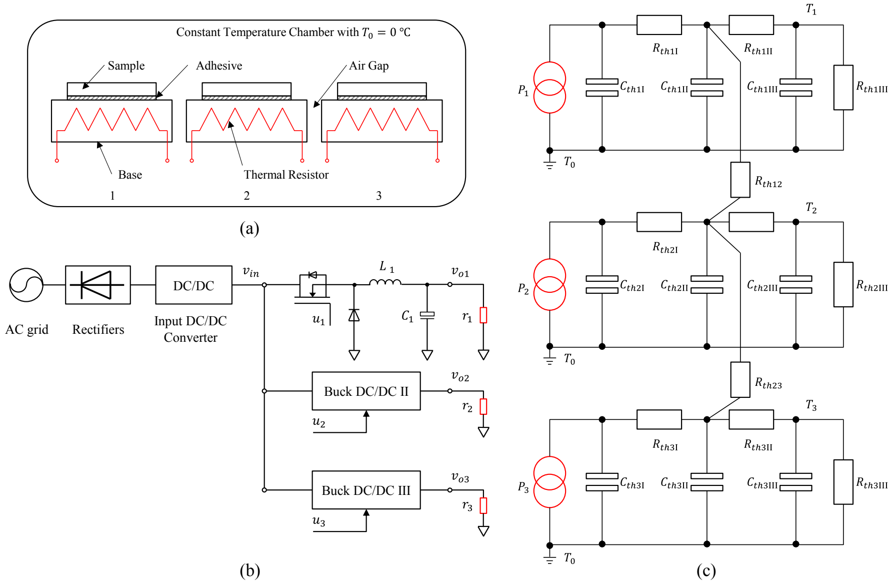


Fig. 5. The 3-level heating system. (a) The thermal structure. (b) The electrical system. (c) The thermal equivalent circuit model for the thermal structure.

TABLE II
SYSTEM PARAMETERS FOR CASE 2

Parameters	Values
r_1, r_2, r_3	10 Ω
v_{in}	20 V
P_{max}	20 W
C_{th1I}, C_{th2I}, C_{th3I}	0.152 J/K
R_{th1I}, R_{th2I}, R_{th3I}	2.66 K/W
C_{th1II}, C_{th2II}, C_{th3II}	0.002 J/K
R_{th1II}, R_{th2II}, R_{th3II}	0.625 K/W
C_{th1III}, C_{th2III}, C_{th3III}	0.06 J/K
R_{th1III}, R_{th2III}, R_{th3III}	20.8 K/W
R_{th12}, R_{th23}	8000 K/W

TABLE III
CONTROL PARAMETERS FOR CASE 2

Parameters	Values	Parameters	Values
c_1, c_2, c_3	40	T_{1ref}	150 °C
β	20	T_{2ref}	100 °C
k_1, k_2, k_3	0.001	T_{3ref}	50 °C
k	1000	-	-

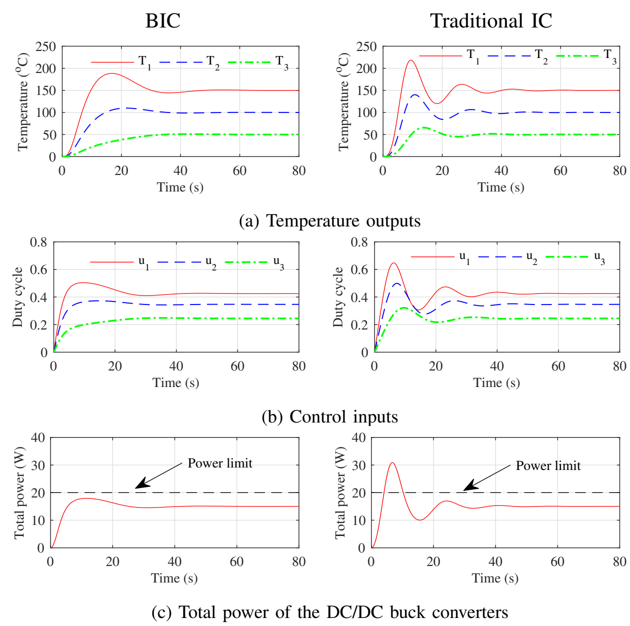


Fig. 6. Thermal control results with the BIC (left column) and the traditional IC (right column).

B. Case 2: Multilevel heating system with power constraint

A 3-level heating system is shown in Fig. 5, including the thermal structure in Fig. 5(a), the electrical system in Fig. 5(b), and the thermal equivalent circuit model for the thermal structure in Fig. 5(c). In Fig. 5(a), it includes three parallel thermal subsystems located in a constant temperature chamber with $T_0 = 0$ °C. In each subsystem, the thermal resistor is embedded in the thermal base, and the sample is stuck on the base with some thermal conductive adhesive. There are some air gaps between different bases. The electrical system is shown in Fig. 5(b), where the thermal resistors r_1, r_2, r_3 are powered by three DC/DC buck converters individually. The input voltage v_{in} is from a AC grid with the rectifiers, and the input DC/DC converter is used to stabilize the input voltage v_{in}, wherein the maximum power of the input DC/DC converter is P_{max}. In the thermal equivalent circuit model shown in Fig. 5(c), the P_1, P_2, P_3 are heat power generated by the thermal resistors. The bases, the adhesives, and the samples can be represented with the thermal resistance R_{ths} and thermal capacitors C_{ths}, respectively. Also, the air gaps are represented as thermal resistances. The system parameters are shown in Table II.

The control aims are the output temperatures of the three samples T_1, T_2, T_3, and the control inputs are the duty cycles u_1, u_2, u_3 of DC/DC buck converters. A power constraint of the input DC/DC converter is considered in this system. According to characteristics of the DC/DC buck converter, the power constraint of the input DC/DC converter can be approximated as the total output power of the DC/DC buck converters, which can be expressed as

$$\frac{(v_{in}u_1)^2}{r_1}+\frac{(v_{in}u_2)^2}{r_2}+\frac{(v_{in}u_3)^2}{r_3}\leq P_{max}.$$

This 3-level heating system is built in Matlab/Simulink/SimPowerSystems, including both electrical system and thermal equivalent circuit model. The BIC is provided for validation. In order to show the advantages of the BIC, the traditional IC is also provided for the comparison. The control parameters are shown in Table III.

The system responses are shown in Fig. 6 with the BIC at left column and the traditional IC at right column. Compared with the traditional IC, the BIC achieves much smaller overshoots for temperatures control with less settling times, as shown in Fig. 6(a). And the traditional IC has larger control inputs with more oscillations than the BIC. The BIC can always regulate the total power of the DC/DC buck converters within the power limit, whereas the traditional IC violates the power constraint with more power consumption in transient states. Therefore, the BIC can always fulfill the system power constraint and demonstrate better performance than the traditional IC.

V. Conclusions

In this paper, a BIC has been developed to handle the sum of squares constraint of control inputs for nonlinear MIMO systems. The sum of squares constraint of control inputs have been achieved through bounded design with the nonlinear Lyapunov analysis. The closed-loop stability with ISpS has been analyzed through the small-gain theorem. Two simulation cases with voltage limitation of vector control, and power limitation of the 3-level heating system, have verified the effectiveness and the advantages of the BIC design.

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