2018 IEEE Conference on Decision and Control (CDC)
Miami Beach, FL, USA, Dec. 17-19, 2018

Bounded UDE-based Controller for Systems with Input Constraints

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***Abstract*—The uncertainty and disturbance estimator (UDE)-based control is a robust control method, which is proposed as a replacement of the time-delay controller (TDC). With the filter design in the UDE-based controller, the challenging problem of designing a robust controller is converted into designing a filter. In this paper, a bounded UDE-based control is developed to deal with systems subject to uncertainties, disturbances and input constraints. The bounded controller output is achieved through nonlinear Lyapunov analysis, and an additional time-varying variable is introduced into the error dynamics to naturally avoid the integrator windup. The boundedness design is embedded into the existing UDE framework to form a bounded UDE-based controller without integrator windup via a simple structure and clear guidelines of parameter selections. Both theoretical analysis and simulation studies are provided to validate the proposed design.**

***Index Terms*—Uncertainty and disturbance estimator (UDE)-based controller, bounded control, input constraint, anti-windup.**

I. Introduction

A robust control method, the uncertainty and disturbance estimator (UDE)-based controller, was proposed in [1] to handle uncertainties and disturbances for linear time-invariant (LTI) systems as a replacement of the time-delay controller (TDC) [2]. Compared to the TDC [2], the UDE-based controller does not need to measure the derivative of the states, and no oscillations exist in control signal [1]. In the UDE design, a filter is adopted to estimate and compensate uncertainties and disturbances, then the challenging problem of designing a robust controller is converted into the design of a filter. In recent years, the UDE-based control demonstrates excellent performances in broad practical applications, e.g., variable-speed wind turbine control [3], solar system control [4], current control for permanent-magnet synchronous motor (PMSM) drives [5], power electronics control [6]–[9], quadrotors [10], and robot manipulator tracking [11], etc. The idea of UDE is also extended to the sliding-mode control [12], [13]. Some further theoretical work about the UDE-based control is conducted in [14]–[17]. A two-degree-of-freedom nature of the UDE-based controller is disclosed in [14]. A rough first-order plus time delay (FOPDT) model is introduced into the UDE-based controller in [15] to handle apparent lag and time delay. The asymptotic reference tracking and disturbance rejection of the UDE-based controller is achieved in [16] based on the internal model principle. The tradeoff between the tracking and disturbance rejection under finite bandwidth constraints is investigated in the UDE-based controller [17].

The constraint of the system input or controller output is a common problem in practice, due to the physical limitation of the actuators or the stability requirement. The UDE-based controller faces the same challenges. However, it is not an easy task to design the boundedness for the UDE-based controller, as it includes the integral terms to eliminate the tracking error. If a saturation unit is simply applied to the UDE-based controller, this often leads to the instability due to the problem of the integrator windup [18], [19]. To overcome this issue, the anti-windup design [19] is usually adopted for the controllers with integral property, but this still can not guarantee system stability in the original form or require additional knowledge of the system structure and parameters [18], [20]. Recently, a bounded integral controller (BIC) is proposed in [18] to handle input constraint with integral control, which autonomously provides the boundedness of the controller output without any switches. The BIC is further applied to many applications, e.g. the bounded voltage and frequency of virtual synchronous machines [20], bounded droop controller [21], and solar systems [4]. Though the BIC can handle both input constraint and integrator windup simultaneously, its robust performance is still limited by the single integral control. Moreover, the bounded-input bounded-output (BIBO) stability of original system is required for the BIC.

Motivated by the BIC in [18], a boundedness design is proposed in this paper for the conventional UDE-based controller to deal with input constraints. The boundedness of the final controller output is investigated through nonlinear Lyapunov analysis. With an additional time-varying variable introduced into the error dynamics, the dynamic integral gain of the modified UDE-based controller is achieved. The integral property will converge to zero, when the controller output converges to its limits, which naturally avoids the integrator windup subject to the input constraint. Compared to the existing anti-windup designs using the auxiliary systems [22], [23], the proposed boundedness design is embedded into the conventional UDE structure, and the whole controller becomes a bounded UDE-based controller. Compared to the conventional UDE-based controller, only two additional design parameters are introduced in the boundedness design.

This work was supported in part by the National Science Foundation under Grant CMMI-1728255.

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TABLE I
FILTER DESIGN FOR THE UDE-BASED CONTROLLER

G_f	$\frac{1}{1-G_f(s)}$	$\frac{sG_f(s)}{1-G_f(s)}$
$G_{f1}(s)=\frac{a_0}{s+a_0}$	$1+\frac{a_0}{s}$	a_0
$G_{f2}(s)=\frac{a_0}{s^2+a_1 s+a_0}$	$1+\frac{1}{s}\cdot\frac{a_0}{s+a_1}$	$\frac{a_0}{s+a_1}$
$G_{f3}(s)=1-\frac{s(s^2+\omega_0^2)}{(s+a_0)(s^2+a_1 s+\omega_0^2)}$	$1+\frac{a_0}{s}+\frac{a_1(s+a_0)}{s^2+\omega_0^2}$	$a_0+a_1+\frac{a_1(a_0 s-\omega_0^2)}{s^2+\omega_0^2}$

This bounded UDE-based controller inherits the merits of the conventional UDE method with a simple structure and clear guidelines of parameter selections, and the BIBO assumption in [18] is relaxed.

II. PROBLEM FORMULATION

A. Overview of the UDE-based controller

Consider a class of LTI systems

$$\dot{x}=Ax+f(x)+Bu(t)+d(t) \tag{1}$$

where $x(t)=[x_1, x_2, ..., x_n]\in R^n$ is the system state, $u(t)\in R$ is the system input, $A\in R^{n\times n}$ is the known system matrix, $B\in R^n$ is known control vector, $f(x)\in R^n$ is nonlinear part or unknown dynamics, and $d(t)\in R^n$ is the bounded external disturbance. $f(x)$ is assumed with

$$\|f(x)\|\le F\|x\| \tag{2}$$

where $\|\cdot\|$ denotes the Euclidean norm, F is a positive constant. Then, a stable reference model is selected as

$$\dot{x}_m(t)=A_m x_m(t)+B_m c(t) \tag{3}$$

where $x_m(t)\in R^n$ is the reference state vector, $c(t)=[c_1, c_2, ..., c_r]\in R^r$ is a piecewise continuous and uniformly bounded command for the reference model, $A_m\in R^{n\times n}$ and $B_m\in R^{n\times r}$ are selected to meet the desired specification.

The objective is to design the control law $u(t)$ such that the state $x(t)$ can asymptotically track its reference $x_m(t)$, where the tracking error

$$e_x(t)=x_m(t)-x(t) \tag{4}$$

satisfies the following error dynamics

$$\dot{e}_x=(A_m+K)e_x \tag{5}$$

where $K\in R^{n\times n}$ is a constant error feedback gain matrix and (A_m+K) should be Hurwitz.

Combining (1)-(5), there is

$$A_m x_m+B_m c-Ax-Bu-f(x)-d=(A_m+K)e_x.$$

The control law $u(t)$ is designed as

$$Bu=A_m x_m+B_m c-Ax-u_d-(A_m+K)e_x \tag{6}$$

where the lumped term

$$u_d=f(x)+d$$

consists of the unknown term $f(x)$ and disturbance $d(t)$. According to the system dynamics (1), u_d can be written as

$$u_d=f(x)+d=\dot{x}-Ax-Bu.$$

Following the UDE procedures in [1], u_d can be approximated as

$$\begin{aligned}\hat{u}_d &= L^{-1}\{G_f(s)\}*u_d\\ &= L^{-1}\{G_f(s)\}*(\dot{x}-Ax-Bu)\end{aligned}$$

where $*$ is the convolution operator and $G_f(s)$ is the UDE filter with both strictly proper stable manner and appropriate bandwidth to cover the spectrum of u_d. L^{-1} means inverse Laplace transformation. Replacing u_d with $\hat{u}_d$ in (6), there is

$$\begin{aligned}Bu &= A_m x_m+B_m c-Ax-(A_m+K)e_x\\ &\quad -L^{-1}\{G_f(s)\}*(\dot{x}-Ax-Bu).\end{aligned}$$

Then, the final UDE-based control law is formulated as

$$\begin{aligned}u &= B^{+}\Bigg[-Ax+L^{-1}\left\{\frac{1}{1-G_f(s)}\right\}\\ &\quad *[A_m x_m+B_m c-(A_m+K)e_x]\\ &\quad -L^{-1}\left\{\frac{sG_f(s)}{1-G_f(s)}\right\}*x\Bigg]\end{aligned} \tag{7}$$

where $B^{+}=(B^T B)^{-1}B^T$. The scheme of the conventional UDE-based controller is shown in Fig. 2.

B. Integral property in the UDE-based controller

In the UDE-based controller (7), there are two terms involving the filter, $\frac{1}{1-G_f(s)}$ and $\frac{sG_f(s)}{1-G_f(s)}$. As pointed out in [14], [16], the filter design plays a very important role to achieve the good performances of the UDE-based controller, e.g., asymptotic reference tracking and disturbance rejection. Based on the internal model principle [16], the low-pass filter with $G_f(0)=1$ is required to handle step disturbance and step reference. Table I lists three frequently-used UDE filters, $G_{f1}(s)$ in [1], [3], [4], $G_{f2}(s)$ in [6], and G_{f3} in [7], [16]. It is interesting to notice that the integral property is included in the $\frac{1}{1-G_f(s)}$ term to provide good steady-state performances. However, the $\frac{sG_f(s)}{1-G_f(s)}$ term does not include the integral property due to the s term in the numerator.

Though the integral property in $\frac{1}{1-G_f(s)}$ can handle step disturbance and step reference, it might cause the integrator windup, if the system input is subject to a constraint. The anti-windup designs [19] are commonly adopted to deal with this issue, however, system structure and parameters are usually required to guarantee the closed-loop system stability [18], [20]. Furthermore, anti-windup designs with auxiliary systems [22], [23] will become very complex with the number of design parameters increasing, if the system order increases. Is it possible to have a simple bounded design for the UDE-based controller to handle the integrator windup and to guarantee the closed-loop system stability?

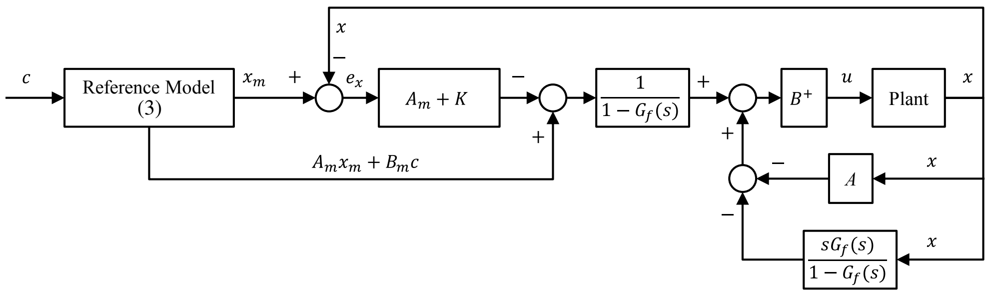


Fig. 1. The scheme of the conventional UDE-based controller [1].

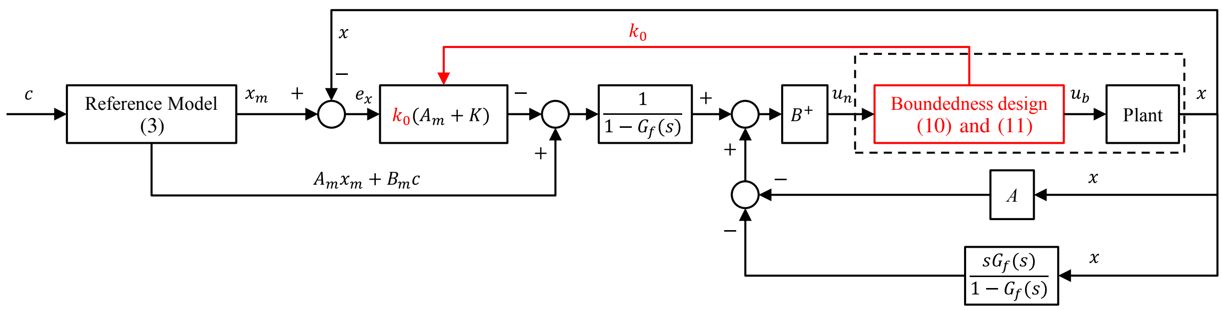


Fig. 2. The scheme of the proposed bounded UDE-based controller.

III. Bounded UDE-based Controller

Inspired by the BIC in [18], a boundedness design is proposed for the UDE-based controller to handle input constraint without integrator windup. Unlike the auxiliary system design, this boundedness design is embedded into the conventional UDE structure to result in a bounded UDE-based controller. The stability of the closed-loop system is investigated.

A. Design of the bounded UDE-based controller

Instead of the error dynamics in equation (5), a new error dynamics is designed as

$$\dot{e}_x = k_0(t)(A_m + K)e_x \tag{8}$$

where $0 < k_0(t) \leq 1$ is an additional time-varying variable to be determined. Following the same UDE design procedures in Section II, the new modified UDE-based control law can be obtained as

$$\begin{aligned} u_n \;=\; & B^+\Big[-Ax + L^{-1}\left\{\frac{1}{1-G_f(s)}\right\} \\ & * [A_m x_m + B_m c - k_0(A_m+K)e_x] \\ & - L^{-1}\left\{\frac{sG_f(s)}{1-G_f(s)}\right\} * x\Big]. \end{aligned} \tag{9}$$

Inspired by the BIC in [18], a boundedness design is added to (9) to regulate the final controller output $u_b(t)$ inside a given range of $(-u_{max},\, u_{max})$,

$$\dot{u}_b = -k_1 u_b\left(\frac{u_b^2}{u_{max}^2} + k_0^2 - 1\right) - k_2 k_0^2 (u_b - u_n) \tag{10}$$

$$\dot{k}_0 = -k_1 k_0\left(\frac{u_b^2}{u_{max}^2} + k_0^2 - 1\right) + \frac{u_b k_2 k_0}{u_{max}^2}(u_b - u_n) \tag{11}$$

where $k_1 > 0$ and $k_2 > 0$ are positive constants. $k_0(t)$ is further introduced into new error dynamics (8).

Lemma 1. *Through the boundedness design in (10) and (11), the final controller output $u_b(t)$ is regulated within a given range, i.e. $u_b \in (-u_{max},\, u_{max})$.*

Proof: Consider the following Lyapunov function candidate

$$V = \frac{u_b^2}{u_{max}^2} + k_0^2.$$

Taking the derivative of V along (10) and (11), it yields

$$\begin{aligned} \dot{V} &= \frac{2u_b}{u_{max}^2}\dot{u}_b + 2k_0\dot{k}_0 \\ &= -\frac{2k_1 u_b^2}{u_{max}^2}\left(\frac{u_b^2}{u_{max}^2} + k_0^2 - 1\right) - \frac{2u_b k_2 k_0^2}{u_{max}^2}(u_b - u_n) \\ &\quad -2k_1 k_0^2\left(\frac{u_b^2}{u_{max}^2} + k_0^2 - 1\right) + \frac{2u_b k_2 k_0^2}{u_{max}^2}(u_b - u_n) \\ &= -2k_1\left(\frac{u_b^2}{u_{max}^2} + k_0^2\right)\left(\frac{u_b^2}{u_{max}^2} + k_0^2 - 1\right) \\ &= -2k_1 V^2 + 2k_1 V. \end{aligned} \tag{12}$$

Then, solving (12) gives

$$\begin{aligned} V(t) &= \frac{e^{2k_1 t}V(0)}{e^{2k_1 t}V(0) - V(0) + 1} \\ &= \frac{1}{1 - e^{-2k_1 t}(1 - \frac{1}{V(0)})}. \end{aligned} \tag{13}$$

Through the initial design with $k_0(0) = 1$, and $u_b(0) = 0$, i.e., $V(0) = 1$, then

$$V(t) = 1, \,\forall\, t \geq 0.$$

Consequently, it always holds that $\frac{u_b^2}{u_{max}^2} + k_0^2 = 1$. So, $u_b(t)$ is kept within a given range of $(-u_{max},\, u_{max})$. ■

The scheme of the proposed bounded UDE-based controller is shown in Fig. 2. Compared to the conventional UDE-based controller [1], shown in Fig. 1, it can be seen

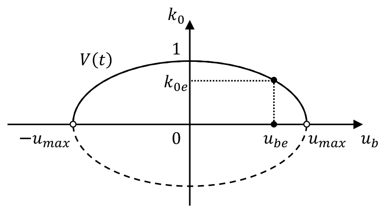


Fig. 3. An illustration of the bounded controller output u_b and the additional variable k_0 with the proposed bounded UDE-based controller (10) and (11).

that the boundedness design is well embedded into the conventional UDE framework with a simple structure. Note that if there are any numerical errors or parameter drifts in $V(t)$, $V(t)$ will still converge to 1 with $t \to \infty$, according to equation (13), and the rate of convergence can be adjusted by the parameter k_1. In this way, with the introduction of the additional controller state $k_0(t)$, the controller states, $u_b(t)$ and $k_0(t)$ will start and always remain on the ellipse

$$\frac{u_b^2}{u_{max}^2} + k_0^2 = 1 \tag{14}$$

which indicates that $u_b(t)$ is bounded in the given range $u_b \in (-u_{max},\, u_{max})$, and $k_0 \in (0,\, 1]$, no matter how $u_n(t)$ in (9) changes. In the steady-state, $\dot{u}_b(t)$ and $\dot{k}_0(t)$ will be regulated to 0 with zero tracking error. When $\dot{u}_b = 0$ and $\dot{k}_0 = 0$, both controller states $u_b(t)$ and $k_0(t)$ will converge to an equilibrium point $(u_{be},\, k_{0e})$, as shown in Fig. 3.

Remark 2. With $V(t) = 1$, the boundedness design (10) and (11) are reduced to

$$\begin{aligned} \dot{u}_b &= -k_2 k_0^2 (u_b - u_n) \\ \dot{k}_0 &= \frac{u_b k_2 k_0}{u_{max}^2}(u_b - u_n). \end{aligned}$$

When the final controller output $u_b(t)$ is not close to the maximum value and $k_0(t)$ does not converge to 0, $u_b(t)$ will converge to $u_n(t)$, and the rate of convergence can be adjusted by the parameter k_2. If $u_b(t)$ does not converge to $u_n(t)$, the non-zero $\dot{u}_b$ will drive u_b to u_n. So, the parameter k_2 should be well designed and big enough to cover all the effective bandwidths of the modified controller output u_n. When $u_b(t)$ is close to the maximum value, i.e., $u_b(t) \to \pm u_{max}$, the variable $k_0(t)$ will converge to 0. Then, the term $L^{-1}\left\{\frac{1}{1-G_f(s)}\right\} * [k_0(A_m + K)e_x]$ with integral property in (9) will converge to zero as $k_0(t) \to 0$. This means that the integral property in (9) slows down and converges to zero, when $u_b(t)$ goes to the limits, which can naturally prevent the integrator windup problem.

Compared to the conventional UDE-based controller [1], this bounded UDE-based controller (9), (10) and (11) only introduce two additional design parameters, k_1, and k_2, and both of them have clear selection guidelines.

B. Stability analysis

Theorem 3. *Consider the closed-loop system shown in Fig. 2. Given any compact set* $\Omega_x = \left\{x(t) \mid \|x(t)\|^2 < q,\, q > 0\right\}$, *if the initial state* $x(0)$ *is within this set* Ω_x, and *the system input within the range* $u \in (-u_{max},\, u_{max})$ *is capable to stabilize the system (1) within the set* Ω_x, *i.e.,* $k_0(t)$ *does not converge to 0, the closed-loop system is stable in the sense of boundedness.*

Proof: As shown in Fig. 2, the boundedness design (10) and (11) and the plant can be combined together as a new plant (inside the dashed box) with the system input $u_n(t)$, where both $x(t)$ and $k_0(t)$ are system states. Then, the boundedness design can be treated as a disturbance $\Delta \in R^n$ for the new plant as

$$\dot{x} = Ax + f(x) + Bu_n(t) + d(t) + \Delta \tag{15}$$

where $\Delta = B(u_b - u_n)$. And, the lumped term becomes $u_d = f(x) + d + \Delta$.

Consider the following Lyapunov function candidate

$$V_x(x) = x^T x.$$

Taking the derivative of $V_x(x)$ along with the new plant (15) and the modified UDE-based control law (9), there is

$$\begin{aligned} \dot{V}_x(x) &= \dot{x}^T x + x^T \dot{x} \\ &= \left[x^T A^T + f^T(x) + u_n^T B^T + d^T + \Delta^T\right] x \\ &\quad + x^T \left[Ax + f(x) + Bu_n + d + \Delta\right] \\ &= k_0 x^T (A_m^T + K^T + A_m + K) x \\ &\quad + (1 - k_0)\left(x_m^T A_m^T x + x^T A_m x_m\right) \\ &\quad - k_0\left(x_m^T K^T x + x^T K x_m\right) + c^T B_m^T x + x^T B_m c \\ &\quad + L^{-1}\left\{1 - G_f(s)\right\} * \left[f^T(x) x + x^T f(x)\right] \\ &\quad + L^{-1}\left\{1 - G_f(s)\right\} \\ &\quad * \left[\left(d^T + \Delta^T\right) x + x^T (d + \Delta)\right] \\ &\le k_0 \lambda_{max}(Q) \|x\|^2 + 2(1 - k_0)\|A_m\|\|x_m\|\|x\| \\ &\quad + 2k_0\|K\|\|x_m\|\|x\| + 2\|B_m c\|\|x\| \\ &\quad + L^{-1}\left\{1 - G_f(s)\right\} * (2F\|x\|^2) \\ &\quad + L^{-1}\left\{1 - G_f(s)\right\} * \left[2\|x\|(\|d\| + \|\Delta\|)\right] \\ &\le \left[k_0 \lambda_{max}(Q) + 2F\right]\|x\|^2 + 2\zeta\|x\| \end{aligned} \tag{16}$$

where F is defined in (2), $Q = A_m^T + K^T + A_m + K$ is negative semi-definite with the Hurwitz matrix $(A_m + K)$, $\lambda_{max}(Q) < 0$ is the maximum eigenvalue of Q, and $\zeta = (1 - k_0)\|A_m\|\|x_m\| + k_0\|K\|\|x_m\| + \|B_m c\| + \|d\| + \|\Delta\|$. Within the set Ω_x, $\|\Delta\|$ is bounded, according to Remark 2. Therefore, ζ has an upper bound $p > 0$, which is a function of q.

By applying the Young's inequality to (16), there is

$$\begin{aligned} \dot{V}_x(x) &\le \left[k_0 \lambda_{max}(Q) + 2F + \varepsilon^2\right]\|x\|^2 + \frac{\zeta^2}{\varepsilon^2} \\ &\le -\lambda_1 V_x(x) + \frac{p^2}{\varepsilon^2} \end{aligned} \tag{17}$$

where $\lambda_1 = -\left[\min_t(k_0)\lambda_{max}(Q) + 2F + \varepsilon^2\right] > 0$, and $\varepsilon > 0$ is a tuning coefficient to determine the value of $\frac{p^2}{\varepsilon^2}$. The proper error feedback gain matrix $(A_m + K)$ can be chosen to fulfill $\lambda_1 > \frac{p^2}{q\varepsilon^2}$, if $k_0(t)$ does not converge to 0. Then, $\dot{V}_x(x) < 0$ when $V_x(x) \ge q$. In other words, the set Ω_x

is an invariant set. Therefore, $\|x(t)\|^2 < q$ for all $t > 0$ if $\|x(0)\|^2 \le q$. The closed-loop system is stable in the sense of boundedness. ■

It is worth noting that the closed-loop system still could be stable when $k_0(t)$ converges to 0, if the plant (1) is BIBO stable, according to [18].

C. Performance analysis

For the new plant (15) and the modified UDE-based control law (9), as illustrated in Fig. 2, the performances of the tracking error will be analyzed.

When the UDE filter is used to estimate the uncertain term u_d, and u_d is replaced with $\hat{u}_d$ for the modified UDE-based control law (9), the actual error dynamics (8) becomes

$$\dot{e}_x = k_0(t)(A_m + K)e_x - \tilde{u}_d \tag{18}$$

where $\tilde{u}_d \triangleq u_d - \hat{u}_d$ is the estimation error of the lumped term $u_d = f(x) + d + \Delta$. The estimation error is

$$\tilde{u}_d = L^{-1}\{1 - G_f(s)\} * u_d. \tag{19}$$

Consider the following Lyapunov function candidate

$$V_e(e_x) = e_x^T e_x.$$

Then,

$$\begin{aligned} \dot{V}_e(e_x) &= k_0 e_x^T Q e_x - \tilde{u}_d^T e_x - e_x^T \tilde{u}_d \\ &\le k_0 \lambda_{max}(Q) \|e_x\|^2 + 2\|\tilde{u}_d\| \|e_x\| \qquad (20) \\ &\le [k_0 \lambda_{max}(Q) + 1] \|e_x\|^2 + \|\tilde{u}_d\|^2 \\ &\le -\lambda_3 V_e(e_x) + \lambda_4 \qquad (21) \end{aligned}$$

where $\lambda_3 = -[\min_t(k_0)\lambda_{max}(Q) + 1]$, $\lambda_4 = \max_t\left(\|\tilde{u}_d\|^2\right) > 0$. Then, solving (21) gives

$$0 \le V_e(e_x) \le V_e(e_x(0))e^{-\lambda_3 t} + \frac{\lambda_4}{\lambda_3}(1 - e^{-\lambda_3 t}). \tag{22}$$

Therefore,

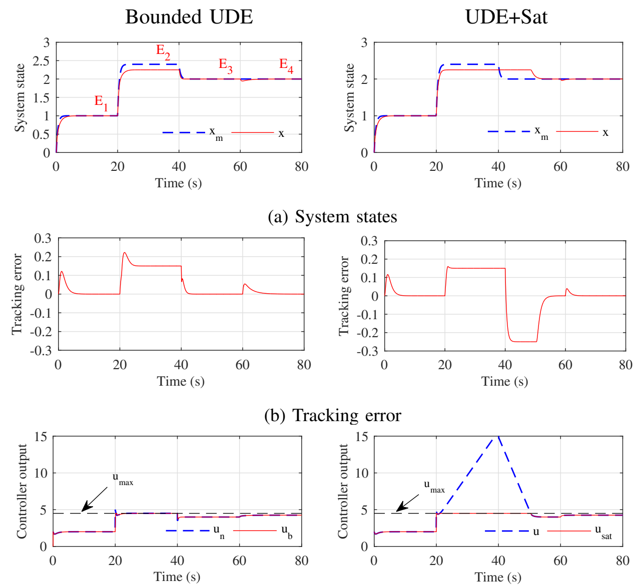
$$0 \le \|e_x\| \le \sqrt{\|e_x(0)\|^2 e^{-\lambda_3 t} + \frac{\lambda_4}{\lambda_3}(1 - e^{-\lambda_3 t})}.$$

If $k_0(t)$ does not converge to zero, the error feedback gain matrix $(A_m + K)$ can be well designed to fulfill $\lambda_3 > 0$. The term $\|e_x(0)\|^2 e^{-\lambda_3 t}$ will gradually decay and converge to zero, and the term $\frac{\lambda_4}{\lambda_3}(1 - e^{-\lambda_3 t})$ will gradually increase and converge to $\frac{\lambda_4}{\lambda_3}$, when $t \to \infty$. Therefore, $\|e_x\|$ will finally converge to $\sqrt{\frac{\lambda_4}{\lambda_3}}$. The value of $\sqrt{\frac{\lambda_4}{\lambda_3}}$ can be adjusted through the design of the error feedback gain matrix $(A_m + K)$. If the filter $G_f(s)$ is well designed [16] as a strictly proper stable filter with unity gain and appropriate bandwidth to cover the spectrum of the lumped term $u_d(t)$, the estimation error (19) will be close to zero, so as λ_4. Then, the tracking error e_x will converge to zero.

Note that the steady-state performances cannot be guaranteed if $k_0(t)$ converges to 0, even though the closed-loop system is stable given the BIBO stability of the plant (1).

TABLE II
CONTROL PARAMETERS

Parameters	Values	Parameters	Values
a_m, b_m	2	k_1	2000
$a_m + k$	2	k_2	2000
$G_f(s)$	$\frac{2}{s+2}$	u_{max}	4.5



(c) Controller output

Fig. 4. Simulation results with the bounded UDE-based controller (left column) and with the conventional UDE-based controller [1] plus a saturation unit (UDE+Sat) (right column).

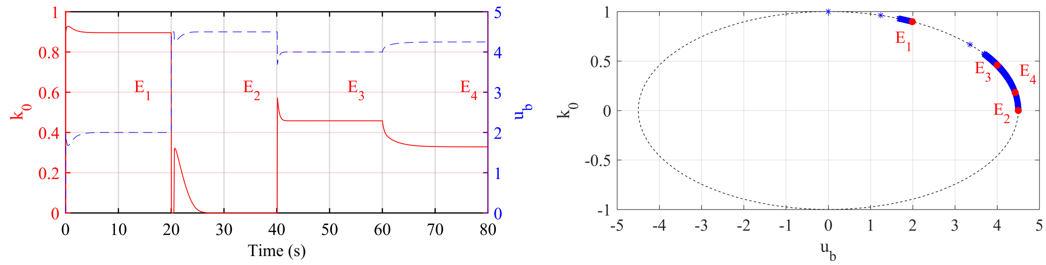
IV. SIMULATION STUDIES

In order to verify the proposed boundedness design for the UDE-based controller, a numerical simulation is provided. Consider the following system:

$$\dot{x} = ax + Fx + u + d$$

where $a = -1$, $F = -1$, and d is a step disturbance with $d = -0.25$ applied at $t = 60$ s. The bounded system input is required within the range of $u \in (-4.5, 4.5)$. The reference model is chosen as $\dot{x}_m = -a_m + b_m c$, where $c(t)$ is a multiple step command with $c = 1$ at $t = 0$ s, $c = 2.4$ at $t = 20$ s, and $c = 2$ at $t = 40$ s. And the error dynamics is designed as $\dot{e} = -k_0(t)(a_m + k)e$. The control parameters for the proposed bounded UDE-based controller (9)-(11) are shown in Table II. How to choose parameters for the UDE filter and the error dynamics in the UDE-based controller can refer to [14], [16]. In order to show the advantages of the proposed boundedness design, the conventional UDE-based controller (7) [1] by adding a saturation unit is also provided for the comparison.

The system responses are shown Fig. 4. Before $t = 20$ s, both the proposed bounded UDE and the conventional UDE plus a saturation unit have the same performances in reference tracking when the controller output is not constrained. The tracking errors are almost zero at steady-state. When the reference is changed to 2.4 at $t = 20$s, both controllers cannot reach the reference target because of the input constraint.



(a) Within time-domain (b) Within the $u_b - k_0$ ellipse

Fig. 5. The illustration of the controller states u_b and k_0 with the proposed boundedness design.

The tracking errors of both controllers are about 0.15 at steady-state in Fig. 4(a) and (b). From Fig. 4(c), it shows that the proposed bounded UDE can guarantee the input constraint without any integrator windup, and u_n in equation (9) stops increasing and converges to steady-state, because of the effects from $k_0(t) \to 0$. However, the conventional UDE plus a saturation unit suffers from the integrator windup, because of the integral property in the UDE-based controller. After $t = 40$ s, the bounded UDE still can achieve reference tracking quickly, whereas the conventional UDE plus a saturation unit cannot achieve reference tracking till about $t = 50$ s due to the effects of the integrator windup. After $t = 60$ s, both controllers can reject the disturbance with similar performances. Therefore, the proposed bounded UDE control design can handle the input constraint without the integrator windup. The controller states $u_b(t)$ and $k_0(t)$ always remain on the ellipse, which is clearly illustrated in Fig. 5, where the equilibrium points E_1, E_2, E_3, and E_4 represent the four steady-states of the system, at 5 s $\sim$ 20 s, 25 s $\sim$ 40 s, 45 s $\sim$ 60 s, and 65 s $\sim$ 80 s respectively.

V. Conclusion

In this paper, a boundedness design has been embedded into the conventional UDE-based controller to form a bounded UDE-based controller. An additional time-varying variable has been introduced into the design of error dynamics to handle the integrator windup issue. The proposed bounded UDE-based control has a simple structure with clear guidelines of parameter selections. Both theoretical analysis and simulation studies have demonstrated the effectiveness of the proposed approach.

References

[1] Q.-C. Zhong and D. Rees, "Control of uncertain LTI systems based on an uncertainty and disturbance estimator," *ASME Trans. J. Dyn. Sys. Meas. Control*, vol. 126, no. 4, pp. 905–910, Dec. 2004.

[2] K. Youcef-Toumi and O. Ito, "A time delay controller for systems with unknown dynamics," *ASME Trans. J. Dyn. Sys. Meas. Control*, vol. 112, no. 1, pp. 133–142, Mar. 1990.

[3] B. Ren, Y. Wang, and Q.-C. Zhong, "UDE-based control of variable-speed wind turbine systems," *International Journal of Control*, vol. 90, no. 1, pp. 121–136, Jan. 2017.

[4] Y. Wang and B. Ren, "Fault ride-through enhancement for grid-tied PV systems with robust control," *IEEE Trans. Ind. Electron.*, vol. 65, no. 3, pp. 2302–2312, Mar 2018.

[5] J. Ren, Y. Ye, G. Xu, Q. Zhao, and M. Zhu, "Uncertainty-and-disturbance-estimator-based current control scheme for PMSM drives with a simple parameter tuning algorithm," *IEEE Trans. Power Electron.*, vol. 32, no. 7, pp. 5712–5722, Jul. 2017.

[6] Y. Wang, B. Ren, and Q.-C. Zhong, "Robust power flow control of grid-connected inverters," *IEEE Trans. Ind. Electron.*, vol. 63, no. 11, pp. 6887–6897, Nov. 2016.

[7] Q.-C. Zhong, Y. Wang, and B. Ren, "UDE-based robust droop control of inverters in parallel operation," *IEEE Trans. Ind. Electron.*, vol. 64, no. 9, pp. 7552–7562, Sept. 2017.

[8] S. Y. Gadelovits, Q. C. Zhong, V. Kadirkamanathan, and A. Kuperman, "UDE-based controller equipped with a multi-band-stop filter to improve the voltage quality of inverters," *IEEE Trans. Ind. Electron.*, vol. 64, no. 9, pp. 7433–7443, Sept. 2017.

[9] Y. Ye and Y. Xiong, "UDE-based current control strategy for LCCL-type grid-tied inverters," *IEEE Trans. Ind. Electron.*, vol. 65, no. 5, pp. 4061–4069, May 2018.

[10] R. Sanz, P. Garcia, Q.-C. Zhong, and P. Albertos, "Robust control of quadrotors based on an uncertainty and disturbance estimator," *Journal of Dyn. Sys. Meas. Con.Trans. ASME*, vol. 138, no. 7, p. 071006, May 2016.

[11] J. P. Kolhe, M. Shaheed, T. S. Chandar, and S. E. Taloe, "Robust control of robot manipulators based on uncertainty and diturbance estimation," *International Journal of Robust and Nonlinear Control*, vol. 23, no. 1, pp. 104–122, Jan. 2013.

[12] S. Talole and S. Phadke, "Model following sliding mode control based on uncertainty and disturbance estimator," *ASME Trans. J. Dyn. Sys. Meas. Control*, vol. 130, pp. 1–5, 2008.

[13] D. D. Dhadekar and B. M. Patre, "UDE-based decoupled full-order sliding mode control for a class of uncertain nonlinear MIMO systems," *Nonlinear Dynamics*, vol. 88, no. 1, pp. 263–276, Apr. 2017.

[14] Q.-C. Zhong, A. Kuperman, and R. Stobart, "Design of UDE-based controllers from their two-degree-of-freedom nature," *International Journal of Robust and Nonlinear Control*, vol. 21, pp. 1994–2008, Nov. 2011.

[15] L. Sun, D. Li, Q. C. Zhong, and K. Y. Lee, "Control of a class of industrial processes with time delay based on a modified uncertainty and disturbance estimator," *IEEE Trans. Ind. Electron.*, vol. 63, no. 11, pp. 7018–7028, Nov. 2016.

[16] B. Ren, Q.-C. Zhong, and J. Dai, "Asymptotic reference tracking and disturbance rejection of UDE-based robust control," *IEEE Trans. Ind. Electron.*, vol. 64, no. 4, pp. 3166–3176, Apr. 2017.

[17] I. Aharon, D. Shmilovitz, and A. Kuperman, "Uncertainty and disturbance estimator-based controllers design under finite control bandwidth constraint," *IEEE Trans. Ind. Electron.*, vol. 65, no. 2, pp. 1439–1449, Feb. 2018.

[18] G. C. Konstantopoulos, Q.-C. Zhong, B. Ren, and M. Krstic, "Bounded integral control of input-to-state practically stable non-linear systems to guarantee closed-loop stability," *IEEE Trans. Autom. Control*, vol. 61, no. 12, pp. 4196–4202, Dec. 2016.

[19] S. Tarbouriech and M. Turner, "Anti-windup design: an overview of some recent advances and open problems," *IET Contr. Theory Appl.*, vol. 3, no. 1, pp. 1–19, Jan. 2009.

[20] Q.-C. Zhong, G. C. Konstantopoulos, B. Ren, and M. Krstic, "Improved synchronverters with bounded frequency and voltage for smart grid integration," *IEEE Trans. Smart Grid*, vol. 9, no. 2, pp. 786–796, Mar. 2018.

[21] G. C. Konstantopoulos, Q.-C. Zhong, B. Ren, and M. Krstic, "Bounded droop controller for parallel operation of inverters," *Automatica*, vol. 53, pp. 320 – 328, Mar. 2015.

[22] K. Esfandiari, F. Abdollahi, and H. A. Talebi, "Adaptive control of uncertain nonaffine nonlinear systems with input saturation using neural networks," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 26, no. 10, pp. 2311–2322, Oct. 2015.

[23] J. Zhou and C. Wen, *Adaptive Backstepping Control of Uncertain Systems: Nonsmooth Nonlinearities, Interactions or Time-Variations*. Springer, 2008.