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Synchronised tracking control of multi-agent system with high-order dynamics

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Abstract: This study is concerned with the synchronised tracking control for multiple agents with high-order dynamics, whereas the desired trajectory is only available for a portion of the team members. Using the weighted average of the neighbours' states as the reference signal, adaptive neural network (NN) control is designed for each agent in both full-state and output feedback cases. It is proved that the adaptive NN control law guarantees that the tracking error of each agent converges to an adjustable neighbourhood of the origin for both cases although some of them do not access the desired trajectory directly. Two simulation examples are provided to demonstrate the performance of the proposed approaches.

1 Introduction

Control of multi-agent systems with applications to the cooperation of robots, unmanned aerial vehicles (UAVs), autonomous underwater vehicles (AUVs), scheduling of automated highway systems have been intensively studied in recent years [1–15]. Various control strategies for multi-agent systems can be roughly assorted into two architectures: centralised and decentralised. In the decentralised control, local control for each agent is designed using locally available information so it requires less computational effort and is more scalable with respect to the swarm size [5, 13].

Through local neighbour-to-neighbour information exchange among the agents, current distributed synchronised control for multiple agents primarily focus on agents with single-integrator kinematics [2, 16], double-integrator dynamics [4, 17] and Euler–Lagrange mechanical systems [18]. Model reference consensus algorithms for high-order system $x^{(\ell)} = u$, $\ell > 3$, was studied in [19]. Consensus algorithms with switching topology and time delays was proposed in [20], leader-following consensus problem for multi-agent system with fixed and switching topologies was studied in [21], where the agent is modelled by linear system $\dot{x} = Ax + Bu$ with (A, B) a stabilised pair. The proposed methods in these works are restricted to the control of agents with known dynamics.

The multi-agent systems considered in this paper have the following features: (i) the agent dynamics is high order ($\geq$3) and with unknown dynamics; (ii) the desired trajectory is only available to a portion of the agents; and (iii) the leadership of the leader itself is unknown to all the others, and the leader can only affect the agents who can sense the leader. The problem studied in this paper is similar to the consensus problem with time-varying reference state [2], where each agent is designed to track the desired trajectory by using only neighbour-to-neighbour information exchange among the agents. Most of recent related works are concerned with the multi-agent systems with known dynamics [5, 19–21]. However, for real-world applications, practical systems usually have complicated non-linear dynamics and there are usually uncertainties in the dynamics. Adaptive control was studied for both consensus problem and leader-following problem for a class of multi-agent systems in [6], where there were noise and one unknown parameter in the agent dynamics. Robust adaptive control for the multi-agent system consensus was studied in [22]. In [13], hidden layer leader-following problem with discrete uncertain dynamics was considered, where the communication graph should be strongly connected.

In contrast, this paper considers the general case that each agent is of uncertain non-linear dynamics and only a portion of the agents can access the desired trajectory. Unlike the leader–follower strategy (e.g. [21]), where the information only flows from the leader to the followers, the problem studied in this paper takes into account the general cases where information flow from any agent to any other agent. There are multiple agents who can access the desired trajectory, therefore increase redundancy and robustness for the whole team [2].

The main contribution of this work lies in three aspects:

(i) We prove that if the extended communication graph contains a spanning tree with the virtual agent as its root, then its Laplacian will be positive definite [12]. This property facilitates the subsequent stability proof. In the

physical view, this property also means that there is a 'path' from the virtual agent to each agent, so that the agent can obtain the information of the desired trajectory directly or indirectly.
(ii) Owing to the fact that only the neighbours' information is available to each agent, we use the weighted average of neighbours' states as the reference state of the agent in the control design. Moreover, to deal with the unknown dynamics and the unknown disturbances, the neural network (NN) approximation is used in the control design for each agent.
(iii) On the adaptive NN control designed, high-gain observer is synthesised and augmented into the controlled system with the aim to estimate the unavailable signals that are required for control design. In case that only outputs are available, the signals provided by the high-gain observer render the estimated values in lower noises than those directly computed by time derivatives of measured outputs. The mathematical proof of stability for both state feedback and output feedback cases are provided. As such, we provide a framework for synchronised tracking control of a general class of high-order single-input-single-output (SISO) systems with unknown dynamics, zero dynamics and disturbance which can be described by (1).

The remainder of the paper is organised as follows: In Section 2, some preliminaries are presented. The synchronised tracking control problem of multi-agent system is given in Section 3. Next, we design the synchronised tracking control for each agent in Section 4 in both full-state feedback and output-feedback cases. We take two simulation examples, including one numerical example and one on the synchronised altitude tracking of multiple helicopters, in Section 5. Concluding remarks are given in Section 6.

2 Preliminaries

2.1 Agent dynamics

In this work, we considered a class of agents with non-linear dynamics which can be described as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ &\vdots \\ \dot{x}_{\rho-1} &= x_\rho \\ \dot{x}_\rho &= f(\eta, x) + g(x,\eta)(u+d) \\ \dot{\eta} &= q(x,\eta) \end{aligned} \tag{1}$$

where $x = [x_1, \ldots, x_\rho]^{\mathrm{T}} \in \mathbb{R}^\rho$ and $\eta = [\eta_1, \ldots, \eta_{n-\rho}]^{\mathrm{T}} \in \mathbb{R}^{n-\rho}$ are the states of the agent; ρ is the relative degree and n is the states dimension of the system, respectively; $y = x_1 \in \mathbb{R}$ is the output of the system, and $u \in \mathbb{R}$ is the input of the system; the mapping $f: \mathbb{R}^n \to \mathbb{R}$ denotes the unmodelled dynamics and uncertainties, which is an unknown smooth function; $g: \mathbb{R}^n \to \mathbb{R}$ stands for the open-loop control gain of the system, which is an unknown function with certain properties, and d is the external disturbance in the input channel; The last differential equation, $\dot{\eta} = q(x,\eta)$, is the internal dynamics of the system, where $q: \mathbb{R}^n \to \mathbb{R}^{n-\rho}$ is a partially unknown vector field satisfying certain properties, which will be described shortly.

Assumption 1: The internal dynamics of system (1), given by $\dot{\eta} = q(x,\eta)$, are exponentially stable. In addition, $q(x,\eta)$ is Lipschitz in x, that is, there exist positive constants a_q and a_x such that $\|q(x,\eta) - q(0,\eta)\| \le a_x\|x\| + a_q,\ \forall (x,\eta) \in \mathbb{R}^n$.

Under the assumption that the zero dynamics are stable, by the converse Lyapunov theorem, there exists a Lyapunov function $V_0(\eta)$ which satisfies the following Lyapunov inequalities for $(x,\eta) \in \mathbb{R}^n$: $\gamma_1\|\eta\|^2 \le V_0(\eta) \le \gamma_2\|\eta\|$, $(\partial V_0/\partial \eta)q(0,\eta) \le -\lambda_a\|\eta\|^2$ and $\|\partial V_0/\partial\eta\| \le \lambda_b\|\eta\|$, where γ_1, γ_2, λ_a and λ_b are positive constants.

Assumption 2: The external disturbances d are uncertain bounded functions $d \in L_\infty$. That is, there exists unknown positive constants ϱ such that $|d(t)| \le \varrho < \infty$, where ϱ can be arbitrarily large.

Assumption 3: There exist smooth functions $\bar{g}(x,\eta)$ and a positive constant $\underline{g} > 0$, such that $\bar{g}(x,\eta) \ge g(x,\eta) > \underline{g} > 0,\ \forall (x,\eta) \in \mathbb{R}^n$. There exists a positive function $g_0(x,\eta)$ satisfying $|\dot{g}(x,\eta)/2g(x,\eta)| \le g_0(x,\eta),\ \forall (x,\eta) \in \mathbb{R}^n$ as well. Without loss of generality, it is further assumed that the sign of $g(x,\eta)$ is positive $\forall (x,\eta) \in \mathbb{R}^n$.

Remark 1: $g(x,\eta)$ stands for the control gain of the open-loop system. Assumption 3 is reasonable for many physical systems as follows: (i) there exist lower and upper bounds of the control gain, and (ii) the states of some physical systems cannot change too fast within a small time internal in open-loop because of the 'inertia' of the systems. This does not pose a strong restriction upon the class of systems such as the altitude regulation system of the helicopter as shown in Section 5. The reason is that if the controller is continuous, the situation in which a finite input causes an infinitely large effect upon the system rarely happen in such systems owing to the smoothness of $g(x,\eta)$.

Remark 2: The non-linear dynamics considered in this paper represent a general case, and many agent dynamics fall into the form (1), such as the full actuated robotics, diving/heading control of the unmanned vehicles etc. The agent dynamics presented in [4, 19, 22] can be considered as the special cases of (1).

2.2 Neural network approximation

In this paper, linearly parametrised NN is used to approximate the unknown continuous function $f_i(Z_i) : \mathbb{R}^q \to \mathbb{R}$ for ith agent [23, 24]

$$f_i(Z_i) = \theta_i^{\mathrm{T}}\psi_i(Z_i) + \varepsilon_i(Z_i) \tag{2}$$

where the input vector $Z_i^{\mathrm{T}} \in \mathbb{R}^q$, weight vector $\theta_i \in \mathbb{R}^l$, the NN node number $l > 1$ and $\psi_i(Z_i) \in \mathbb{R}^l$. Universal approximation results indicate that, if l is chosen sufficiently large, $\theta_i^{\mathrm{T}}\psi_i(Z_i)$ can approximate any continuous function, $f_i(Z_i)$, to any desired accuracy over a compact set $Z_i^{\mathrm{T}} \in \Omega_{xi}$ to arbitrary any degree of accuracy as

$$f_i(Z_i) = \theta_i^{*\mathrm{T}}\psi_i(Z_i) + \varepsilon_i(Z_i), \quad \forall Z_i \in \Omega_{xi} \subset \mathbb{R}^p \tag{3}$$

where θ_i^* are the ideal constant weight vectors, and $\varepsilon(Z_i)$ is the approximation error which is bounded over the compact set, that is, $|\varepsilon_i(Z_i)| \le \varepsilon_i^*$, $\forall Z_i \in \Omega_{xi}$ with $\varepsilon_i^* > 0$ is an unknown constant. The ideal weight vector θ_i^* is

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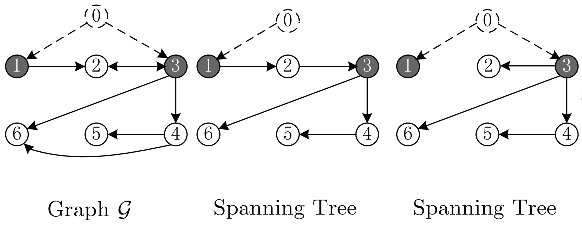

$$L = \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2/3 & 1 & -1/3 & 0 & 0 & 0 \\ -3/4 & 0 & -1/4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1/2 & -1/2 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

Matrix L

Fig. 1 *Sample graph and its Laplacian*

an artificial quantity required for analytical purposes. θ_i^* is defined as the value of θ_i that minimises $|\varepsilon_i|$ for all $Z_i \in \Omega_{xi} \subset \mathbb{R}^p$, that is

$$\theta_i^* := \arg\min_{\theta_i \in \mathbb{R}^l} \left\{ \sup_{Z_i \in \Omega_{xi}} |f_i(Z_i) - \theta_i^{\mathrm{T}} \psi_i(Z_i)| \right\} \tag{4}$$

In this paper, we choose $\psi_i(Z_i) = \exp[-(Z_i - \mu_i)^{\mathrm{T}}(Z_i - \mu_i)/\varsigma_i^2]$, $i = 1, \ldots, l$, where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{iq}]^{\mathrm{T}}$ is the centre of the receptive field and ς_i is the width of the Gaussian function. For the Gaussian radial basis function (RBF) networks, the following lemma provides an upper bound on the two-norm of vector $\psi_i(Z_i)$.

Lemma 1 [25]: Consider the above Gaussian RBF networks, let $\rho := \frac{1}{2}\min_{i \neq j} \|\mu_i - \mu_j\|$, and let q be the dimension of input Z_i, and σ be the width of Gaussian function. Then we may take an upper bound of $\|\psi_i(Z_i)\|$ as

$$\|\psi_i(Z_i)\| \leq \sum_{k=0}^{\infty} 3q(k+2)^{q-1} e^{-2\rho^2 k^2/\sigma^2} := m_\psi^* \tag{5}$$

3 Synchronisation tracking of multiple agents

In this work, weighted directed graph is used to model communication among the agents [18]. In addition, we introduce a virtual agent, denoted as v_0, whose motion follows the desired trajectory strictly, then for the N agent system, the graph $\mathcal{G}$ contains a node set $\mathcal{V} = \{v_0, v_1, \ldots, v_N\}$, and a weighted adjacent matrix $A^* = [a_{ij}^*] \in \mathbb{R}^{(N+1)\times(N+1)}$, where $a_{ij}^* > 0$ means that agent i can obtain the information from agent j, otherwise $a_{ij}^* = 0$. Define a diagonal matrix $\Delta(\mathcal{G}) \in \mathbb{R}^{(N+1)\times(N+1)}$ with elements $\delta_{jj} = \sum_k a_{jk}^*$, and the normalised Laplacian of $\mathcal{G}$ as $L = I - A$, where the elements in the normalised adjacent matrix A are defined as $a_{ij} = a_{ij}^*/\delta_{ii}$, in case $\delta_{ii} \neq 0$, and $a_{ij} = a_{ij}^*$, otherwise. With adding a virtual agent in the system, we call the graph $\mathcal{G}$ the *extended communication graph*. For each agent, the neighbour set of v_i is defined as $\mathcal{N}_i = \{v_j \in \mathcal{V} | a_{ij} > 0\}$.

Theorem 1: If the extended communication graph $\mathcal{G}$ contains a spanning tree with v_0 as its root. Then the normalised adjacent matrix A is sub-stochastic, and $L = I - A$ is positive definite, whose inverse is given by $L^{-1} = \sum_{l=0}^{\infty} A^l$.

Proof: By introducing the virtual agent v_0, we know that $\mathcal{N}_0 = \varnothing$ and all the elements of the first row of A are zero, as it does not accept any other agents information. Since $\mathcal{G}$ has a spanning tree and v_0 is the root, then each agent has at least one neighbour, the sum of any other row of A equals to 1, then A is a sub-stochastic matrix.

It is clear that all the diagonal elements of L are 1, and all the row sum of A is 1 except the first row, that means L is a diagonal dominant matrix and there exist a set $J = \{0\}$ satisfies $|\ell_{00}| > \sum_{j \neq i} |\ell_{ij}|$. Furthermore, $\mathcal{G}$ contains a spanning tree with v_0 as the root also means that there is a path from v_0 to any agent $v_i \in \mathcal{V}$, then for every element $i \neq 0$, there exists a sequence of non-zero elements form $\ell_{ii_1}, \ell_{i_1 i_2}, \ldots, \ell_{i_s 0}$. Then L satisfies all the conditions of the diagonally dominant matrix with non-zero elements chain [26]. Since L is real and $\ell_{ij} < 0, i \neq j$, $\ell_{ii} = 1$, L is a non-singular M-matrix [26]. By using Gerschgorin disc theory, we also know that all the eigenvalues of L lie in the right part of the complex plane, then we can conclude that L is positive definite. Furthermore, it follows from the fact that the spectral radius of A, $\rho(A) < 1$, then $\lim_{l\to\infty} A^l = 0$, and $(I - A)(I + A + A^2 + \cdots) = (I + A + A^2 + \cdots) - (A + A^2 + A^3 + \cdots) = I$. We obtain $L^{-1} = \sum_{l=0}^{\infty} A^l$. This completes the proof. □

Example 1: To explain Theorem 1 clearly, let us see the sample graph shown in Fig. 1, both v_1 and v_3 can access the desired trajectory, and two spanning trees of $\mathcal{G}$ with node 0 as its root are shown as well. We can find that in the Laplacian matrix L, there exists a sequence ℓ_{54}, ℓ_{43}, ℓ_{32}, ℓ_{21}, $\ell_{10} \neq 0$, or ℓ_{54}, ℓ_{43}, $\ell_{30} \neq 0$ from node 5 to node 0.

We studied the synchronised altitude tracking problem of multiple agents as follows: Considering a group of agents, the desired trajectory, $y_d(t)$, and its derivations up to ρth order is bounded, and is available to a portion of the agents. For each agent, we design a control, (i) using the full states of its neighbours and itself, and (ii) using the outputs of neighbours and itself, such that the tracking error converges to a neighbourhood of zero, that is, $\lim_{t\to\infty} |y_i(t) - y_d(t)| = \bar{\varepsilon}$, where $\bar{\varepsilon} > 0$. At the same time, all closed-loop signals are to be kept bounded.

The desired trajectory $y_d(t)$ is generated by the following reference model: $\dot{x}_{dj} = x_{dj+1}$, $j = 1, \ldots, \rho - 1$, $\dot{x}_{d\rho} = f_d(x_d, t)$, with $y_d = x_{d1}$, where $\rho \geq 2$ is a constant index, $x_d = [x_{d1}, \ldots, x_{d\rho}]^{\mathrm{T}} \in \mathbb{R}^\rho$ are the states of reference system, $y_d \in \mathbb{R}$ is the system output.

Assumption 4: The reference trajectory $y_d(t)$ and its ρth derivatives remain bounded, that is, $x_d \in \Omega_d \subset \mathbb{R}^\rho$, $\forall t \geq 0$.

Assumption 5: The extended communication graph $\mathcal{G}$ has a spanning tree with the virtual agent as the root, and this virtual agent follows the desired trajectory strictly.

The following lemma is useful for analysis of the internal dynamics of the agent.

Lemma 2 [27]: Denote positive constants $a_1 = (\lambda_b a_x)/\lambda_a$ and $a_2 = (\lambda_b a_q)/\lambda_a$. If Assumptions 1 and 4 are satisfied, there exists a positive time constant T_0 such that the trajectories $\eta(t)$ of the internal dynamics satisfy $\|\eta\| \leq a_1\|x(t)\| + a_2$, $\forall t > T_0$.

4 Synchronised tracking control design

In this section, we design the synchronised tracking control for each agent based on its neighbours' states. Feedforward approximators are used to compensate for unknown non-linear functions. Full-state feedback controller, in case that the neighbours' full-states are available for control design, will be derived first. Based on this, output feedback controllers, in case only the neighbours' outputs are available, will be subsequently designed via certainty equivalence approach for each agent, with the unavailable output derivative estimated by a high-gain observer.

4.1 Control design with full information

Since only a portion of the agents can access the information of the desired trajectory, the tracking control is designed based on the relative states with respect to its neighbours. Define the following error variables for the agents

$$z_{i,1} = y_{i,1} - y_{ir}, \quad z_{i,2} := \dot{z}_{i,1} = x_{i,2} - \dot{y}_{ir}, \ldots,$$
$$z_{i,\rho} := z_{i,1}^{(\rho)} = x_{i,\rho} - y_{ir}^{(\rho)} \tag{6}$$

with $y_{ir}(t) = \sum_{j\in\mathcal{N}_i} a_{ij}y_j(t)$, $y_{ir}^{(k)}(t) = \sum_{j\in\mathcal{N}_i} a_{ij}y_j^{(k)}(t)$, $k = 1, \ldots, \rho - 1$, where a_{ij} is the element of the normalised adjacent matrix A of the extended communication graph $\mathcal{G}$.

For each agent, define vectors $\bar{z}_i = [z_{i,1}, \ldots, z_{i,\rho}]^{\mathrm{T}} \in \mathbb{R}^\rho$, and the filtered tracking error $s_i = [\Lambda^{\mathrm{T}} \ \ 1]\bar{z}_i$, where $\Lambda = [\lambda_1, \ldots, \lambda_{\rho-1}]^{\mathrm{T}}$ satisfies $p^{\rho-1} + \lambda_{\rho-1}p^{\rho-2} + \cdots + \lambda_1$ is Hurwitz. Then the dynamics of s_i is written as

$$\dot{s}_i = f_i(x_i, \eta_i) + g_i(u_i + d_i) + [0 \quad \Lambda^{\mathrm{T}}]\bar{z}_i - y_{ir}^{(\rho)} \tag{7}$$

Considering the Lyapunov function candidate $V_{si} = (1/2g_i)s_i^2$, we have

$$\dot{V}_{si} = -\left(g_0 + \frac{\dot{g}_i}{2g_i^2}\right)s_i^2 + s_i(u_i + d_i) + s_i\frac{f_i(x_i, \eta_i) + [0 \quad \Lambda^{\mathrm{T}}]\bar{z}_i - y_{ir}^{(\rho)} + g_ig_0s_i}{g_i} \tag{8}$$

Owing to the existence of the uncertain items, we use the parameter linearised NN to approximate the unknown non-linear function $\bar{f}_i(x_i, \eta_i, \bar{z}_i) = (f_i(x_i, \eta_i) + [0 \quad \Lambda^{\mathrm{T}}]\bar{z}_i - y_{ir}^{(\rho)} + g_ig_0s_i)/g_i$, which can be described as $\bar{f}_i(Z_i) = \theta_i^{*T}\varphi_i(Z_i) + \bar{\varepsilon}_i$, where θ^* is the ideal weighted vector and $Z_i = [x_i, \eta_i, \bar{z}_i]^{\mathrm{T}}$.

Considering the Lyapunov function candidate

$$V_i = V_{si} + \frac{1}{2\gamma_2}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i + \frac{1}{2\gamma_1}\tilde{\varphi}_i^2 \tag{9}$$

where γ_1 and γ_2 are the positive constants, $\tilde{\theta}_i = \hat{\theta}_i - \theta_i^*$ and $\tilde{\varphi}_i = \hat{\varphi} - \varphi_i^*$ are the estimated errors of parameters and the error bound, where $\hat{\theta}_i$ and $\hat{\varphi}_i$ are the estimation of θ_i^* and $\varphi_i^* = (\varrho_i + \bar{\varepsilon}_i)^2$, respectively. Then we have

$$\dot{V}_i = -\frac{\dot{g}_i}{2g_i^2}s_i^2 + \frac{1}{g_i}s_i\dot{s}_i + \frac{1}{\gamma_2}\tilde{\theta}_i\dot{\hat{\theta}}_i + \frac{1}{\gamma_1}\tilde{\varphi}_i\dot{\hat{\varphi}}_i$$
$$= -\left(g_0 + \frac{\dot{g}_i}{2g_i^2}\right)s_i^2 + s_i(u_i + d_i) + s_i[\theta_i^{*\mathrm{T}}\psi_i(Z_i) + \bar{\varepsilon}_i] + \frac{1}{\gamma_2}\tilde{\theta}_i\dot{\hat{\theta}}_i + \frac{1}{\gamma_1}\tilde{\varphi}_i\dot{\hat{\varphi}}_i \tag{10}$$

Remark 3: The NN is constructed to approximate $\bar{f}_i(x_i, \eta_i, \bar{z}_i, y_{ir}^{(\rho)}) = (f_i(x_i, \eta_i) + [0 \ \ \Lambda^{\mathrm{T}}]\bar{z}_i - y_{ir}^{(\rho)} + g_ig_0s_i)/g_i$ on a whole, which avoids the possible singularity of the direct approximation of g_i.

Select the following control u_i for each agent

$$u_i = -\hat{\theta}_i^{\mathrm{T}}\psi_i - k_is_i - \frac{1}{2}\hat{\varphi}_is_i, \quad i = 1, \ldots, N \tag{11}$$

The update laws for the parameters are designed as

$$\dot{\hat{\varphi}}_i = -\gamma_1\left[-\frac{1}{2}(1 - \varpi_\varphi)s_i^2 + \sigma_1\hat{\varphi}_i\right]$$
$$\dot{\hat{\theta}}_i = -\gamma_2(-\psi_is_i + \sigma_2\hat{\theta}_i) \tag{12}$$

where $\varpi_{\varphi_i} = 0$ if $|\hat{\varphi}_i| \leq M_{\varphi_i}$ with M_{φ_i} is a designed positive constant, or 1 otherwise,

By using Young's inequality, we have $-\sigma_2\tilde{\theta}_i^{\mathrm{T}}\hat{\theta}_i \leq -(\sigma_2/2)\|\tilde{\theta}_i\|^2 + (\sigma_2/2)\|\theta_i^*\|^2$, $-\sigma_1\tilde{\varphi}_i\hat{\varphi}_i \leq -(\sigma_1/2)\tilde{\varphi}_i^2 + (\sigma_1/2)\varphi_i^{*2}$, and $(\varrho_i + \bar{\varepsilon}_i)s_i \leq \frac{1}{2} + \frac{1}{2}s_i^2\varphi_i^*$. Considering (11) and (12), the time derivative of V_i can be written as

$$\dot{V}_i = -\left(g_0 + \frac{\dot{g}_i}{2g_i^2}\right)s_i^2 - k_is_i^2 + s_i\left(\bar{\varepsilon}_i + d_i - \frac{1}{2}\hat{\varphi}_i\right) - \sigma_2\tilde{\theta}_i^{\mathrm{T}}\hat{\theta}_i + \frac{1}{2}(1 - \varpi_\varphi)s_i\tilde{\varphi}_is_i - \sigma_1\tilde{\varphi}_i\hat{\varphi}_i$$
$$\leq -k_is_i^2 + \frac{1}{2} + \frac{1}{2}s_i^2\varphi_i^* - \frac{1}{2}s_i^2\hat{\varphi}_i - \frac{\sigma_2}{2}\|\tilde{\theta}_i\|^2 + \frac{\sigma_2}{2}\|\theta_i^*\|^2 + \frac{1}{2}\tilde{\varphi}_is_i^2 - \frac{\sigma_1}{2}\tilde{\varphi}_i^2 + \frac{\sigma_1}{2}\varphi_i^{*2}$$
$$= -k_is_i^2 - \frac{\sigma_1}{2}\tilde{\varphi}_i^2 - \frac{\sigma_2}{2}\|\tilde{\theta}_i\|^2 + c_{2i} \tag{13}$$

where $c_{2i} = (\sigma_2/2)\|\theta_i^*\|^2 + (\sigma_1/2)\varphi_i^{*2} + \frac{1}{2}$.

Now define $\Omega_{si} = \left\{s_i \middle| |s_i| \leq \sqrt{c_{2i}/k_i}\right\}$, $\Omega_{\theta_i} = \left\{(\tilde{\theta}_i, \tilde{\varphi}_i) \middle| \|\tilde{\theta}_i\| \leq \sqrt{2c_{2i}/\sigma_2}, |\tilde{\varphi}_i| \leq \sqrt{2c_{2i}/\sigma_1}\right\}$, $\Omega_{ei} = \left\{(s_i, \tilde{\theta}_i, \tilde{\varphi}_i) \middle| k_is_i^2 + (\sigma_2/2)\tilde{\theta}_i^T\tilde{\theta}_i + (\sigma_1/2)\tilde{\varphi}_i^2 \leq c_{2i}\right\}$. Since c_{1i}, σ_1, σ_2 and k_i are positive constants, we know that Ω_{si}, Ω_{θ_i} and Ω_{ei} are compact sets. Equation (13) shows that $\dot{V}_i \leq 0$ once the errors are outside the compact set Ω_{ei}. According to the standard Lyapunov theorem, we conclude that s_i, $\tilde{\theta}_i$ and $\tilde{\varphi}_i$ are bounded. From (13), it can be seen that $\dot{V}_i$ is strictly negative as long as s_i is outside the compact set Ω_{si}. Therefore there exists a constant T_1 such that for $t > T_1$, the filtered tracking error s_i converges to Ω_{si}, that is to say, $s_i \leq \beta_{si}(k_i, \gamma_1, \gamma_2, \sigma_1, \sigma_2, \theta_i^*, \varphi_i^*, \varepsilon_i^*) = \sqrt{c_{2i}/k_i}$.

Now we will show that all the agents will track the desired trajectory although some of them cannot access the desired trajectory. Define the error between ith agent and the desired trajectory as $\tilde{y}_i(t) = y_i(t) - y_d(t) = y_i(t) - y_0(t)$, and the auxiliary states of each agent $\xi_i(t) = [\Lambda^{\mathrm{T}} \ \ 1]Y_i$ with $Y_i = [y_i, y_i^{(1)}, \ldots, y_i^{(\rho-1)}]^{\mathrm{T}}$. The filtered error is denoted as $\tilde{\xi}_i(t) = \xi_i(t) - \xi_d(t) = \xi_i(t) - \xi_0(t)$.

Using the fact that $s_i(t) = \xi_i(t) - \sum_{j\in\mathcal{N}_i} a_{ij}\xi_j(t)$, we have $\tilde{\xi}_i = \xi_i - \xi_0 = \sum_{j\in\mathcal{N}_i} a_{ij}\xi_j + s_i - \xi_0$, $i = 1, \ldots, N$, and in

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the vector form $\dot{\tilde{\xi}} = A\xi + s - \xi_0\mathbf{1}$, where $\mathbf{1} = [1, \dots, 1]^{\mathrm{T}}$, $s = [s_0, s_1, \dots, s_N]^{\mathrm{T}}$ and A is the normalised adjacency matrix of the extended communication graph. Note that all elements in the first row of A are equal to 0, and the other row summations of the matrix A are 1, we have $[0, 1, \dots, 1]^{\mathrm{T}} = A[0, 1, \dots, 1]^{\mathrm{T}}$. Then

$$\begin{aligned}\tilde{\xi} &= A(\tilde{\xi} + \xi_0\mathbf{1}) + s + [1, 0 \dots, 0]^{\mathrm{T}}\xi_0 - \xi_0\mathbf{1}\\ &= A\tilde{\xi} + [0, 1, \dots, 1]^{\mathrm{T}}\xi_0 + s + [1, 0 \dots, 0]^{\mathrm{T}}\xi_0 - \xi_0\mathbf{1}\\ &= A\tilde{\xi} + s \end{aligned}\tag{14}$$

Under Assumption 5, we know that L is an invertible matrix according to Theorem 1, then we have $\tilde{\xi} = L^{-1}s$.

Remark 4: To show that the tracking error for each agent converges to a compact set, we establish the relationship between $\tilde{\xi}$ and s by introducing the extended formation graph. Thanks to Theorem 1, under Assumption 5, we can obtain $\tilde{\xi} = L^{-1}s$, which facilities the subsequent stability proof of our results.

Define vectors $\mathcal{Y} = [Y_0^{\mathrm{T}}, Y_1^{\mathrm{T}}, \dots, Y_N^{\mathrm{T}}]^{\mathrm{T}}$, $\tilde{\mathcal{Y}} = [\tilde{Y}_0^{\mathrm{T}}, \tilde{Y}_1^{\mathrm{T}}, \dots, \tilde{Y}_N^{\mathrm{T}}]^{\mathrm{T}}$, $X = [X_0^{\mathrm{T}}, X_1^{\mathrm{T}}, \dots, X_{\rho-1}^{\mathrm{T}}]^{\mathrm{T}}$ and $\tilde{X} = [\tilde{X}_0, \tilde{X}_1, \dots, \tilde{X}_{\rho-1}]^{\mathrm{T}}$, where $X_j = [X_{0,j}, X_{1,j}, \dots, X_{N,j}]^{\mathrm{T}}$, $\tilde{X}_j = X_j - X_{jd} = X_j - y_0^{(j)}\mathbf{1}$, $\tilde{Y}_i = Y_i - Y_d = Y_i - Y_0$. Then we have $\dot{\tilde{\mathcal{Y}}} = \bar{A}_p\tilde{\mathcal{Y}} + \bar{b}\tilde{\xi}$, where $\bar{A}_p = I_{N+1} \otimes A_p$ and $\bar{b} = I_{N+1} \otimes b$ with

$$A_p = \begin{bmatrix} 0 & 1 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & 1\\ -\lambda_1 & -\lambda_2 & \cdots & -\lambda_{\rho-1}\end{bmatrix}$$

and

$$b = [\underbrace{0, \dots, 0}_{\rho-2}, 1]^{\mathrm{T}}$$

The symbol '$\otimes$' stands for the Kronecker product of the matrices.

Considering (14), the error dynamics can be written as

$$\dot{\tilde{\mathcal{Y}}} = \bar{A}_p\tilde{\mathcal{Y}} + \bar{b}\tilde{\xi} = \bar{A}_p\tilde{\mathcal{Y}} + \bar{b}L^{-1}s \tag{15}$$

Lemma 3: For some constant time T_1, define $s_{i,\max} = \sup_{0\le\tau\le t}|s_i(t)|$, $\beta_{s_i} = \sup_{t>T_1|s_i(t)|}$ and $s_{\max,i}(t) = \max_i \sup_{0\le\tau\le t}|s_i(t)|$, then the following equations hold

$$\|\tilde{\mathcal{Y}}(t)\| \le k_0 e^{-\lambda_0 t}\|\tilde{\mathcal{Y}}(0)\| + \frac{k_0}{\lambda_0}\left[N\lambda_{\max}(L^{-1})\right]s_{\max,i}(t)$$

and

$$\|\tilde{\mathcal{Y}}(t)\| \le k_0 e^{-\lambda_0 t}\left(\|\tilde{\mathcal{Y}}(0)\| + \frac{e^{\lambda_0 T_1}}{\lambda_0}\beta_s(T_1)\right) + \frac{k_0}{\lambda_0}\beta_{s_T}$$

where $\beta_s(t) = N\lambda_{\max}(L^{-1})s_{\max,i}(t)$ and $\beta_{s_T} = N\lambda_{\max}(L^{-1})\sup_{T_1\le t}s_{\max,i}(t)$ with constants $\lambda_0 > 0$ and $k_0 > 0$.

Proof: From (15) and A_p is Hurwiz, we have $\tilde{\mathcal{Y}}(t) = \tilde{\mathcal{Y}}(0)e^{\bar{A}_p t} + \int_0^t e^{\bar{A}_p(t-\tau)}\bar{b}L^{-1}s\,d\tau$, and $\|e^{\bar{A}_p t}\| \le k_0 e^{-\lambda_0 t}$

$$\begin{aligned}&\|\tilde{\mathcal{Y}}(t)\|\\ &\le k_0 e^{-\lambda_0 t}\|\tilde{\mathcal{Y}}(0)\| + \int_0^t e^{-\lambda_0(t-\tau)}\left\|\bar{b}L^{-1}s\right\|\,d\tau\\ &\le k_0 e^{-\lambda_0 t}\|\tilde{\mathcal{Y}}(0)\| + k_0 e^{-\lambda_0 t}\left[N\lambda_{\max}(L^{-1})\right]s_{\max,i}(t)\frac{e^{\lambda_0 t}-1}{\lambda_0}\\ &\le k_0 e^{-\lambda_0 t}\|\tilde{\mathcal{Y}}(0)\| + \frac{k_0}{\lambda_0}\left[N\lambda_{\max}(L^{-1})\right]s_{\max,i}(t)\end{aligned}\tag{16}$$

where $\lambda_{\max}(\cdot)$ is the maximum eigenvalue of the matrix.

Noting the above equation and that

$$\begin{aligned}\int_0^t e^{-\lambda_0(t-\tau)}\left\|\bar{b}(L^{-1}s)\right\|\,d\tau &= \int_0^{T_1} e^{-\lambda_0(t-\tau)}\left\|\bar{b}(L^{-1}s)\right\|\,d\tau\\ &\quad + \int_{T_1}^t e^{-\lambda_0(t-\tau)}\left\|\bar{b}(L^{-1}s)\right\|\,d\tau\end{aligned}\tag{17}$$

we have (16) as follows

$$\begin{aligned}\|\tilde{\mathcal{Y}}(t)\| &\le k_0 e^{-\lambda_0 t}\|\tilde{\mathcal{Y}}(0)\| + k_0 e^{-\lambda_0 t}\frac{e^{\lambda_0 T_1}-1}{\lambda_0}\beta_s(T_1)\\ &\quad + k_0 e^{-\lambda_0 t}\frac{e^{\lambda_0 t_0} - e^{\lambda_0 T_1}}{\lambda_0}\beta_{s_T}\\ &\le k_0 e^{-\lambda_0 t}\left(\|\tilde{\mathcal{Y}}(0)\| + \frac{e^{\lambda_0 T_1}}{\lambda_0}\beta_s(T_1)\right) + \frac{k_0}{\lambda_0}\beta_{s_T}\end{aligned}\tag{18}$$

This completes the proof. □

Now we will show that for a proper choice of the control parameters, the trajectories of each agent do remain in the compact set. From the fact that $L^{-1}s = ([\Lambda^{\mathrm{T}}\ 1] \otimes I_{N+1})\tilde{\bar{X}}$, where $\tilde{\bar{X}} = [\tilde{X}^{\mathrm{T}}\ \tilde{x}_\rho^{\mathrm{T}}]^{\mathrm{T}}$, we can see that $\tilde{x}_\rho = L^{-1}s - (\Lambda^{\mathrm{T}} \otimes I_N)\tilde{X}$. Therefore

$$\begin{aligned}\|\tilde{\bar{X}}\| &\le \|\tilde{X}\| + \|\tilde{x}_\rho\|\\ &\le (1 + \|\Lambda\|)\|\tilde{X}\| + \|L^{-1}\|\|s\|\\ &\le \left(1 + \|\Lambda\|\right)\|\tilde{\mathcal{Y}}\| + \lambda_{\max}(L^{-1})\|s\|\end{aligned}\tag{19}$$

It follows from (18) and the fact that s_i will converge to Ω_{si}, we know that $\|\tilde{\bar{X}}\| \le k_a\|\tilde{\mathcal{Y}}(0)\| + k_b\beta_{s_T} + k_c$, $\forall t \ge T_1$, with $k_a = (1 + \|\Lambda\|)k_0$, $k_b = (k_a/\lambda_0) + 1$ and $k_c = k_a(e^{\lambda_0 T_1}/\lambda_0)\beta_s(T_1)$, Hence

$$\begin{aligned}\|\bar{X}(t)\| &\le \|\tilde{\bar{X}}(t)\| + \|\bar{X}_d(t)\mathbf{1}\|\\ &\le k_a\|\tilde{\mathcal{Y}}(0)\| + k_b\beta_{s_T} + k_c + c,\ \forall t \ge T_1\end{aligned}\tag{20}$$

We now provide the conditions that guarantee $\bar{X} \in \Omega_{\bar{X}}$, $\forall t \ge 0$. Define the compact set

$$\begin{aligned}\Omega_0 := \Big\{\bar{X}(0)\Big|\{\bar{X}|\|\bar{X}(t)\| < k_a\|\tilde{\mathcal{Y}}(0)\|\} \subset \Omega_{\bar{X}},\\ \lambda_{\max}(L^{-1})\|s(0)\| < \beta_{s_T}\Big\}\end{aligned}\tag{21}$$

the positive constant $c^* := \sup_{c\in\mathbb{R}^+}\left\{c\,\middle|\,\left\{\bar{X}|\|\bar{X}\| < k_a\|\tilde{\mathcal{Y}}(0)\| + k_c + c,\ \bar{X}(0) \in \Omega_0\right\} \subset \Omega_{\bar{X}}\right\}$.

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Theorem 2: Consider a group of agents with dynamics (1) and the extended communication graph containing a spanning tree, where the virtual agent is the root, under Assumptions 1–4, control (11) and parameters update law (12) for each agent. For initial conditions $\bar{X}(0), \eta(0), \tilde{\theta}_i(0)$ and $\tilde{\varphi}_i(0)$ starting in any compact set, all closed-loop signals of the system are semiglobal uniform bounded, and the total tracking error of the agents $\tilde{\bar{X}}$ converges to a neighbourhood of the origin.

Proof: From (20), we know that the overall system state $\bar{X}(t)$ will stay in $\Omega_{\bar{X}}$ for all time. Furthermore, since the NN weights have been proven bounded for any bounded $\hat{\theta}_i(0)$ and $\hat{\varphi}_i(0)$, and from Lemma 2, it can be seen that η_i is bound if x_i is bounded. As a result, the states of the internal dynamics of the agent will converges to the compact set $\Omega_{\eta_i} = \{\eta_i \in \mathbb{R}^p | \, \|\eta_i\| \le a_1(\sqrt{2c_2/c_1} + \|X_d\|) + a_2\}$, where $a_1 = \lambda_b a_x/\lambda_a$ and $a_2 = \lambda_b a_q/\lambda_a$ are positive constants. As the control signal $u_i(t)$ is a function of the weights $\hat{\theta}_i$ and $\hat{\varphi}_i$, the states η_i, x_i and the filtered tracking error s_i, we know that it is also bounded. Therefore we have all the closed-loop signals are semi-global uniform bound. This completes the proof. □

4.2 Control design with partial information

In this section, we assume that each agent can only access its neighbours' output information y_{ir}. With the adaptive NN controller designed in Section 4.1, high-gain observer is synthesised and augmented into the controlled system to estimate the other required states for control design.

In the following lemma, high-gain observer [28] is presented, which will be used to estimate the unkown states.

Lemma 4 [27]: Consider the following linear system

$$\begin{aligned} \epsilon\dot{\pi}_i &= \pi_{i+1}, \quad i = 1, 2, \ldots, \rho - 1 \\ \epsilon\dot{\pi}_\rho &= -\bar{\gamma}_1\pi_\rho - \bar{\gamma}_2\pi_{\rho-1} - \ldots - \bar{\gamma}_{\rho-1}\pi_2 - \pi_1 + \chi(t) \end{aligned} \tag{22}$$

where ϵ is a small positive constant and the parameters $\bar{\gamma}_1$ to $\bar{\gamma}_{\rho-1}$ are chosen such that the polynomial $s^\rho + \bar{\gamma}_1 s^{\rho-1} + \cdots + \bar{\gamma}_{\rho-1}s + 1$ is Hurwitz. Suppose the states $\chi(t)$ and its first n derivatives are bounded, so that $\chi^{(k)} < \varpi_k$ with positive constants ϖ_k. Then the following property holds

$$\tilde{\chi}^{(k)} := \pi_k/\epsilon^{k-1} - \chi^{(k)} = -\epsilon\zeta^{(k)}, \quad k = 1, 2, \ldots, \rho \tag{23}$$

where $\zeta := \pi_p + \bar{\gamma}_1\pi_{\rho-1} + \cdots + \bar{\gamma}_{\rho-1}\pi_1$ and $\zeta^{(k)}$ denotes the kth derivative of ζ. Furthermore, there exist positive constants h_k and t^* such that for all $t > t^*$ we have $|\zeta^{(k)}| \le \epsilon h_k$, $k = 2, 3, \ldots, \rho$.

Note that π_{k+1}/ϵ^k asymptotically converges to $\zeta^{(k)}$, with a small time constant provided that ζ and its k derivatives are bounded. Hence, π_{k+1}/ϵ^k is a suitable observer to estimate the output derivatives up to the ρth order.

To prevent peaking [29], saturation functions are used on the observer signals whenever they are outside the domain of interest Ω as $\pi^s_{i,j} = \bar{\pi}_{i,j}\phi\left(\pi_{i,j}/\bar{\pi}_{i,j}\right)$, $\bar{\pi}_{i,j} \ge \max_{(\tilde{y}_i, s_i, \tilde{\theta}_i, \tilde{\varphi}_i)\in\Omega}(\pi_{i,j})$, where $\phi(a) = -1$ while $a < -1$, $\phi(a) = a$ while $|a| < 1$, and $\phi(a) = 1$ while $a > 1$.

Now we revisit the control law (11) and adaption law (12). From the certainty equivalence approach, we modify them by replacing the reference signal $y^{(k)}_{ir}, k = 1, \ldots, \rho$ which depends on the neighbours' output with estimates

$$\hat{y}^{(k)}_{ir} = \frac{\pi_{i,k}}{\epsilon^k}, \quad i = 2, \ldots, N \text{ and } k = 1, \ldots, \rho \tag{24}$$

Then, $\hat{z}_{i,k} = x_{i,k} - \hat{y}^{(k)}_{ir}$ and $\hat{s}_i = [\Lambda^{\mathrm{T}} \quad 1]\hat{\bar{z}}_i$, where $\hat{\bar{z}}_i = [z_{i,1}, \hat{z}_{i,2}, \ldots, \hat{z}_{i,\rho}]^{\mathrm{T}}$. $\tilde{z}_{i,k} = y^{(k-1)}_i - \hat{y}^{(k-1)}_i = \epsilon\chi^{(k-1)}_i$ and $\tilde{y}^{(\rho)}_{ir} = \hat{y}^{(\rho)}_{ir} - y^{(\rho)}_{ir}$.

From the certainty equivalence approach, we modify the control law (11) and adaption laws (12) by replacing the partially available quantities with their estimates, which can be written as

$$u_i = -\hat{\theta}^{\mathrm{T}}_i\psi_i(\hat{Z}_i) - k_i\hat{s}_i - \frac{1}{2}\hat{\varphi}_i\hat{s}_i, \quad i = 1, \ldots, N \tag{25}$$

The update law of parameters are designed as

$$\begin{aligned} \dot{\hat{\varphi}}_i &= -\gamma_1\left[-\frac{1}{2}(1 - \varpi_\varphi)\hat{s}^2_i + \sigma_1\hat{\varphi}_i\right] \\ \dot{\hat{\theta}}_i &= -\gamma_2\left(-\psi_i\hat{s}_i + \sigma_2\hat{\theta}_i\right) \end{aligned} \tag{26}$$

where γ_1, γ_2, σ_1 and σ_2 are positive constants, and $\varpi_{\varphi_i} = 0$ while $|\hat{\varphi}_i| \le M_{\varphi_i}$, otherwise $\varpi_{\varphi_i} = 1$, where M_{φ_i} is a designed positive constant. Consider Lyapunov function candidate

$$V_{ie} = \frac{1}{2}s^2_i + \frac{1}{2\gamma_2}\tilde{\theta}^{\mathrm{T}}_i\tilde{\theta}_i + \frac{1}{2\gamma_1}\tilde{\varphi}^2_i \tag{27}$$

The following lemma is useful for handling the terms containing the estimation errors.

Lemma 5 [27]: There exist positive constants F_{ik} which are independent of ϵ_i, such that for $t > t^*$, the estimate $\hat{y}^{(k)}_{ir}$, $i = 1, \ldots, N$, $k = 1, \ldots, \rho$, and satisfy the following inequalities: $|\tilde{y}^{(k)}_{ir}| = |\hat{y}^{(k)}_{ir} - y^{(k)}_{ir}| \le \epsilon_i F_{ik}$.

Since s_i is the linear combination of Y_i and Y_j, $j \in \mathcal{N}_i$, we know that there exist positive constants G_{is} which are independent of ϵ_i satisfy $|\tilde{s}_i| \le \epsilon_i G_{is}$. Taking the time derivative of V_i along the closed-loop trajectory and using the property $\psi_i(\hat{Z}_i) - \psi_i(Z_i) = \epsilon_i\psi_{ti}$, where ψ_{ti} is a bounded vector function [23], we have

$$\begin{aligned} \dot{V}_{ie} &= -\left(\frac{\dot{g}_i}{2g^2_i} + g_0\right)s^2_i - k_i s^2_i - k_i s_i\tilde{s}_i - s_i\hat{\theta}^{\mathrm{T}}_i\psi_i(\hat{Z}_i) \\ &\quad - \frac{1}{2}\hat{\varphi}_i s_i\hat{s}_i + s_i(d_i + \bar{\varepsilon}_i) + s_i\theta^{*\mathrm{T}}\psi_i(Z_i) + \frac{1}{\gamma_2}\tilde{\theta}_i\dot{\hat{\theta}}_i + \frac{1}{\gamma_1}\tilde{\varphi}_i\dot{\hat{\varphi}}_i \\ &\le -\frac{k_i}{2}(s^2_i + \tilde{s}^2_i) + \frac{1}{2}\left(-\hat{\varphi}_i s_i\hat{s}_i + \varphi_i s^2_i + \tilde{\varphi}_i\hat{s}^2_i\right) - s_i\hat{\theta}^{\mathrm{T}}_i\psi_i(\hat{Z}_i) \\ &\quad + s_i\theta^{*\mathrm{T}}\psi_i(Z_i) + \hat{s}_i\tilde{\theta}^{\mathrm{T}}_i\psi_i(\hat{Z}_i) - \sigma_2\tilde{\theta}^{\mathrm{T}}_i\hat{\theta}_i - \sigma_1\tilde{\varphi}_i\hat{\varphi}_i + \frac{1}{2} \end{aligned}$$

By using Young's inequations and Lemma 1, after long heavy but direct computation, we arrive at $-\hat{\varphi}_i s_i\hat{s}_i + \varphi_i s^2_i + \tilde{\varphi}_i\hat{s}^2_i \le s^2_i + \epsilon^2_i G^2_{is}(\hat{\varphi}^2_i + \frac{1}{2}\varphi^2_i + 1)$, and $-s_i\hat{\theta}^{\mathrm{T}}_i\psi_i(\hat{Z}_i) + s_i\theta^{*\mathrm{T}}\psi_i(Z_i) + \hat{s}_i\tilde{\theta}^{\mathrm{T}}_i\psi_i(\hat{Z}_i) \le \frac{1}{2}\tilde{\theta}^{\mathrm{T}}_i\tilde{\theta}_i + \frac{1}{2}\epsilon^2_i G^2_{is}m^{*2}_\psi + \frac{1}{2}s^2_i + \frac{1}{2}\epsilon^2_i \|\psi_{ti}\|^2\|\theta^*_i\|^2$.

Then the time derivation of V_{ie} can be written as

$$\dot{V}_{ie} \le -\frac{1}{2}(k_i - 2)s^2_i - \frac{\sigma_2 - 1}{2}\tilde{\theta}^{\mathrm{T}}_i\tilde{\theta}_i - \frac{\sigma_1}{2}\tilde{\varphi}^2_i + c_{2ie} \tag{28}$$

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with constant $c_{2ie} = \frac{1}{2}\epsilon_i^2 G_{is}^2(m_{\varphi}^{*2}+1) + (\epsilon_i^2\|\psi_{ti}\|^2+\sigma_2)/2\|\theta_i^*\|^2 + (\frac{3}{4}\epsilon_i^2 G_{is}^2 + (\sigma_1/2))\varphi_i^{*2} + (k_i/2)\epsilon_i^2 + \frac{1}{2}$.

Now, we choose $k_i > 2$, $\sigma_2 > 1$ and define

$$\Omega_{eie} = \left\{(s_i, \tilde{\theta}_i, \tilde{\varphi}_i) \,|\, \frac{1}{2}(k_i - 2)s_i^2 + \frac{\sigma_2 - 1}{2}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i + \frac{\sigma_1}{2}\tilde{\varphi}_i^2 \le c_{2ie}\right\}$$

$$\Omega_{sie} = \left\{s_i \,|\, |s_i| \le \sqrt{2c_{2ie}/(k_i - 2)}\right\}$$

and

$$\Omega_{\theta_{ie}} = \left\{(\tilde{\theta}_i, \tilde{\varphi}_i) \,|\, \|\tilde{\theta}_i\| \le \sqrt{2c_{2ie}/(\sigma_2 - 1)}, |\tilde{\varphi}_i| \le \sqrt{2c_{2ie}/\sigma_1}\right\}$$

Since c_{2ie}, σ_1, $\sigma_2 - 1$, and $k_i - 2$ are all positive constants, we know that Ω_{sie}, $\Omega_{\theta_i e}$ and Ω_{eie} are compact sets. Equation (28) shows that $\dot{V}_{ie} \le 0$ once the errors are outside the compact set Ω_{ei}. According to the standard Lyapunov theorem, we conclude that s_i, $\tilde{\theta}_i$ and $\tilde{\varphi}_i$ are bounded. From (28), it can be seen that V_{ie} is strictly negative as long as s_i is outside the compact set Ω_{sie}. Therefore there exists a constant T_1 such that for $t > T_1$, the filtered tracking error s_i converges to Ω_{sie}, that is to say, $s_i \le \beta_{sie}$, with $\beta_{sie}(k_i, \gamma_1, \gamma_2, \sigma_1, \sigma_2, \theta_i^*, \varphi_i^*, \epsilon_i) = \sqrt{2c_{2ie}/(k_i - 2)}$.

Theorem 3: Consider a group of agents with dynamics (1) and the extended communication graph containing a spanning tree with the virtual agent as the root, under Assumptions 1–4, the control law (25), parameters update law (26), and the high-gain observer (22), which is turned on at time t^* in advance. For initial conditions $\bar{X}(0)$, $\eta(0)$, $\tilde{\theta}_i(0)$ and $\tilde{\varphi}_i(0)$ staring from any compact set, and the desired trajectory with its derivations up to ρth is bounded, then all closed-loop signals of the system are semi-global uniform bounded, and the total tracking error of the agents $\tilde{\bar{X}}$ converges to a neighbourhood of origin.

Proof: We have concluded that s_i converges to a compact set Ω_{sie}, then following Lemma 3, it can be concluded that $\|\tilde{\mathcal{Y}}\| \le k_0 e^{-\lambda_0 t}(\|\tilde{\mathcal{Y}}(0)\| + (e^{\lambda_0 T_1}/\lambda_0)\beta_s(T_1)) + (k_0/\lambda_0)\beta_{s_T}$, and from (20), we can find $\|\tilde{\bar{X}}\|$ is also bounded. Following the same procedure in the full-state feedback control, we can complete the proof. □

Remark 5: In this paper, we proposed a rigorous theoretical treatment of the output feedback problem for multiple agents tracking using high-gain observers, assuming that measurements are perfect. The rationale for choosing a high-gain observer lies in its simplicity and the fact that it does not require the model of the agents or disturbances, in line with the proposed non-model-based control method. There is a tradeoff between the speed of state reconstruction and the immunity to measurement noise. The value of the parameter ϵ is usually detuned small enough such that the convergence speed of the observer is much faster than the targeted convergence speeds of states of the controlled system, resulting in assurance of the expected control performance by the adaptive NN control designed in Section 4.1, where no observer is assumed. On the other hand, note that the smallness of ϵ in fact leads to large values of $1/\epsilon, \ldots, 1/\epsilon^{(\rho-1)}$, explaining why the observer is named the high-gain observer. Such high-gains may excite unmodelled high-frequency dynamics and amplify measurement noise, and might not be implementable in practice. In practice, the presence of measurement noise necessitates careful implementation, and places a lower limit on the size of the parameter ϵ in the observer (22), with possible degradation of transient performance.

Fig. 2 *Extended communication graph in the numerical example*

5 Simulation studies

In this section, two simulation examples are presented to demonstrate the effectiveness of the proposed synchronised tracking controller. It is noted that RBF neural networks are used to approximate the unknown functions in this work. In practice, the selection of the NN parameters, that is, the centres and widths of RBF, has a great influence on the performance of the designed controller. According to [30], Gaussian RBF NNs arranged on a regular lattice on $\mathbb{R}^{|Z_i|}$, where $|Z_i|$ denotes the dimension of Z_i, can uniformly approximate sufficiently smooth functions on closed bounded subsets. Accordingly, in the following simulation studies, the centres and widths are chosen on a regular lattice in the respective compact sets. The details of the NN parameters of each simulation example will be given independently.

5.1 Numerical example

Consider a network of four agents described in Fig. 2, and the agent dynamics has relative degree 3 given by

$$\begin{aligned} &\dot{x}_1 = x_2,\ \dot{x}_2 = x_3 \\ &\dot{x}_3 = 3\eta\cos x_1 - x_3\sin x_2 + x_2 x_3 + (2 - \sin\eta)(u + d) \\ &\dot{\eta} = -\eta + x_3\cos(x_1 + x_2) \\ &y = x_1 \end{aligned} \tag{29}$$

As shown in Fig. 2, the network has the leader–follower structure, where agent 1 can their access the desired trajectory and the others follow their precursors, respectively.

The desired trajectory y_d is generated by $y_d = 8/(s^3 + 6s^2 + 12s + 8)y_{\mathrm{ref}}$, where

$$y_{\mathrm{ref}}(t) = \begin{cases} 0 & \text{if } 0 \le t \le 10 \\ 1 & \text{if } 10 < t \le 20 \\ 0 & \text{if } 20 < t \le 30 \\ 1 & \text{if } t > 30 \end{cases} \tag{30}$$

In the simulation, both the non-linear item, $f(\eta, x) = 3\eta\cos x_1 - x_3\sin x_2 + x_2 x_3$, and the open-loop control gain, $g(\eta, x) = (2 - \sin\eta)$, are considered as unknown. The external disturbance in the input channel of each agent is generated as $d_i(t) = 0.1\sin(\pi t/2i)$, $i = 1, \ldots, 4$. The control and observer design parameters, and initial conditions are chosen as $\Lambda = [2, 3]^{\mathrm{T}}$, $k_i = 3$, $\epsilon_i = 0.004$, $i = 1, \ldots, 4$, $\bar{\gamma}_1 = \bar{\gamma}_2 = 4$, $\bar{\pi}_2 = \bar{\pi}_3 = 4$, $x_1(0) = [1.5, 0, 0, 0]^{\mathrm{T}}$, $x_2(0) = [-0.4, 0, 0, 0]^{\mathrm{T}}$, $x_3(0) = [0.2, 0, 0, 0]^{\mathrm{T}}$, $x_4(0) = [-0.6, 0, 0, 0]^{\mathrm{T}}$, $\hat{\theta}_i(0) = 0$ and $\hat{\psi}_i(0) = 0$. The saturation limits of the control are ± 20. In this example, there are eight inputs of the NN for each agent, $x_{i,k}$, $z_{i,k}$, $k = 1, 2, 3$, η_i and $y_{ir}^{(3)}$, and we use two nodes for each input dimension of $\theta_i^{\mathrm{T}}\psi(Z_i)$; thus we end up with 256 nodes (i.e. $l = 256$) with centres $\mu_k = 1.0$, $k = 1, \ldots, l$,

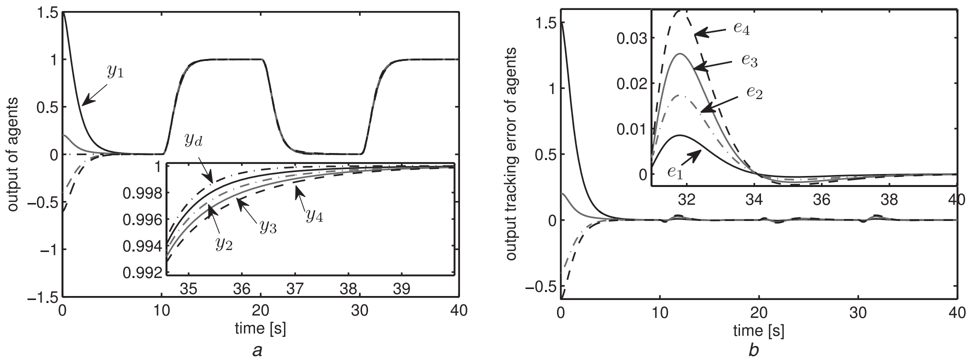

Fig. 3 *Output and tracking error of each agent (output feedback)*
a Output of each agent
b Output tracking error of each agent

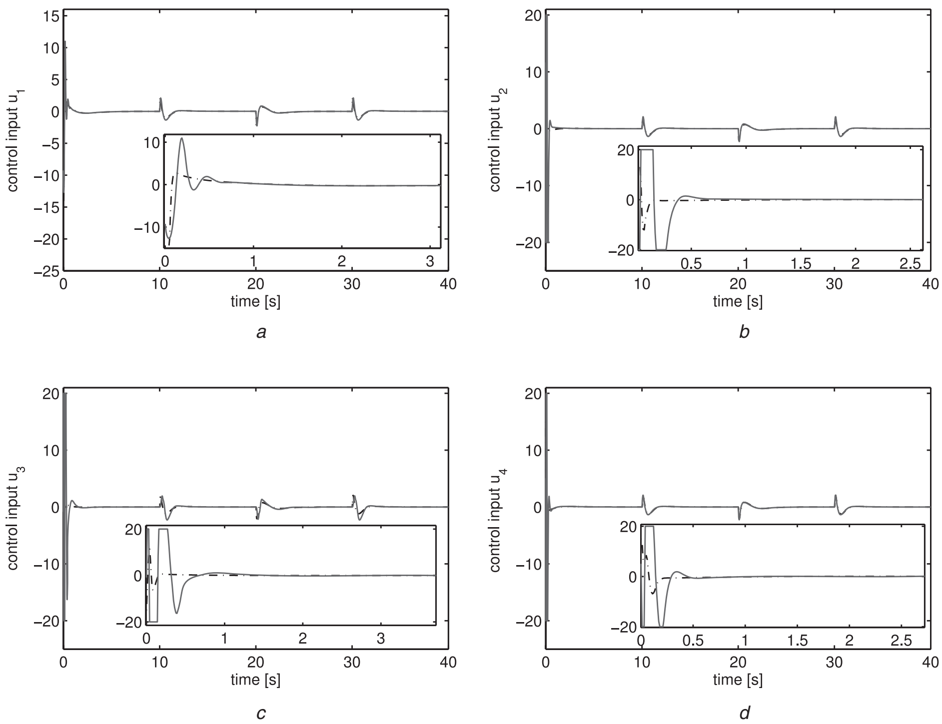

Fig. 4 *Control input of each agent under full-state (dash-dot) and output (solid) feedback control*
a Control input of agent 1
b Control input of agent 2
c Control input of agent 3
d Control input of agent 4

evenly spaced in $[-3.0, 3.0] \times [-3.0, 3.0] \times [-3.0, 3.0] \times [-3.0, 3.0] \times [-3.0, 3.0] \times [-3.0, 3.0] \times [-3.0, 3.0] \times [-3.0, 3.0]$. The other NN control parameters are chosen as $\sigma_1 = 0.05$, $\gamma_1 = 1$, $\sigma_2 = 9 \times 10^{-4}$ and $\gamma_2 = 10^3$.

Simulation results are shown in Figs. 3–5. From Fig. 3, we find that good tracking performance is achieved and the tracking errors of all the agents converge to a small neighbourhood of zero although some of the agents cannot access the desired trajectory directly. The control inputs, and norm of the neural weights of the agents in both state feedback and output cases are bounded as shown in Fig. 5. We also find that agent 1 has the smallest tracking error e_1, and $e_{i+1} > e_i$, $i = 1, 2, 3$, as shown in Fig. 3, this is due to the fact that agent 1 follows the virtual agent directly and the others follows their precursors (leaders), respectively. The results reflect that there exists error propagation in the leader–follower pair of the agents.

5.2 Application to synchronised vertical flight of X-cell 50 helicopters

In this subsection, we adopted our algorithm to control six X-cell 50 helicopters in vertical flight, whose communication graph is shown in Fig. 1. The dynamics of the altitude tracking for the X-cell 50 helicopter can be written as follows [31]

$$\begin{aligned}
\dot{\zeta}_1 &= \zeta_2 \\
\dot{\zeta}_2 &= a_0 + a_1\zeta_2 + a_2\zeta_2^2 + (a_3 + a_4\zeta_4 - \sqrt{a_5 + a_6\zeta_4})\zeta_3^2 \\
\dot{\zeta}_3 &= a_7 + a_8\zeta_3 + (a_9 \sin\zeta_4 + a_{10})\zeta_3^2 + a_{\text{th}} \\
\dot{\zeta}_4 &= \zeta_5 \\
\dot{\zeta}_5 &= a_{11} + a_{12}\zeta_4 + a_{13}\zeta_3^2 \sin\zeta_4 + a_{14}\zeta_5 - K_1 u
\end{aligned} \tag{31}$$

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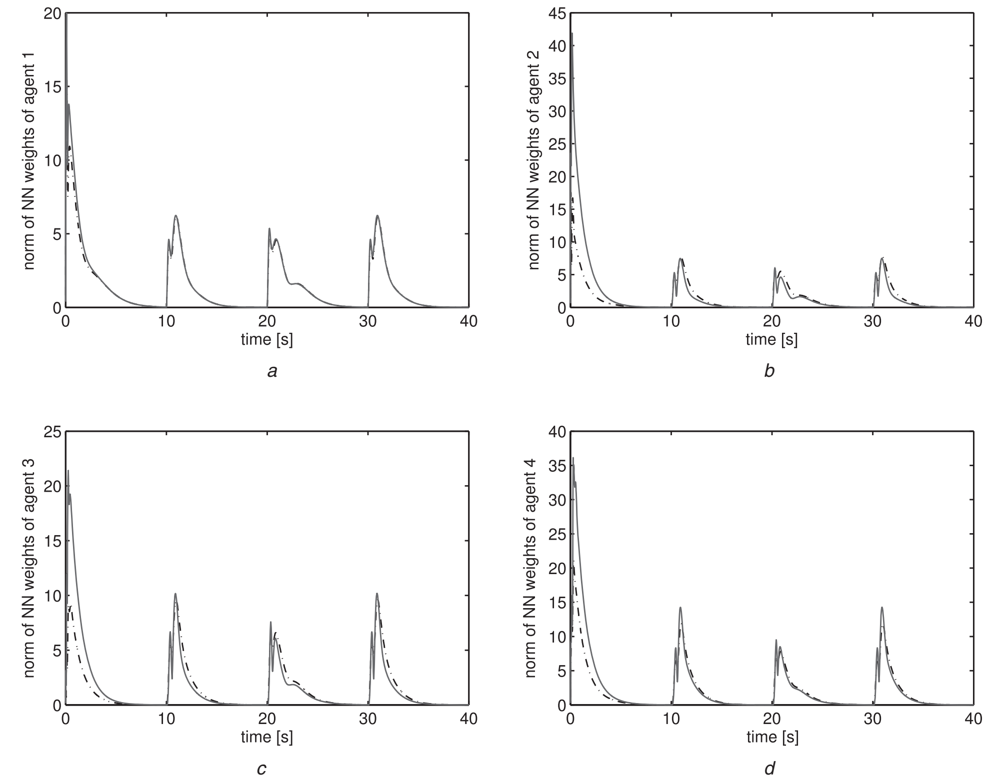

Fig. 5 *Norm of neural weights of agents under full-state (dash-dot) and output (solid) feedback control*
a Norm of neural weights, $\|\hat{\theta}_1\|$
b Norm of neural weights, $\|\hat{\theta}_2\|$
c Norm of neural weights, $\|\hat{\theta}_3\|$
d Norm of neural weights, $\|\hat{\theta}_4\|$

where ζ_1 denotes altitude (m), ζ_2 the altitude rate (m/s), ζ_3 the rotational speed of the rotor blades (rad/s), ζ_4 the collective pitch angle (rad), ζ_5 the collective pitch rate (rad/s), $a_{\text{th}} = 111.69\,\text{s}^{-2}$ the constant input to the throttle and u the input to the collective servomechanisms.

Let y be the altitude ζ_1. By restricting the throttle input to be constant, we obtain a SISO in which u is the only input variable forcing the output y to track a desired trajectory y_d, which is generated by

$$y_d = \frac{150.056}{s^4 + 12.6s^3 + 64.19s^2 + 154.35s + 150.056} h_{\text{ref}} \quad (32)$$

Similar to [32], we impose the take off time at $t_{\text{off}} = 5$ s, and define $h_{\text{ref}} = 0$, in case $0 \le t \le t_{\text{off}}$, $h_{\text{ref}} = -3[e^{-(t-t_{\text{off}})^2/20} - 1]$, in case $t_{\text{off}} < t \le t_a$, $h_{\text{ref}} = 4 - \cos[(t - t_a)/2]$, in case $t_a < t \le t_b$, otherwise, $h_{\text{ref}} = 3$, where $t_a = 18$ and $t_b = 4\pi + t_a$.

It can be shown that the system has strong relative degree 4. Through the coordinate transformation, we can arrive at the following x system

$$\begin{aligned} \dot{x}_i &= x_{i+1}, \quad i = 1, 2, 3 \\ \dot{x}_4 &= b(x) + g(x)u \end{aligned} \quad (33)$$

where $g(x) = -K_2\zeta_3^2\left(a_4 - a_6/2\sqrt{a_5 + a_6\zeta_4}\right)$.

The derivation of $b(x)$ in (33) is omitted, and we proceed to verify that the system indeed satisfy the assumptions supported in the control design. To verify Assumption 3, we first note, from a practical standpoint, that the collective pitch angle, ζ_4, is stricter with a range, typically from 0 to 0.44 rad [33]. It can be verified that the bracketed terms in $g(x)$ are virtually constant: they take values in the range $[1.4, 1.5] \times 10^{-3}$. Thus the control coefficient $g(x)$ is always negative. Together with the fact that rotor speed, ζ_3, is non-zero during flight, it can be concluded that there does not exist any control singularities or zero crossings of $g(x)$. Therefore the first part of Assumption 3 is satisfied. The second part of the assumption, that $g(x) > 0$, does not correspond to this example, there is no less of applicability of the theoretical results. The control is still valid under a simple change of sign. Lastly, it is not difficult to verify the existence of a function

$$\begin{aligned} g_0(x) = 2\Bigg(&|a_8| + |a_9 \sin\zeta_4 + a_{10}||\zeta_3| \\ &+ \frac{a_6(a_5 + a_6\zeta_4)^{-1.5}}{8\left[a_4 - 0.5a_6(a_5 + a_6\zeta_4)^{-0.5}\right]}\Bigg) \\ &> 0, \forall \zeta_3 \in \mathbb{R}^+, \quad \zeta_4 \in [0, 0.44] \end{aligned} \quad (34)$$

which fulfil Assumption 3. Note that this function needs not to be known; we only need to show its existence.

In the simulation, the agents are assigned different parameter values $a_1, \ldots, a_{14}$ with adding maximum $\pm 5\%$ tolerance to the nominal value [31], and the external disturbance in the input channel of the agents are generated to satisfy $|d_i(t)| \le 5$ m rad. Also, the sensor noise chosen for each agent is bounded by $\pm 5\%$ of the output. The control and observer parameters are chosen as $\Lambda = [64, 48, 12]^{\text{T}}$, $k_i = 3$, $\epsilon_i = 0.07$, $i = 1, \ldots, 6$, $\bar{\gamma}_1 = 4$, $\bar{\gamma}_2 = 6$, $\bar{\gamma}_3 = 4$, $\bar{\pi}_2 = 0.1$, $\bar{\pi}_3 = 0.15$ and $\bar{\pi}_4 = 0.025$. The saturation limits of the control are ± 430 m rad. The

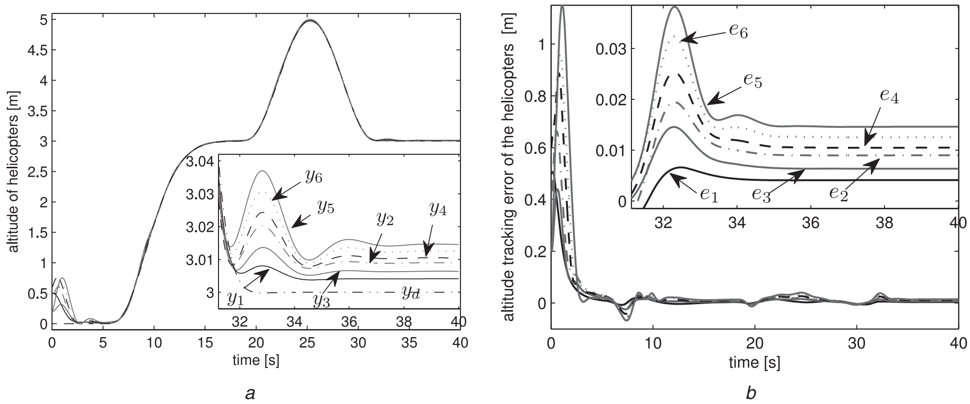

Fig. 6 *Altitude trajectory and error of each helicopter (output feedback)*
a Altitude trajectory of each helicopter
b Altitude tracking error of each helicopter

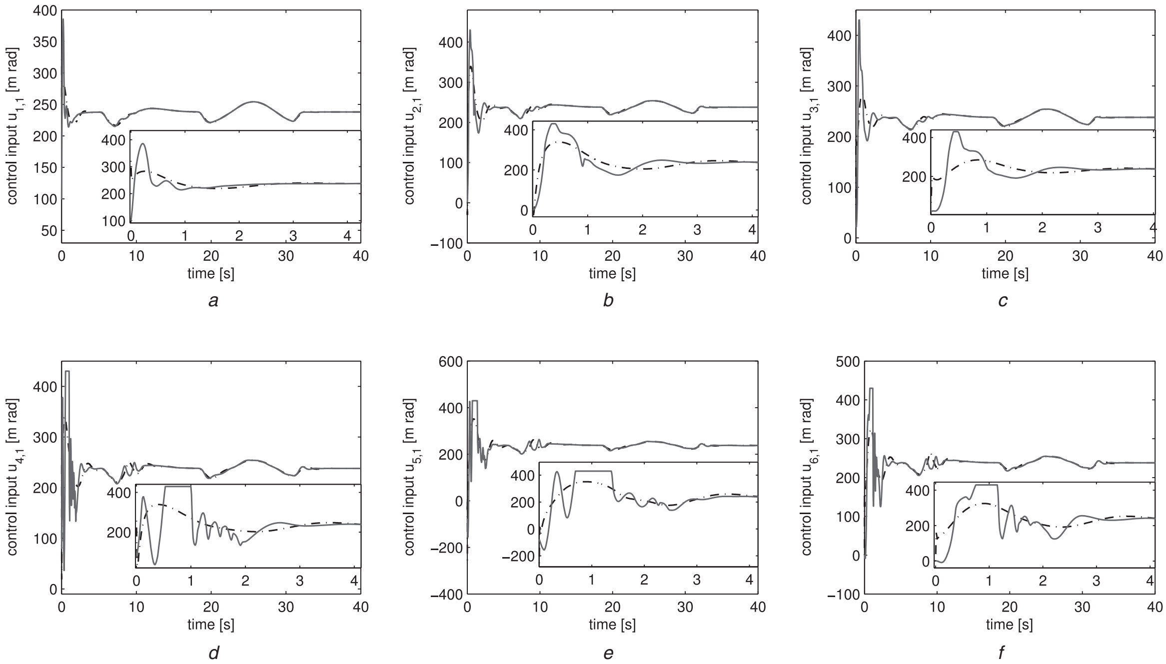

Fig. 7 *Control input of helicopters under full-state (dash-dot) and output (solid) feedback control*
a Control input of helicopter 1
b Control input of helicopter 2
c Control input of helicopter 3
d Control input of helicopter 4
e Control input of helicopter 5
f Control input of helicopter 6

initial conditions are $\zeta_1(0) = [0.5, 0.0, 95.3567, 0.222, 0.0]^{\mathrm{T}}$, $\zeta_2(0) = [0.4, 0.0, 95.3567, 0.3, 0.0]^{\mathrm{T}}$, $\zeta_3(0) = [0.2, 0.0, 95.4, 0.22, 0.0]^{\mathrm{T}}$, $\zeta_4(0) = [0.6, 0.0, 95.3567, 0.3, 0.0]^{\mathrm{T}}$, $\zeta_5(0) = [0.2, 0.0, 95.3567, 0.3, 0.0]^{\mathrm{T}}$, $\zeta_6(0) = [0.4, 0.0, 95.4, 0.24, 0.0]^{\mathrm{T}}$, $\hat{\theta}_i(0) = 0$ and $\hat{\varphi}_i(0) = 0$ for each agent.

In this application, there are ten inputs of the NN for each helicopter, $x_{i,k}$, $k = 1, \ldots, 5$, $z_{i,k}$, $k = 1, \ldots, 4$ and $y_{ir}^{(4)}$, and we use two nodes for each input dimension of $\theta_i^{\mathrm{T}} \psi(Z_i)$; thus we end up with 1024 nodes (i.e. $l = 1024$) with centres $\mu_k = 1.0$, $k = 1, \ldots, l$, evenly spaced in $[-8.0, 8.0] \times [-8.0, 8.0] \times [-8.0, 8.0] \times [-8.0, 8.0] \times [-8.0, 8.0] \times [-8.0, 8.0] \times [-8.0, 8.0] \times [-8.0, 8.0] \times [-8.0, 8.0] \times [-8.0, 8.0]$. The other NN control parameters are chosen as $\sigma_1 = 0.05$, $\gamma_1 = 1$, $\sigma_2 = 0.01$ and $\gamma_2 = 100$.

Simulation results are shown in Figs. 6–9. From Fig. 6, which shows the altitudes and tracking errors of the helicopters in output feedback case, we can find that the initial output errors of all helicopters are sufficiently reduced and the altitude trajectories lie in close proximity of the desired trajectory. Helicopter 1 has the smallest error e_1, this is largely due to it can access the desired trajectory directly and its motion is not affected by any other agents. Output error of helicopter 3, e_3, is a little larger than e_1 at steady state as its motion is affected by the virtual agent as well as helicopter 2. The motion of helicopter 2 is affected by helicopters 1 and 3, both of which can access the desired trajectory, then e_2 is smaller than others excluding e_1 and e_3. The motion of helicopter 4 is purely affected by helicopter 3, whereas the motion of helicopter 6 is affected

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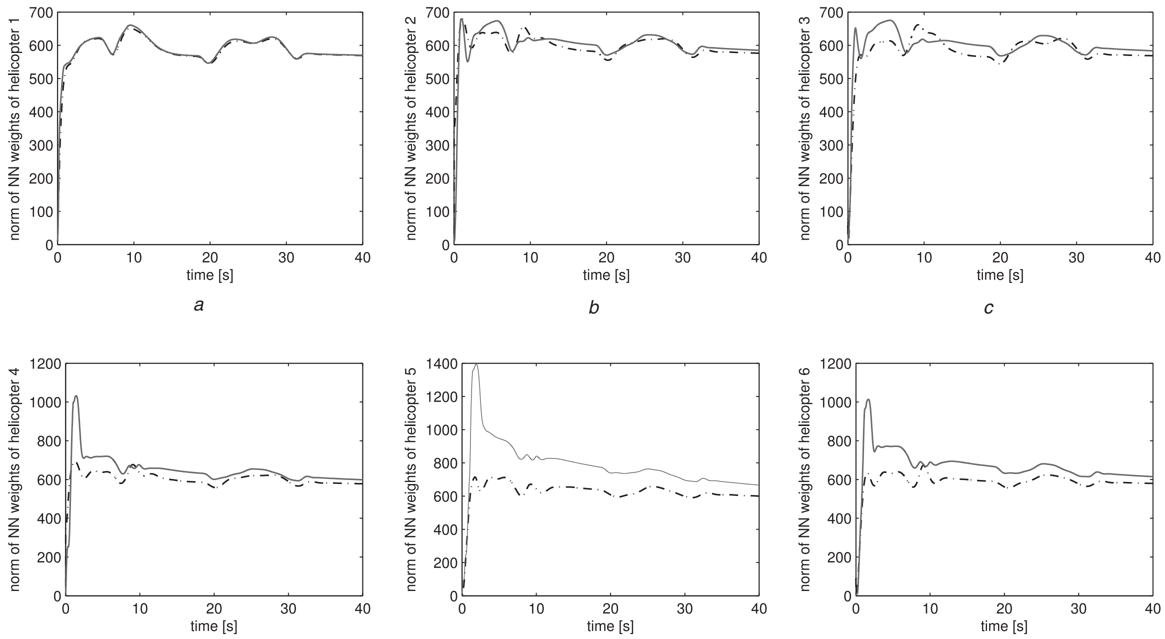

Fig. 8 *Norm of neural weights under full-state (dash-dot) and output (solid) feedback control*
a Norm of neural weights, $\|\hat{\theta}_1\|$
b Norm of neural weights, $\|\hat{\theta}_2\|$
c Norm of neural weights, $\|\hat{\theta}_3\|$
d Norm of neural weights, $\|\hat{\theta}_4\|$
e Norm of neural weights, $\|\hat{\theta}_5\|$
f Norm of neural weights, $\|\hat{\theta}_6\|$

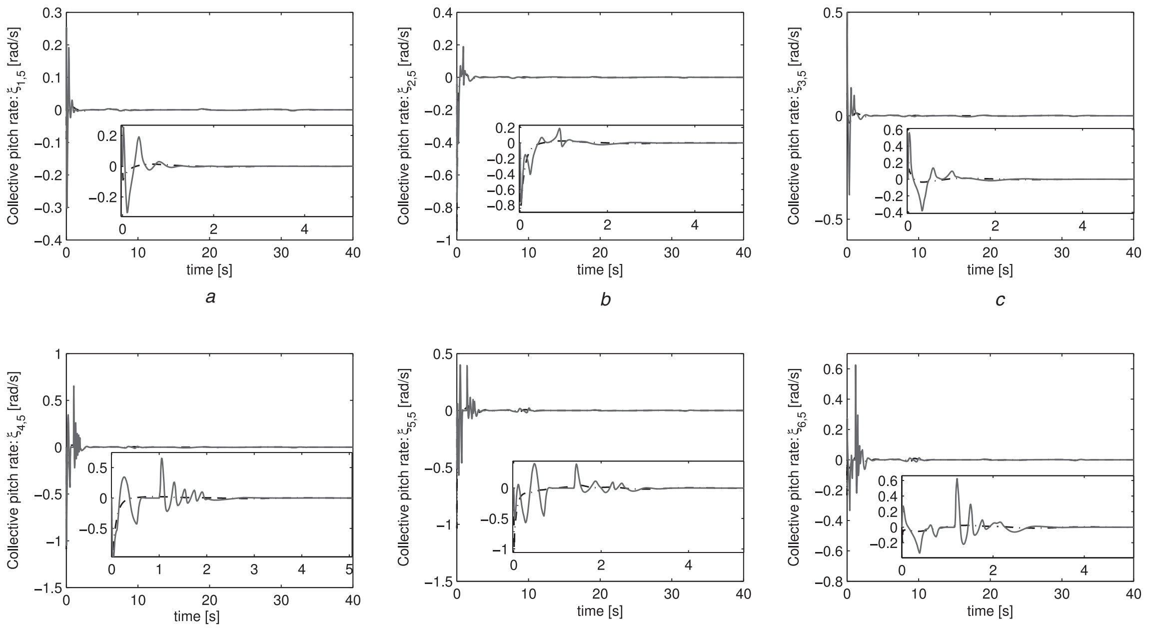

Fig. 9 *Collective pitch angle rate of each helicopter under full-state (dash-dot) and output (solid) feedback control*
a Collective pitch angle rate of helicopter 1
b Collective pitch angle rate of helicopter 2
c Collective pitch angle rate of helicopter 3
d Collective pitch angle rate of helicopter 4
e Collective pitch angle rate of helicopter 5
f Collective pitch angle rate of helicopter 6

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by both helicopters 3 and 4, then e_4 is little smaller than e_6. Helicopter 5, which is furthest from the virtual agent in the extended communication graph, has the largest output error compared to other agents. The results reflect the fact that there exists error propagation in the control of multi-agent system as well.

It is noted that all the parameters in (31) will lie in $b(x)$ and $g(x)$ after coordinate transformation. In this problem, owing to the parametric uncertainties, both $b(x)$ and $g(x)$ are considered as unknown, the controller is shown to be able to compensate for the unmodelled uncertainties and robust to the sensor noise and the disturbances as shown in Fig. 6.

The control inputs of the agents are all bounded as shown in Fig. 7. The output feedback case exhibit control input fluctuations at the initial stage since the high-gain observer is used. It is noted that in the output feedback case, control input, u_3, u_4, u_5, u_6 all hit the saturation limit of control input at the initial stage. This is because at this stage, the observer is in the transient stage and does not stabilised, consequently, adequate control effort should be supplied. The norm of the NN parameters and the internal states of all the agents are bounded as shown in Figs. 8 and 9.

6 Conclusion

In this paper, we have studied the synchronised tracking problem of multiple agents. Under the condition that the Laplacian matrix of the extended communication graph contains a spanning tree with the virtual agent as the root, by using the weighted average of neighbours' states as the reference signal, adaptive NN tracking control law has been designed for each agent. It has been shown that the tracking error of each agent converges to an adjustable neighbourhood of origin although the desired trajectory is not available to some of them. Simulation results have shown the effectiveness of the proposed methods.

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