

# Robust Control for a Class of Nonaffine Nonlinear Systems Based on the Uncertainty and Disturbance Estimator

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**Abstract**—In this paper, the uncertainty and disturbance estimator (UDE)-based robust control is applied to the control of a class of nonaffine nonlinear systems. This class of systems is very general and covers a large range of nonlinear systems. However, the control of such systems is very challenging because the input variables are not expressed in an affine form, which leads to the failure of using feedback linearization. The proposed UDE-based control method avoids the inverse operator construction, which might result in the control singularity problem. Moreover, the general assumption on the uncertainty and disturbance term is relaxed, and only its bandwidth information is required for the control design. The asymptotic stability of the closed-loop system is established. The proposed approach is easy to be implemented and tuned while bringing very good robust performance. The important features and performance of the proposed approach are demonstrated through both simulation studies and experimental validation on a servo system with nonaffine uncertainties.

**Index Terms**—Disturbance, nonaffine systems, nonlinear systems, robust control, uncertainty, uncertainty and disturbance estimator (UDE).

## I. INTRODUCTION

CONTROL of nonlinear systems using feedback linearization has been extensively studied for affine systems, of which the model is linear in input variables, and many significant developments have been achieved. In the early stage of the research, several important results on adaptive control have been developed for systems [1]–[4], where the nonlinearities are linearly parameterized with unknown parameters (structured uncertainties). To handle structured/unstructured uncertainties and/or reject external disturbances, some other control techniques have been also proposed, for example, adaptive robust control [5]–[7], sliding-mode control [8],

[9], disturbance-observer-based control (DOBC) [10]–[12], extended-state-observer-based control [13]–[15], active disturbance rejection control (ADRC) [16], [17], robust nonlinear predictive controller [18], [19], and equivalent-input-disturbance (EID) approach [20].

Many practical systems, e.g., chemical reactions and PH neutralization, are inherently nonlinear, and the input variables may not be expressed in an affine form. Indeed, the control of these nonaffine nonlinear systems is not only of practical interest but also academically challenging, because of the lack of mathematical tools. In fact, it is impossible to handle the control problem of the nonaffine nonlinear systems directly because, in general, even if it is known that the inverse exists, it is difficult to construct it analytically. For example, for systems with input hysteresis, even if the hysteresis model is available, it is quite difficult to find the inverse hysteresis model due to the complexity of the hysteresis characteristics [21], particularly because of the multivalued and nonsmoothness features. Consequently, no control system design is possible along the lines of conventional model-based control.

For control of nonaffine nonlinear systems, several researchers have suggested the use of neural networks (NNs) as emulators of inverse systems [22], [23]. The main idea is that for a system with finite relative degree, the mapping between a system input and the system output is one-to-one, thus allowing the construction of a left-inverse of the nonlinear system using NNs. Hence, if the controller is an inverse operator of the nonlinear system, the reference input to the controller will produce a control input to the plant, which will, in turn, produce an output identical to the reference input. Based on the implicit function theory, the NN control methods proposed in [24] and [25] have been used to emulate the inverse controller to achieve tracking control objectives [26].

In this paper, the uncertainty and disturbance estimator (UDE)-based robust control is developed for a general class of nonaffine nonlinear systems. The UDE control algorithm, which was proposed in [27] as a replacement of the time-delay control in [28], is based on the assumption that an engineering signal can be approximated and estimated by using a filter with the appropriate bandwidth. The UDE-based control does not require a completely known system model or a disturbance model and is robust against structured/unstructured uncertainties (e.g., modeling error and parameter variation) and external disturbances. The UDE-based robust control has demonstrated

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its excellent performance in handling uncertainties and disturbances yet a simple control scheme and has been successfully applied to robust input–output linearization [29], [30] and combined with sliding-mode control [31], [32], and further extended to uncertain systems with state delays, for both linear systems [33] and nonlinear systems [34], recently in variable-speed wind turbines [35]. The two-degree-of-freedom nature of the UDE-based control has been revealed in [36], which enables the decoupled design of the reference model and the filter. The major contribution of this paper is to apply this control strategy to a general class of nonaffine nonlinear systems, establish the asymptotic stability criteria, and validate it with simulation and experimental results. It can be shown that the asymptotic stability of the closed-loop system is established when the UDE is chosen appropriately under very mild conditions. The most important features of the approach are as follows: 1) Instead of constructing an inverse operator which might cause the controller singularity problem, the employment of UDE makes it possible to estimate the lumped uncertain term, which is a function of control inputs, states, and disturbances; 2) it relaxes the general assumption on the uncertainty and disturbance term, and only its bandwidth information is required for the control design; 3) it is easy to be implemented and tuned while bringing good robust performance.

Strictly speaking, the aforementioned ADRC, DOBC, and EID approaches are all very powerful and might be applied to this challenging problem as well. The reasons why the UDE-based control strategy is chosen in this paper are highlighted as follows.

- The ADRC [16] takes the total disturbance as an additional state variable and employs the extended state observer and nonlinear feedback to handle disturbance estimation and rejection. However, the boundedness of the derivative of the disturbance is required in the ADRC design as it is rigorously proved in [37]. The UDE-based control strategy does not need this assumption on the disturbance and only requires the bandwidth of the disturbance for the filter design.
- The DOBC [10] and the UDE-based control are similar in the sense that the disturbance is extracted from the system dynamics and then estimated via a low-pass filter. However, the frequency-domain DOBC requires the nominal plant model and involves taking the inverse of the plant model, whereas the UDE does not; hence, the UDE-based control is applicable to a wider class of systems and is easy to be implemented and tuned while bringing very good robust performance.
- The EID approach [20] requires that the disturbance be applied at or interpreted to the input channel, whereas the UDE-based control does not have this restriction.

The rest of this paper is organized as follows. Section II formulates the problem. In Section III, a UDE-based controller is constructed for nonlinear nonaffine systems, and the stability of the closed-loop system is established. The effectiveness of the proposed approach is demonstrated through simulation results in Section IV for an unstable nonaffine system

and experimental studies in Section V for a servo system with nonaffine uncertainties. Concluding remarks are made in Section VI.

## II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following class of nonaffine nonlinear systems:

$$\dot{x}(t) = g(x) + bu(t) + f(x, u) + d(t) \quad (1)$$

where  $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T \in R^n$  is the state vector,  $u = [u_1(t), u_2(t), \dots, u_r(t)]^T \in R^r$  is the control input vector,  $g(x)$  is a known smooth nonlinear vector function of the state vector,  $f(x, u)$  is an unknown smooth nonaffine nonlinear vector function of the state vector  $x$  and the control input  $u$ ,  $d(t)$  is an unknown disturbance vector, and  $b \in R^{n \times r}$  is a known constant control matrix with full-column rank  $r$ .

**Assumption 1:** The nonlinear function  $(\partial[f(x, u) + bu])/ \partial u \neq 0$ , for all  $(x, u) \in R^n \times R^r$ .

Assume the following stable linear reference model:

$$\dot{x}_m(t) = A_m x_m(t) + B_m c(t) \quad (2)$$

where  $x_m(t) \in R^n$  is the reference state vector,  $c(t) = [c_1(t), c_2(t), \dots, c_r(t)]^T \in R^r$  is a piecewise continuous and uniformly bounded command to the system,  $A_m \in R^{n \times n}$  and  $B_m \in R^{n \times r}$  are chosen to meet the desired specification of the closed-loop control system. The objective is to design a controller  $u(t)$  such that  $x(t)$  asymptotically tracks the reference trajectory  $x_m(t)$ , i.e., the tracking error  $e(t) = x_m(t) - x(t)$  asymptotically converges to zero.

In this paper, the desired error dynamics are specified as

$$\dot{e}(t) = (A_m + K)e(t) \quad (3)$$

where  $K \in R^{n \times n}$  is an error feedback gain matrix. Since the reference model is chosen to be stable,  $K$  may be chosen as 0. If a different error dynamics is desired or required to guarantee the stability or to meet the dynamic performance, then common control strategies, e.g., pole placement, can be used to choose  $K$ . Another option is to follow the guidelines in [36], where the two-degree-of-freedom nature of the UDE-based control has been revealed. It is worth noting that the dimension of  $c(t)$  does not have to be the same as that of  $u(t)$ , as demonstrated in [34]. This provides extra freedom for the choice of  $B_m$ .

## III. CONTROL DESIGN AND STABILITY ANALYSIS

In system (1), input  $u$  is not expressed in an affine form, which leads to the failure of using feedback linearization. Here, the UDE-based control will be applied after lumping the nonaffine term into the uncertainty and disturbance term.

### A. Control Design

Combining (1)–(3), we have

$$A_m x(t) + B_m c(t) - g(x) - bu(t) - f(x, u) - d(t) = K e(t). \quad (4)$$

Based on (4), the control signal  $u$  should satisfy

$$bu(t) = A_m x(t) + B_m c(t) - g(x) - u_d(x, u, t) - Ke(t) \quad (5)$$

where

$$u_d(x, u, t) = f(x, u) + d(t)$$

denotes the unknown terms in (4), including the nonaffine uncertainty term  $f(x, u)$  and the external disturbance term  $d(t)$ . According to the system dynamics in (1),  $u_d(x, u, t)$  can be represented as

$$u_d(x, u, t) = f(x, u) + d(t) = \dot{x}(t) - g(x) - bu(t)$$

which indicates that the unknown dynamics and disturbances can be obtained from the known dynamics of the system and control signal. However, it cannot be directly used to formulate a control law. The UDE-based robust control strategy [27] adopts an estimation of this signal so that a control law is derived, based on the assumption that a signal can be approximated and estimated using a filter with the appropriate bandwidth. For example, if the filter has a wide enough bandwidth, the UDE is able to accurately and quickly estimate the lumped uncertainty term  $u_d$ , which is a function of control inputs, states, and disturbances.

Following the procedures provided in [27], assume that  $g_f(t)$  is the impulse response of a strictly proper stable filter  $G_f(s)$  with the unity gain and zero phase shift over the spectrum of  $u_d(x, u, t)$  and zero gain elsewhere. Then,  $u_d(x, u, t)$  can be accurately approximated by

$$\hat{u}_d(x, u, t) = u_d(x, u, t) * g_f(t) = (\dot{x}(t) - g(x) - bu(t)) * g_f(t) \quad (6)$$

where  $\hat{u}_d(x, u, t)$  is an estimate of  $u_d(x, u, t)$ , and “\*” is the convolution operator. Therefore,  $\hat{u}_d(x, u, t) = u_d(x, u, t)$  if the bandwidth of the filter is chosen appropriately to cover the spectrum of  $u_d(x, u, t)$ .

Replacing  $u_d(x, u, t)$  with  $\hat{u}_d(x, u, t)$  in (5) results in

$$\begin{aligned} bu(t) &= A_m x(t) + B_m c(t) - Ke(t) - g(x) - \hat{u}_d(x, u, t) \\ &= A_m x(t) + B_m c(t) - Ke(t) - g(x) \\ &\quad - (\dot{x}(t) - g(x) - bu(t)) * g_f(t). \end{aligned} \quad (7)$$

This eliminates the nonaffine term and leads to the UDE-based control law, i.e.,

$$\begin{aligned} u(t) = b^+ \left[ L^{-1} \left\{ \frac{1}{1 - G_f(s)} \right\} * (A_m x(t) + B_m c(t) - Ke(t)) \right. \\ \left. - g(x) - L^{-1} \left\{ \frac{sG_f(s)}{1 - G_f(s)} \right\} * x(t) \right] \end{aligned} \quad (8)$$

where  $b^+ = (b^T b)^{-1} b^T$  is the pseudoinverse of  $b$ , and  $L^{-1}\{\cdot\}$  is the inverse Laplace transform operator. It is observed that the unknown nonaffine dynamics and disturbances are removed from the control signal.

## B. Stability Analysis

The asymptotic stability of the closed-loop system is described in the following theorem.

**Theorem 1:** Consider the closed-loop system consisting of the nonaffine nonlinear system (1) satisfying Assumption 1, the reference model (2), and the UDE-based controller (8). If the filter  $G_f(s)$  is chosen appropriately as a strictly proper stable filter with unity gain and zero phase shift over the spectrum of the lumped uncertain term  $u_d(x, u, t) = f(x, u) + d(t)$  and zero gain elsewhere, then the closed-loop system is asymptotically stable. Moreover, the tracking error dynamics of the state converges according to (3).

*Proof:* If the lumped uncertain term  $u_d(x, u, t)$  in (5) is known, then the desired error dynamics (3) is obtained after substituting (5) into (1) and calculating the derivative of the error signal  $e(t) = x_m(t) - x(t)$ .

When  $u_d(x, u, t)$  is unknown, the estimated term  $\hat{u}_d(x, u, t)$  from the UDE in (6) is adopted to estimate  $u_d(x, u, t)$ . Then, the error dynamics becomes

$$\dot{e}(t) = (A_m + K)e(t) - \tilde{u}_d(x, u, t) \quad (9)$$

where

$$\tilde{u}_d(x, u, t) \triangleq u_d(x, u, t) - \hat{u}_d(x, u, t)$$

is the estimated error of the uncertainty and disturbance. According to (6), the estimated error is

$$\tilde{u}_d(x, u, t) = u_d(x, u, t) * L^{-1} \{1 - G_f(s)\}. \quad (10)$$

Since the filter  $G_f(s)$  is designed as a strictly proper stable filter with unity gain and zero phase shift over the spectrum of the lumped uncertain term  $u_d(x, u, t)$  and zero gain elsewhere, we have

$$\tilde{u}_d(x, u, t) = 0.$$

Therefore, the error dynamics in (9) converges to the desired error dynamics in (3), which is asymptotically stable. This completes the proof. ■

This theorem guarantees the asymptotic stability of the closed-loop system if such a low-pass filter  $G_f$  is designed. It can be very difficult to design  $G_f(s)$  to meet the condition fully; however, in practice, it is not a problem at all because it can be designed approximately to meet the major characteristics, and then, the controller itself will take care of the mismatch because of excellent robustness. This would reduce the stability region, but it is enough for most engineering systems. In this paper, the first-order low-pass filter, i.e.,

$$G_f = \frac{1}{\tau s + 1} \quad (11)$$

is adopted, where parameter  $\tau$  is chosen as a small enough positive number to ensure that the bandwidth of  $G_f(s)$  covers the spectrum of  $u_d(x, u, t)$ , and the good performance has been obtained in both simulation and experimental studies.

#### IV. NUMERICAL VALIDATION WITH AN UNSTABLE NONAFFINE NONLINEAR SYSTEM

To demonstrate the effectiveness of the proposed approach, consider the following unstable nonaffine nonlinear system, which is modified based on the nonaffine system investigated in [38]:

$$\dot{x} = 0.5x + 0.01u + \tanh(u+3) + \tanh(u-3) + d(t) \quad (12)$$

where the disturbance  $d(t)$  is the signal  $0.1 \sin(0.2\pi t) + 1(t-30)$  polluted with white noise having the noise power of 0.001.

Comparing the system with the plant model in (1), we have  $g(x) = 0.5x$ ,  $f(x, u) = \tanh(u+3) + \tanh(u-3)$ , and  $b = 0.01$ . It can be verified that Assumption 1 is satisfied. The uncertainty and disturbance term to be estimated is  $u_d = \tanh(u+3) + \tanh(u-3) + d(t)$ . The reference model is chosen as

$$\dot{x}_m = -5x_m + 5c(t)$$

with the reference input chosen as  $c(t) = \sin(0.1\pi t)$ , and the low-pass filter  $G_f$  is chosen as (11) with  $\tau = 0.001$  s. The error feedback gain is chosen as  $K = -2$ .

The simulation results are shown in Fig. 1. Although the uncertainty and disturbance term  $u_d$  is very significant, at the similar scale of the state, the UDE controller (8) is able to deal with it without any problem, achieving a tracking error at the level of  $10^{-4}$ .

#### V. EXPERIMENTAL VALIDATION WITH A ROTARY SERVO SYSTEM HAVING NONAFFINE UNCERTAINTIES

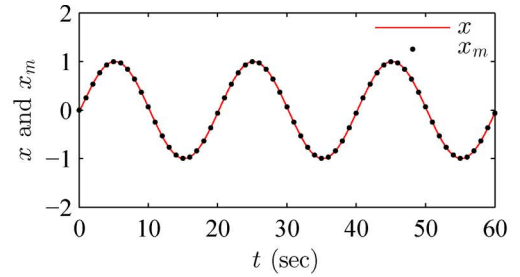
To further demonstrate the important features and performance of the proposed UDE-based robust control, experimental studies were carried out with a modified rotary servo system shown in Fig. 2 having a nonaffine uncertainty term added through software.

##### A. System Model

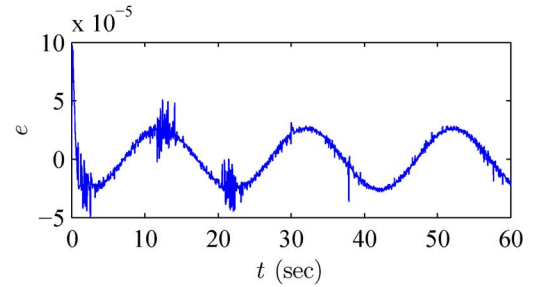
The mathematical model of this servo system with the added nonaffine uncertainty is described as

$$\begin{bmatrix} \dot{\theta} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -25 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 80 \end{bmatrix} u + \begin{bmatrix} 0 \\ f_2(\theta, \dot{\theta}, u) \end{bmatrix} + \begin{bmatrix} 0 \\ d_2(t) \end{bmatrix} \quad (13)$$

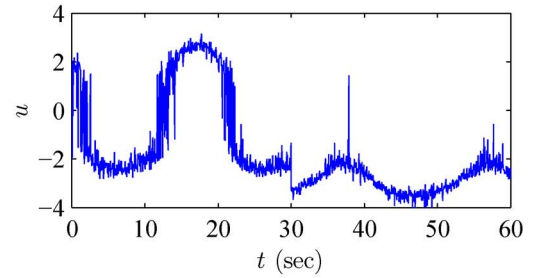
where  $\theta$  and  $\dot{\theta}$  are the system states that represent the angular position and velocity, respectively;  $u$  is the control signal;  $f_2(\theta, \dot{\theta}, u)$  is the added nonaffine uncertainty; and  $d_2(t)$  is the external disturbance. Compared with (1), we have  $x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$ ,  $g(x) = \begin{bmatrix} 0 & 1 \\ 0 & -25 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$ ,  $b = \begin{bmatrix} 0 \\ 80 \end{bmatrix}$ ,  $f(x, u) = \begin{bmatrix} 0 \\ f_2(\theta, \dot{\theta}, u) \end{bmatrix}$ ,



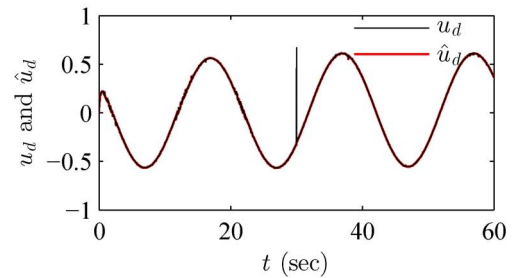
(a)



(b)



(c)



(d)

Fig. 1. Simulation results from the unstable nonaffine nonlinear system (12). (a) System state  $x$  and its reference  $x_m$ . (b) Tracking error  $e = x_m - x$ . (c) Control signal  $u$ . (d) Uncertainty and disturbance term  $u_d$  and its estimation  $\hat{u}_d$ .

and  $d(t) = \begin{bmatrix} 0 \\ d_2(t) \end{bmatrix}$  with  $f_2(\theta, \dot{\theta}, u) = 32(\theta + \dot{\theta} + \arctan(u))$ . It can be easily verified that for (13), Assumption 1 is satisfied because  $(\partial[f(x, u) + bu])/\partial u = \begin{bmatrix} 0 \\ 80 + \frac{32}{1+u^2} \end{bmatrix} \neq 0$ .

The objective is to design the control law  $u$  to make system (13) follow the desired trajectory generated by the following second-order reference system:

$$\begin{bmatrix} \dot{\theta}_m \\ \ddot{\theta}_m \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -900 & -60 \end{bmatrix} \begin{bmatrix} \theta_m \\ \dot{\theta}_m \end{bmatrix} + \begin{bmatrix} 0 \\ 900 \end{bmatrix} c(t) \quad (14)$$

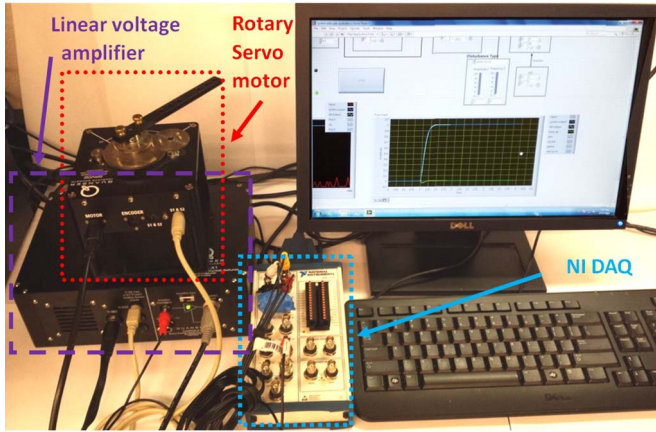


Fig. 2. QUANSER rotary servo motor system with a nonaffine uncertainty term added through software.

where  $\theta_m$  and  $\dot{\theta}_m$  are the reference states, and  $c(t)$  is the command signal to the reference model. Compared with (2), we have  $x_m = \begin{bmatrix} \theta_m \\ \dot{\theta}_m \end{bmatrix}$ ,  $A_m = \begin{bmatrix} 0 & 1 \\ -900 & -60 \end{bmatrix}$ , and  $B_m = \begin{bmatrix} 0 \\ 900 \end{bmatrix}$ .

### B. Filter and Controller Design

Based on the derivation and analysis of the UDE-based control law (8) in Section III, the filter  $G_f(s)$  plays a very important role. As demonstrated in [36], the two-degree-of-freedom nature of the UDE-based control has been revealed, which enables the decoupled design of the reference model and the filters. The filter  $G_f(s)$  can be designed to meet the desired specifications, but in many cases, it is enough to choose  $G_f(s)$  as the first-order low-pass filter given in (11) with the bandwidth wide enough to cover the spectrum of  $u_d$ . Then, the uncertainty estimate error (10) becomes

$$\tilde{u}_d = u_d \cdot \frac{\tau s}{\tau s + 1}. \quad (15)$$

It is obvious that for any bounded uncertain term  $u_d$ , if  $\tau$  is chosen as a small enough positive number to ensure that the bandwidth of  $G_f(s)$  covers the spectrum of  $u_d$ , then  $\tilde{u}_d$  is also small. It is also worth noting that when the low-pass filter (11) is used, we have

$$\frac{1}{1 - G_f(s)} = 1 + \frac{1}{\tau s}$$

$$\frac{sG_f(s)}{1 - G_f(s)} = \frac{1}{\tau}.$$

Therefore, the UDE-based control law (8) is obtained as

$$u(t) = b^+ \left[ -g(x) + (A_m x_m(t) + B_m c(t)) + \frac{1}{\tau} \left( (I - (A_m + K)\tau) e(t) - (A_m + K) \int_0^t e(\xi) d\xi \right) \right].$$

This simplified control law clearly demonstrates the nature of the UDE-based control strategy. It consists of three terms: the

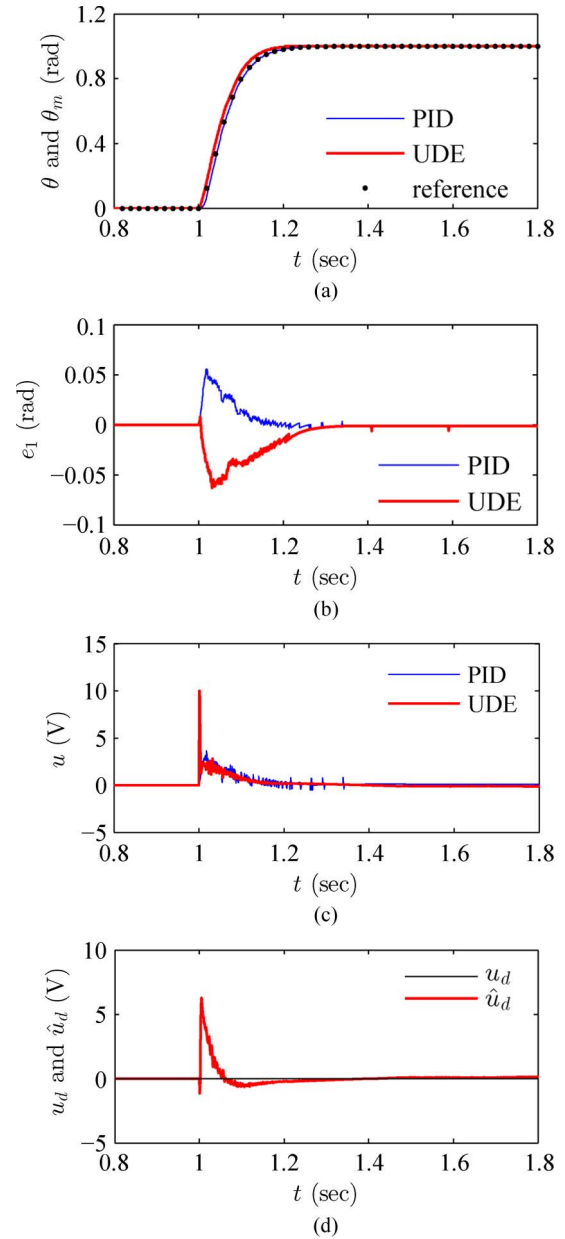


Fig. 3. Experiment results with the UDE-based and PID controllers that are tuned to achieve similar performance when  $f_2(\theta, \dot{\theta}, u) = 0$  and  $d_2(t) = 0$ . (a) System output  $\theta$  and reference  $\theta_m$ . (b) Tracking error  $e_1 = \theta_m - \theta$ . (c) System input  $u$ . (d)  $u_d$  and its estimated value  $\hat{u}_d$ .

first cancels the known system dynamics, the second introduces the desired dynamics given by the reference model, and the third adds a proportional–integral–like controller.

### C. Experimental Results

Here, the experimental results carried out on the test rig shown in Fig. 2 with the proposed UDE-based controller and the conventional proportional–integral–derivative (PID) controller are presented. To compare these two approaches fairly, their parameters are tuned to produce similar tracking performance when both the nonaffine term and the disturbance term are 0, as shown in Fig. 3. For the PID controller:  $K_p = 50$ ,

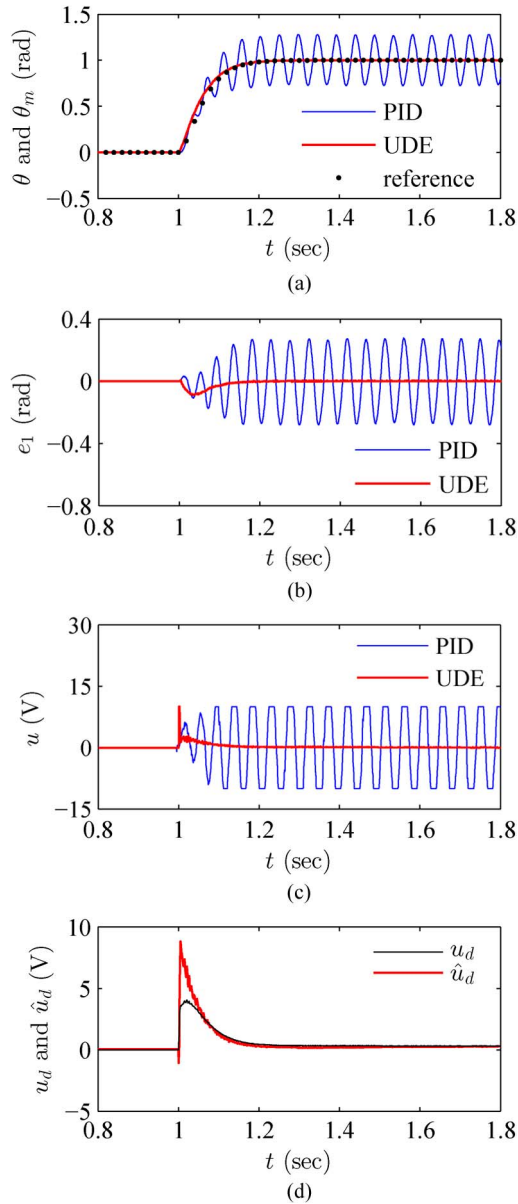


Fig. 4. Experiment results with the UDE-based and PID controllers when tracking a step signal with the external disturbance  $d_2(t) = 0$ . (a) System output  $\theta$  and reference  $\theta_m$ . (b) Tracking error  $e_1 = \theta_m - \theta$ . (c) System input  $u$ . (d)  $u_d$  and its estimated value  $\hat{u}_d$ .

$K_d=0.2$ ,  $K_i=50$ , and for the UDE-based controller:  $K=0$ ,  $\tau=0.01$ . Then, the same parameters are adopted for two case studies with the added nonaffine uncertainty  $f_2(\theta, \dot{\theta}, u) = 32(\theta + \dot{\theta} + \arctan(u))$ : 1) to track a step signal without the external disturbance and 2) to track a sinusoidal signal with the external disturbance  $d_2(t) = 32 \cos(2\pi t)$ . The sampling time is chosen as 0.001 s, and zero initial conditions are set for both the reference model and the system model.

1) *Tracking a Step Signal With the External Disturbance  $d_2(t) = 0$* : The experiment results are shown in Fig. 4. The PID controller cannot handle the nonaffine uncertainty  $f_2(\theta, \dot{\theta}, u) = 32(\theta + \dot{\theta} + \arctan(u))$  unless the design parameters that were adopted are adjusted again. The system output using the PID controller blows up, whereas the output of the PID

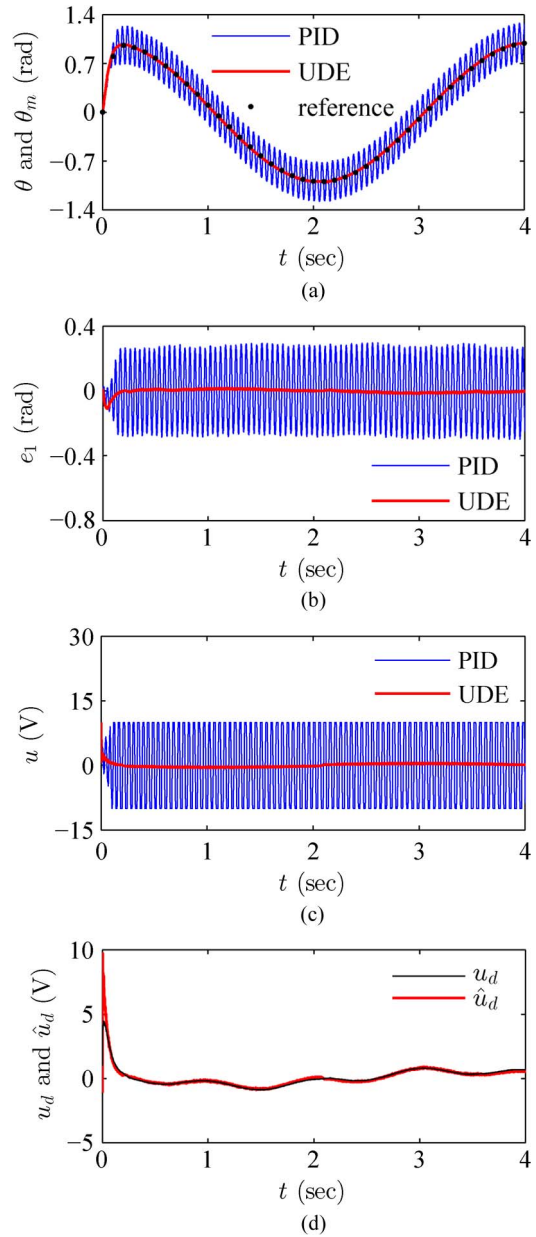


Fig. 5. Experiment results with the UDE-based and PID controllers when tracking a sinusoidal signal with the external disturbance  $d_2(t) = 32 \cos(2\pi t)$ . (a) System output  $\theta$  and reference  $\theta_m$ . (b) Tracking error  $e_1 = \theta_m - \theta$ . (c) System input  $u$ . (d)  $u_d$  and its estimated value  $\hat{u}_d$ .

controller violates the physical input constraints ( $\pm 10$  V). The proposed UDE-based control can still keep very good tracking performance due to its uncertainty estimation function shown in Fig. 4(d). Therefore, the proposed UDE-based controller is much more robust to handle the nonaffine uncertainty than the PID controller.

2) *Tracking a Sinusoidal Signal With the External Disturbance  $d_2(t) = 32 \cos(2\pi t)$* : The experimental results with  $c(t) = \cos(0.5\pi t)$  are shown in Fig. 5. Again, the UDE-based controller has demonstrated excellent performance in tracking the desired state and in rejecting the disturbance, but the PID controller could not handle the uncertainty and the disturbance. Fig. 5(d) shows that the lumped uncertainty and disturbance

term  $f_2(\theta, \dot{\theta}, u) + d_2(t)$  can be estimated fast and accurately by the UDE-based controller, which results in its good control performance.

## VI. CONCLUSION

In this paper, the UDE-based robust control strategy was applied to the control of a class of nonaffine nonlinear systems. The UDE was employed to estimate the lumped uncertain term, which is a function of control input, states, and disturbances. The proposed approach avoids the controller singularity problem, which might be caused by constructing the inverse operator. The common assumptions about the uncertainties and disturbances are relaxed as well. It does not require any knowledge of the uncertainties and disturbances, except information on bandwidth, during the design process. The asymptotic stability of the resulting closed-loop system is achieved. Through both numerical simulation studies and extensive experimental studies, it has been shown that the UDE-based control is easy to be implemented while bringing excellent performance.

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