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Robust attitude control of helicopters with actuator dynamics using neural networks

M. Chen[1,3] *S.S. Ge*[2,3] *B. Ren*[3]

[1]*College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, People's Republic of China*
[2]*Institute of Intelligent Systems and Information Technology & Robotics Institute, University of Eletronic Science and Technology of China, Chengdu 611731, People's Republic of China*
[3]*Department of Electrical & Computer Engineering, National University of Singapore, Singapore 117576*
E-mail: chenmou@nuaa.edu.cn

Abstract: In this study, attitude control is proposed for helicopters with actuator dynamics. For the nominal helicopter dynamics, model-based control is firstly presented to keep the desired helicopter attitude. To handle the model uncertainty and the external disturbance, radial basis function neural networks are adopted in the attitude control design. Using neural network approximation and the backstepping technique, robust attitude control is proposed with full state feedback. Considering unknown moment coefficients and the mass of helicopters, approximation-based attitude control is developed for the helicopter dynamics. In all proposed attitude control techniques, multi-input and multi-output non-linear dynamics are considered and the stability of the closed-loop system is proved via rigorous Lyapunov analysis. Extensive numerical simulation studies are given to illustrate the effectiveness of the proposed attitude control.

1 Introduction

Helicopters have been widely developed and used in various practical areas such as above-ground traffic transport, ground security detection, traffic condition assessment, forest fire monitoring, smuggling prevention and crime precautions [1]. Ensuring the stability of the helicopter flight is the elementary requirement for achieving the above-mentioned tasks. On the other hand, helicopters are inherently unstable without closed-loop control, different from other mechanical systems that are naturally passive or dissipative [2]. In addition, the helicopter dynamics is severely non-linear, time-varying, highly uncertain and strongly coupled. In general, the helicopter flight is characterised by time-varying environmental disturbances and widely changing flight conditions. Owing to the empirical representation of aerodynamic forces and moments, uncertainties exist in helicopter dynamics, especially those covering large operational flight envelopes. Therefore the robust flight control design and development for helicopters is a challenging control problem.

During the past two decades, flight control design of helicopters has attracted an ever increasing interest [3–9]. A large number of effective control techniques have been proposed in the literature for the helicopter flight control including robust adaptive control [6, 10–12], H_∞ control [13–16], state-dependent Riccati equation control [17], sliding mode control [18], trajectory tracking control [19, 20], backstepping control [2, 8, 21], fuzzy control [22, 23] and neural network control [24, 25]. In [10], robust non-linear motion control of a helicopter was developed. The H_∞ loop shaping control was investigated for Yamaha R-50 robotic helicopter in [13]. The flight control approach based on a state-dependent Riccati equation and its application were studied for autonomous helicopters [17]. A synchronised trajectory-tracking control strategy was proposed for multiple experimental three-degrees-of-freedom helicopters [20]. In [8], adaptive trajectory control was proposed for autonomous helicopters. Helicopter trimming and tracking control were investigated using direct neural dynamic programming [25]. In most existing works, the designed flight control technologies are concentrated on the linear helicopter dynamics or single-input/single-output (SISO) non-linear helicopter dynamics. Thus, the robust flight control techniques [26, 27] need to be further developed for the non-linear multi-input and multi-output (MIMO) helicopter dynamics.

To effectively handle strongly coupled non-linearities, model uncertainties and time-varying unknown perturbations, the dynamics of helicopters must be treated as an uncertain MIMO non-linear system in the flight control design. On the other hand, universal function approximators such as neural networks (NNs) have been extensively used in the control design of uncertain non-linear systems because of their universal approximation capabilities. Thus, NNs can be adopted to tackle uncertainties and disturbances in the helicopter dynamics [28]. Approximation-based control was developed for a scale model helicopter mounted on an experimental platform in the presence of model uncertainties, which may be caused by unmodelled dynamics, sensor errors or aerodynamical disturbances from the environment in [2]. In [29], robust adaptive NN control was proposed for helicopters in vertical flight based on the implicit function theorem and the mean value theorem. However, the robust adaptive flight control need to be further considered for uncertain MIMO non-linear helicopter dynamics using NNs.

In order to meet high-accuracy performance on pointing requirement, attitude control is usually applied to helicopters. Specially, helicopter attitude keeping is important for security monitoring and hovering flight. In these cases, the main objective is always keeping the same orientation and altitude, which are usually described by roll, pitch and yaw angles. Attitude control for helicopters is an important control topic in non-linear control system design because of the non-linearity of the dynamics and strong interactions between variables. A fuzzy gain-scheduler for the attitude control was developed for the unmanned helicopter in [22]. In [30], non-linear attitude control was investigated for the Bell 412 Helicopter. In the helicopter flight control system, control signals are produced by the control command via the servo actuator. The flight control performance can be improved if the actuator dynamics is explicitly considered in the control design. In [31], the adaptive output feedback control was studied for the model helicopter under known actuator characteristics including actuator dynamics and saturation. Aggressive control was proposed in the presence of parametric and dynamical uncertainties [1].

This work is motivated by the attitude control of helicopters in the presence of model uncertainty and external disturbance. The backstepping technique combining with NNs is employed to design the robust attitude control for uncertain MIMO non-linear helicopter dynamics. The main contributions of the paper are as follows:

1. To the best of our knowledge, there are few works in the literature, taking into account the actuator dynamics in the helicopter control, which is practically relevant but more challenging as well. In our work, helicopter models are considered as the MIMO non-linear dynamic systems, where the actuator dynamics in the first-order low-pass filter form are considered.

2. The possible singularity problem of the control coefficient matrix for the model-based attitude control case has been tackled effectively by introducing a control gain matrix.

3. Approximation-based attitude control is developed to handle the model uncertainties (e.g. unknown moment coefficients and mass) and external disturbances. Rigorous stability analysis and extensive simulations results show the effectiveness and robustness of the proposed attitude control.

The organisation of the paper is as follows. Section 2 details the problem formulation. Section 3 presents the model-based attitude control for the nominal plant. Robust attitude control is investigated for helicopters with uncertainties and disturbances in Section 4. Section 5 proposes the approximation-based attitude control of helicopters. Simulation studies are shown in Section 6 to demonstrate the effectiveness of our developed approaches, followed by concluding remarks in Section 7.

2 Problem formulation

Assuming that the flight positions and velocities along the x- and y-axes are very small, that is, $x = 0$, $y = 0$, $u = 0$ and $v = 0$, the attitude/altitude dynamics of the helicopter can be derived from the six degrees of freedom [14, 32] and is represented in the non-linear form of

$$\dot{z} = w \cos\phi \cos\theta \tag{1}$$

$$\dot{\phi} = p + q \sin\phi \tan\theta + r \cos\phi \tan\theta \tag{2}$$

$$\dot{\theta} = q \cos\phi - r \sin\phi \tag{3}$$

$$\dot{\psi} = q \sin\phi \sec\theta + r \cos\phi \sec\theta \tag{4}$$

$$\dot{w} = g \cos\phi \cos\theta + Z/m \tag{5}$$

$$\dot{p} = (c_1 r + c_2 p)q + c_3 L + c_4 N \tag{6}$$

$$\dot{q} = c_5 pr - c_6(p^2 - r^2) + c_7 M \tag{7}$$

$$\dot{r} = (c_8 p - c_2 r)q + c_4 L + c_9 N \tag{8}$$

where z is the helicopter altitude and w is the velocity along the z-axis; m is the mass of helicopter; p, q and r are the fuselage coordination system angular velocity components; ϕ, θ and ψ are Euler angles, that is, fuselage attitude angles; Z is the aerodynamic force, and L, M and N are aerodynamic moments about the centre of gravity. The

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coefficients of moment equations are given by [14]

$$c_1 = \frac{(I_{yy} - I_{zz})I_{zz} - I_{xz}^2}{\Gamma}, \quad c_2 = \frac{(I_{xx} - I_{yy} + I_{zz})I_{xz}}{\Gamma}$$
$$c_3 = \frac{I_{zz}}{\Gamma}, \quad c_4 = \frac{I_{xz}}{\Gamma}, \quad c_5 = \frac{I_{zz} - I_{xx}}{I_{yy}}$$
$$c_6 = \frac{I_{xz}}{I_{yy}}, \quad c_7 = \frac{1}{I_{yy}}, \quad c_8 = \frac{I_{xx}(I_{xx} - I_{yy}) + I_{xz}^2}{\Gamma}$$
$$c_9 = \frac{I_{xx}}{\Gamma}, \quad \Gamma = I_{xx}I_{zz} - I_{xz}^2$$

where I_{xx}, I_{xz}, I_{yy} and I_{zz} are the inertia moments of the helicopter.

Moments L, M and N including the contributions from aerodynamics and propulsion can be written as [1, 33]

$$\begin{aligned} L &= L_R + Y_R h_R + Z_R y_R + Y_T h_T \\ M &= M_R - X_R h_R + Z_R l_R \\ N &= N_R - Y_R l_R - Y_T l_T \\ Z &= Z_R \end{aligned} \tag{9}$$

where the subscripts denote the main rotor (R) and the tail rotor (T). (l_R, y_R, h_R) and (l_T, y_T, h_T) are the coordinates of the main rotor and the tail rotor shafts relative to the centre of helicopter mass, respectively. The forces X_R, Y_R, Z_R and Y_T and torques L_R, M_R and N_R can be expressed as [1, 33]

$$\begin{aligned} &X_R = -T_R \sin a_{1_s}, \quad Y_R = -T_R \sin b_{1_s} \\ &Z_R = -T_R \cos a_{1_s} \cos b_{1_s}, \quad Y_T = -T_T \\ &L_R = C_b^R b_{1_s} - Q_R \sin a_{1_s}, \quad M_R = C_a^R a_{1_s} + Q_R \sin b_{1_s} \\ &N_R = -Q_R \cos a_{1_s} \cos b_{1_s} \end{aligned} \tag{10}$$

where a_{1_s} and b_{1_s} are the longitudinal and lateral inclination of the tip path plane of the main rotor; C_a^R and C_b^R are physical parameters modelling the flapping dynamic of the main rotor; Q_R is the total main rotor torque; T_R and T_T are thrusts generated by the main and the tail rotors which can be computed as shown in [1, 33]

$$T_R = K_{T_M}\theta_M w_e^2, \quad T_T = K_{T_T}\theta_T w_e^2 \tag{11}$$

where θ_M and θ_T are the collective pitches of the main and tail rotors, respectively; w_e denotes the angular velocity of the main rotor; and K_{T_M} and K_{T_T} are the aerodynamics constants of the rotor's blades, respectively.

Based on (1)–(11), the attitude dynamics of a helicopter with model uncertainty and external disturbance can be written as

$$\begin{aligned} \dot{x}_1 &= J(x_1)x_2 \\ \dot{x}_2 &= F(x_1, x_2) + H(x_3) + \Delta F_1(x_1, x_2) + D(x_1, x_2, t) \\ y &= x_1 \end{aligned} \tag{12}$$

where

$$x_1 = \begin{bmatrix} z \\ \phi \\ \theta \\ \psi \end{bmatrix}, \; x_2 = \begin{bmatrix} w \\ p \\ q \\ r \end{bmatrix}, \; x_3 = \begin{bmatrix} b_{1_s} \\ a_{1_s} \\ \theta_T \\ \theta_M \end{bmatrix}$$

$$J(x_1) = \begin{bmatrix} \cos\phi\cos\theta & 0 & 0 & 0 \\ 0 & 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & 0 & \cos\phi & -\sin\phi \\ 0 & 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix}$$

$$F(x_1, x_2) = \begin{bmatrix} g\cos\phi\cos\theta \\ (c_1 r + c_2 p)q \\ c_5 pr - c_6(p^2 - r^2) \\ (c_8 p - c_2 r)q \end{bmatrix}$$

$H(x_3) = [H_1(x_3), H_2(x_3), H_3(x_3), H_4(x_3)]^T$, $\Delta F_1(x_1, x_2) = [\Delta f_{11}(x_1, x_2), \Delta f_{12}(x_1, x_2), \Delta f_{13}(x_1, x_2), \Delta f_{14}(x_1, x_2)]^T$, $\Delta f_{1i}(x)$, $i = 1, 2, 3, 4$ are the system modelling uncertainties, $D(x_1, x_2, t) = [D_1(x_1, x_2, t), D_2(x_1, x_2, t), D_3(x_1, x_2, t), D_4(x_1, x_2, t)]^T$, $D_i(x_1, x_2, t)$, $i = 1, 2, 3, 4$ are the system external disturbances such as wind disturbance and y is the system output. $H_1(x_3)$, $H_2(x_3)$, $H_3(x_3)$ and $H_4(x_3)$ are given by

$$\begin{aligned} H_1(x_3) &= -K_{T_M}\theta_M w_e^2 \cos a_{1_s}\cos b_{1_s}/m \\ H_2(x_3) &= c_3 C_b^R b_{1_s} - c_3 Q_R \sin a_{1_s} - c_3 K_{T_M}\theta_M w_e^2 \sin b_{1_s} h_R \\ &\quad - c_3 K_{T_M}\theta_M w_e^2 \cos a_{1_s}\cos b_{1_s} y_R \\ &\quad - c_3 K_{T_T}\theta_T w_e^2 h_T - c_4 Q_R \cos a_{1_s}\cos b_{1_s} \\ &\quad + c_4 K_{T_M}\theta_M w_e^2 \sin b_{1_s} l_R + c_4 K_{T_T}\theta_T w_e^2 l_T \\ H_3(x_3) &= c_7 C_a^R a_{1_s} + c_7 Q_R \sin b_{1_s} + c_7 K_{T_M}\theta_M w_e^2 h_R \\ &\quad - c_7 K_{T_M}\theta_M w_e^2 \cos a_{1_s}\cos b_{1_s} l_R \\ H_4(x_3) &= c_4 C_b^R b_{1_s} - c_4 Q_R \sin a_{1_s} - c_4 K_{T_M}\theta_M w_e^2 \sin b_{1_s} h_R \\ &\quad - c_4 K_{T_M}\theta_M w_e^2 \cos a_{1_s}\cos b_{1_s} y_R \\ &\quad - c_4 K_{T_T}\theta_T w_e^2 h_T - c_9 Q_R \cos a_{1_s}\cos b_{1_s} \\ &\quad + c_9 K_{T_M}\theta_M w_e^2 \sin b_{1_s} l_R + c_9 K_{T_T}\theta_T w_e^2 l_T \end{aligned}$$

From (12), it is difficult to directly design the robust attitude control for helicopters because of the input implicit function $H(x_3)$. To expediently design the model-based attitude control, $H(x_3)$ is separated into two parts including the linear part Gx_3 and the non-linear part $F_2(x_3)$. On the other hand, to improve the closed-loop system control performance, we involve the influence of actuator dynamics in the control design, where the actuator dynamics are assumed to be in the first-order low-pass filter form. Considering (12), the attitude dynamics of a helicopter including the actuator dynamics, the model uncertainties and the external disturbances can be rewritten as

$$\begin{aligned}\dot{x}_1 &= J(x_1)x_2\\ \dot{x}_2 &= F(x_1, x_2) + Gx_3 + \Delta F_1(x_1, x_2) + F_2(x_3) + D(x_1, x_2, t)\\ \dot{x}_3 &= -\lambda(x_3 - u)\\ y &= x_1\end{aligned} \tag{13}$$

where

$$G = \begin{bmatrix} \frac{\tau_1}{m} & \frac{\tau_2}{m} & 0 & \frac{\tau_3}{m} \\ c_3\tau_4 & c_3\tau_5 & c_3\tau_6 + c_4\tau_8 & c_3\tau_7 + c_4\tau_9 \\ 0 & 0 & c_7\tau_8 & c_7\tau_9 \\ c_4\tau_4 & c_4\tau_5 & c_4\tau_6 + c_9\tau_8 & c_4\tau_7 + c_9\tau_9 \end{bmatrix}, \quad \tau_j > 0$$

are control gain parameters and $u = [u_{b_{1s}}, u_{a_{1s}}, u_{\theta_T}, u_{\theta_M}]^T$ is the system command input. The third subequation of (13) is the actuator dynamics and λ is the actuator gain. $F_2(x_3) = [f_{21}(x_3), f_{22}(x_3), f_{23}(x_3), f_{24}(x_3)]^T$, $f_{2i}(x_3)$, $i = 1, 2, 3, 4$ are given by

$$\begin{aligned} F_2(x_3) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_3 & 0 & c_4 \\ 0 & 0 & c_7 & 0 \\ 0 & c_4 & 0 & c_9 \end{bmatrix} \begin{bmatrix} f_{31}(x_3) \\ f_{32}(x_3) \\ f_{33}(x_3) \\ f_{34}(x_3) \end{bmatrix}\\ f_{31}(x_3) &= -(\tau_1 b_{1s} + \tau_2 a_{1s} + \tau_3\theta_M \\ &\quad + K_{T_M}\theta_M w_e^2 \cos a_{1s} \cos b_{1s})/m \\ f_{32}(x_3) &= C_b^R b_{1s} - Q_R \sin a_{1s} \\ &\quad - K_{T_M}\theta_M w_e^2 \sin b_{1s} h_R - K_{T_M}\theta_M w_e^2 \cos a_{1s} \cos b_{1s} y_R \\ &\quad - K_{T_T}\theta_T w_e^2 h_T - \tau_4 b_{1s} - \tau_5 a_{1s} - \tau_6\theta_T - \tau_7\theta_M \\ f_{33}(x_3) &= C_a^R a_{1s} + Q_R \sin b_{1s} + K_{T_M}\theta_M w_e^2 h_R \\ &\quad - K_{T_M}\theta_M w_e^2 \cos a_{1s} \cos b_{1s} l_R - \tau_8\theta_T - \tau_9\theta_M \\ f_{34}(x_3) &= -Q_R \cos a_{1s} \cos b_{1s} + K_{T_M}\theta_M w_e^2 \sin b_{1s} l_R \\ &\quad + K_{T_T}\theta_T w_e^2 l_T - \tau_8\theta_T - \tau_9\theta_M \end{aligned} \tag{14}$$

It is easy to know that $J(x_1)$ is invertible for all $\theta \in (-\pi/2, \pi/2)$ and $\phi \in (-\pi/2, \pi/2)$. To facilitate control system design, we assume that all states of the helicopter attitude dynamics (13) are available. Moreover, the following assumptions are needed for the subsequent developments.

Assumption 1 [34]: For the continuous functions $D_i(x_1, x_2, t){:}\mathcal{R}^4 \times \mathcal{R}^4 \times \mathcal{R} \to \mathcal{R}$, $i = 1, 2, 3, 4$, there exist positive, smooth, non-decreasing functions $d_i^f(x_1, x_2){:}\mathcal{R}^4 \times \mathcal{R}^4 \to \mathcal{R}^+$ and time-dependent functions $d_i^t(t){:}\mathcal{R}^+ \to \mathcal{R}^+$, $i = 1, 2, 3$ such that

$$|D_i(x_1, x_2, t)| \le d_i^f(x_1, x_2) + d_i^t(t) \tag{15}$$

where

$$d_i^t(t) \le \bar{d}_i$$

with unknown constants $\bar{d}_i \in \mathcal{R}^+$, $\forall t > t_0$.

Lemma 1 [34, 35]: For bounded initial conditions, if there exists a C^1 continuous and positive definite Lyapunov function $V(x)$ satisfying $\gamma_1(\|x\|) \le V(x) \le \gamma_2(\|x\|)$, such that $\dot{V}(x) \le -\kappa V(x) + c$, where $\gamma_1, \gamma_2{:}\mathcal{R}^n \to \mathcal{R}$ are class K functions and c is a positive constant, then the solution $x(t)$ is uniformly bounded.

The control objective is to keep the desired altitude/attitude of helicopter in the presence of model uncertainty and environment disturbance. Thus, the proposed altitude control techniques must render the helicopter track a desired attitude x_{1d} such that the tracking errors converge to a very small neighbourhood of the origin, that is, $\lim_{t\to\infty} \|y - x_{1d}\| < \epsilon$ with $\epsilon > 0$.

Assumption 2 [34]: For all $t > 0$, there exist $\delta_{11} > 0$, $\delta_{21} > 0$ and $\delta_{31} > 0$ such that $\|\dot{x}_{1d}(t)\| \le \delta_{11}$, $\|\ddot{x}_{1d}(t)\| \le \delta_{21}$ and $\|x_{1d}^{(3)}(t)\| \le \delta_{31}$.

3 Model-based attitude control for nominal plant

In this section, we assume that the moment coefficients and mass of the helicopter are known and neglect the uncertainty $\Delta F_1(x_1, x_2)$ and external disturbance $D(x_1, x_2, t)$ of system (13). Then, the model-based backstepping attitude control is developed for the nominal dynamics of helicopters. Rigorous analysis through Lyapunov analysis is given to show the stability of the closed-loop system. To develop the model-based attitude control, we define error variables $z_1 = x_1 - x_{1d}$, $z_2 = x_2 - \alpha_1$ and $z_3 = x_3 - \alpha_2$, where $\alpha_1 \in \mathcal{R}^4$ and $\alpha_2 \in \mathcal{R}^4$ are virtual control laws.

Step 1: Considering (13) and differentiating z_1 with respect to time yields

$$\dot{z}_1 = \dot{x}_1 - \dot{x}_{1d} = J(x_1)(z_2 + \alpha_1) - \dot{x}_{1d} \tag{16}$$

Owing to the non-singularity of $J(x_1)$, the virtual control law α_1 is chosen as

$$\alpha_1 = J^{-1}(x_1)(-K_1 z_1 + \dot{x}_{1d}) \tag{17}$$

where $K_1 = K_1^T > 0$.

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Substituting (17) into (16), we obtain

$$\dot{z}_1 = J(x_1)z_2 - K_1 z_1 \tag{18}$$

Consider the Lyapunov function candidate $V_1 = \frac{1}{2} z_1^{\mathrm{T}} z_1$. The time derivative of V_1 is

$$\dot{V}_1 = -z_1^{\mathrm{T}} K_1 z_1 + z_1^{\mathrm{T}} J(x_1) z_2 \tag{19}$$

The first term on the right-hand side is negative, and the second term will be cancelled in the next step.

Step 2: Differentiating z_2 with respect to time yields

$$\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1 = F(x_1, x_2) + Gx_3 + F_2(x_3) - \dot{\alpha}_1 \tag{20}$$

where $\dot{\alpha}_1 = \dot{J}(x_1)^{-1}(-K_1 z_1 + \dot{x}_{1\mathrm{d}}) + J(x_1)^{-1}(-K_1 \dot{z}_1 + \ddot{x}_{1\mathrm{d}})$.

Consider the Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2} z_2^{\mathrm{T}} z_2 \tag{21}$$

Invoking (20), the time derivative of V_2 is

$$\begin{aligned}\dot{V}_2 = & -z_1^{\mathrm{T}} K_1 z_1 + z_1^{\mathrm{T}} J(x_1) z_2 + z_2^{\mathrm{T}} F(x_1, x_2) \\ & + z_2^{\mathrm{T}} G(z_3 + \alpha_2) + z_2^{\mathrm{T}} F_2(x_3) - z_2^{\mathrm{T}} \dot{\alpha}_1\end{aligned} \tag{22}$$

The virtual control law α_2 is proposed as follows

$$\alpha_2 = Q^{+}(Z_1)[\dot{\alpha}_1 - K_2 z_2 - F(x_1, x_2) - J^{\mathrm{T}}(x_1) z_1] \tag{23}$$

where

$$Q^{+} = QG^{\mathrm{T}}(GQG^{\mathrm{T}})^{-1} \tag{24}$$

with Q being chosen such that GQG^{T} is non-singular. When the matrix G is determined, we always can find an appropriate matrix Q render GQG^{T} non-singular.

Substituting (23) and (24) into (22), we have

$$\dot{V}_2 \leq -z_1^{\mathrm{T}} K_1 z_1 - z_2^{\mathrm{T}} K_2 z_2 + z_2^{\mathrm{T}} G z_3 + z_2^{\mathrm{T}} F_2(x_3) \tag{25}$$

Step 3: Differentiating z_3 with respect to time yields

$$\dot{z}_3 = \dot{x}_3 - \dot{\alpha}_2 = -\lambda(x_3 - u) - \dot{\alpha}_2 \tag{26}$$

Consider the Lyapunov function candidate

$$V_3 = V_2 + \frac{1}{2} z_3^{\mathrm{T}} z_3 \tag{27}$$

Considering (25) and (26), the time derivative of V_3 is given by

$$\begin{aligned}\dot{V}_3 \leq & -z_1^{\mathrm{T}} K_1 z_1 - z_2^{\mathrm{T}} K_2 z_2 + z_2^{\mathrm{T}} G z_3 + z_2^{\mathrm{T}} F_2(x_3) \\ & - \lambda z_3^{\mathrm{T}} x_3 + \lambda z_3^{\mathrm{T}} u - z_3^{\mathrm{T}} \dot{\alpha}_2\end{aligned} \tag{28}$$

The input control u is proposed as follows

$$u = \begin{cases} x_3 - \lambda^{-1}\left(K_3 z_3 - \dot{\alpha}_2 + G^{\mathrm{T}} z_2 + \dfrac{z_3 z_2^{\mathrm{T}} F_2(x_3)}{\|z_3\|^2}\right), & \|z_3\| \geq \varepsilon_3 \\ 0, & \|z_3\| < \varepsilon_3 \end{cases} \tag{29}$$

where $K_3 = K_3^{\mathrm{T}} > 0$ and $\varepsilon_3 > 0$ are the design parameters.

The above design procedure can be summarised in the following theorem.

Theorem 1: Considering the nominal attitude dynamics of the helicopter system (13), the model-based control law is designed according to (29). Under the proposed model-based control and for any bounded initial condition, the closed-loop signals z_1, z_2 and z_3 are bounded. Namely, there exist design parameters $K_1 = K_1^{\mathrm{T}} > 0$, $K_2 = K_2^{\mathrm{T}} > 0$ and $K_3 = K_3^{\mathrm{T}} > 0$ such that the overall closed-loop control system is semi-globally stable. Furthermore, the tracking error z_1 converges to a compact set and the control objective is obtained.

Proof: When $\|z_3\| \geq \varepsilon_3$, substituting the first equation of (29) into (28), we obtain

$$\dot{V}_3 \leq -z_1^{\mathrm{T}} K_1 z_1 - z_2^{\mathrm{T}} K_2 z_2 - z_3^{\mathrm{T}} K_3 z_3 \leq -\lambda_{\min}(K_i) V_3 \tag{30}$$

where $i = 1, 2, 3$ and $\lambda_{\min}(.)$ denotes the smallest eigenvalues of a matrix. Therefore we know that the closed-loop system is stable when $\|z_3\| \geq \varepsilon_3$ according to (30). If $\|z_3\| < \varepsilon_3$, z_3 approximates to zero. It means that $x_3 = \alpha_2$. From (14), we know that $F_2(x_3)$ is bounded because of the bounded available deflexion angles of the main rotor and the tail rotor. Based on the boundary of $F_2(x_3)$, we can conclude that all signals of the closed-loop system are bounded according to Lemma 1 if only appropriate design parameters K_1 and K_2 are chosen. This concludes the proof. □

Remark 1: To handle the non-linear term $F_2(x_3)$ in (13), the model-based control is proposed as a discontinuous form which can excite the chattering phenomenon. However, we can adjust design parameter ε_3 to decrease the chattering phenomenon and improve the control performance. Furthermore, the $\dot{\alpha}_2$ is used in the model-based control law (29). From (23), we can see the α_2 is continuous and differentiable in which the design matrix Q is introduced to avoid the potential singularity of G. Since G is known which is independent of system states and can be designed according the control demand, we can always choose an appropriate design matrix Q and parameters τ_i to make GQG^{T} non-singular.

4 Robust attitude control of helicopters with uncertainties and disturbances

In this section, the robust attitude control in combination with radial basis function neural network (RBFNN) to keep the desired attitude of helicopter system (13) in the presence of model uncertainty and external disturbance is considered. Define the error variables $z_1 = x_1 - x_{1d}$, $z_2 = x_2 - \alpha_1$ and $z_3 = x_3 - \alpha_2$ which are the same as the related definitions of variables in Section 3. Since there are no model uncertainty and disturbances in the altitude and attitude angle equations, the design of Step 1 is the same as the case of the model-based attitude control for the nominal helicopter dynamics in Section 3. Here, we present only the design processes of Steps 2 and 3 for the robust attitude control.

Step 2: Considering (13) and differentiating z_2 with respect to time yield

$$\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1 = F(x_1, x_2) + Gx_3 + \Delta F_1(x_1, x_2) + F_2(x_3) + D(x_1, x_2, t) - \dot{\alpha}_1 \quad (31)$$

where α_1 is defined in (17).

Choose the Lyapunov function candidate

$$V_2^* = V_1 + \frac{1}{2} z_2^{\mathrm{T}} z_2 \quad (32)$$

Owing to (19), (31) and Assumption 1, the time derivative of V_2^* is given by

$$\dot{V}_2^* \le -z_1^{\mathrm{T}} K_1 z_1 + z_1^{\mathrm{T}} J(x_1) z_2 + z_2^{\mathrm{T}} F(x_1, x_2) + z_2^{\mathrm{T}} G(z_3 + \alpha_2) + z_2^{\mathrm{T}} F_2(x_3) - z_2^{\mathrm{T}} \rho(Z) - z_2^{\mathrm{T}} \dot{\alpha}_1 \quad (33)$$

where $\rho(Z) = -\Delta F_1(x_1, x_2) - \mathrm{Sgn}(z_2)(d^f(x_1, x_2) + \bar{d})$, $Z = [x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \alpha_1^{\mathrm{T}}]$, $\mathrm{Sgn}(z_2) := \mathrm{diag}\{\mathrm{sgn}(z_{2j})\}$, $d^f(x_1, x_2) := [d_1^f(x_1, x_2), d_2^f(x_1, x_2), d_3^f(x_1, x_2), d_4^f(x_1, x_2)]^{\mathrm{T}}$ and $\bar{d} = [\bar{d}_1, \bar{d}_2, \bar{d}_3, \bar{d}_4]^{\mathrm{T}}$. Since $\Delta F_1(x_1, x_2)$, $d^f(x_1, x_2)$ and $\bar{d}$ are all unknown, the model-based control cannot be directly designed. To overcome this problem, we utilise RBFNNs in [36] to approximate the unknown term $\rho(Z)$ which are expressed as

$$\hat{\rho}(Z) = \hat{\Theta}^{\mathrm{T}} S(Z) \quad (34)$$

where $\hat{\Theta} \in \mathcal{R}^{l \times 4}$ is the approximation parameter, $S(Z) = [s_1(Z), s_2(Z), \ldots, s_l(Z)]^{\mathrm{T}} \in \mathcal{R}^{l \times 1}$ represents the vector of smooth basis function, with the NN node number $l > 1$ and $s_i(Z)$ being chosen as the commonly used Gaussian functions $s_i(Z) = \exp[-(Z - \mu_i)^{\mathrm{T}}(Z - \mu_i)/\eta_i^2]$, $i = 1, 2, \ldots, l$, where μ_i is the centre of the receptive field and η_i is the width of the Gaussian function. $\hat{\Theta}^{\mathrm{T}} S(Z)$ approximates $\Theta^{*\mathrm{T}} S(Z)$ given by

$$\Theta^{*\mathrm{T}} S(Z) + \varepsilon = \rho(Z) \quad (35)$$

where Θ^* is the optimal weight value of RBFNN. ε is the approximation error.

Substituting (35) into (33), we obtain

$$\dot{V}_2^* \le -z_1^{\mathrm{T}} K_1 z_1 + z_1^{\mathrm{T}} J(x_1) z_2 + z_2^{\mathrm{T}} F(x_1, x_2) + z_2^{\mathrm{T}} G z_3 - z_2^{\mathrm{T}} \dot{\alpha}_1 + z_2^{\mathrm{T}} G \alpha_2 + z_2^{\mathrm{T}} F_2(x_3) + z_2^{\mathrm{T}}(-\Theta^{*\mathrm{T}} S(Z) - \varepsilon) \quad (36)$$

The virtual control law α_2 is proposed based on the RBFNNs as follows

$$\alpha_2 = Q^+(Z_1)[\dot{\alpha}_1 - K_2 z_2 + \hat{\rho}_1(Z_1) - F(x_1, x_2) - J^{\mathrm{T}}(x_1) z_1] \quad (37)$$

where Q^+ is defined in (24).

Substituting (37) into (36), we obtain

$$\dot{V}_2^* \le -z_1^{\mathrm{T}} K_1 z_1 - z_2^{\mathrm{T}} K_2 z_2 + z_2^{\mathrm{T}} G z_3 + z_2^{\mathrm{T}} F_2(x_3) + z_2^{\mathrm{T}} \tilde{\Theta}^{\mathrm{T}} S(Z) - z_2^{\mathrm{T}} \varepsilon \quad (38)$$

where $\tilde{\Theta} = \hat{\Theta} - \Theta^*$.

Considering the stability of error signals $\tilde{\Theta}$, the augmented Lyapunov function candidate can be written as

$$V_2 = V_2^* + \frac{1}{2} \mathrm{tr}(\tilde{\Theta}^{\mathrm{T}} \Lambda^{-1} \tilde{\Theta}) \quad (39)$$

where $\Lambda = \Lambda^{\mathrm{T}} > 0$.

The time derivative of V_2 along (38) is

$$\dot{V}_2 \le -z_1^{\mathrm{T}} K_1 z_1 - z_2^{\mathrm{T}} K_2 z_2 + z_2^{\mathrm{T}} G z_3 + z_2^{\mathrm{T}} F_2(x_3) + z_2^{\mathrm{T}} \tilde{\Theta}^{\mathrm{T}} S(Z) - z_2^{\mathrm{T}} \varepsilon + \mathrm{tr}(\tilde{\Theta}^{\mathrm{T}} \Lambda^{-1} \dot{\hat{\Theta}}) \quad (40)$$

Consider the adaptive laws for $\hat{\Theta}$ as

$$\dot{\hat{\Theta}} = -\Lambda(S(Z_1) z_2^{\mathrm{T}} + \sigma \hat{\Theta}) \quad (41)$$

where $\sigma > 0$.

Noting the following facts

$$-z_2^{\mathrm{T}} \varepsilon \le \frac{1}{2} \|\varepsilon\|^2 + \frac{1}{2} \|z_2\|^2 \quad (42)$$

$$2\tilde{\Theta}^{\mathrm{T}} \hat{\Theta} = \|\tilde{\Theta}\|^2 + \|\hat{\Theta}\|^2 - \|\Theta^*\|^2 \ge \|\tilde{\Theta}\|^2 - \|\Theta^*\|^2 \quad (43)$$

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and considering (41), we obtain

$$\dot{V}_2 \leq -z_1^{\mathrm{T}} K_1 z_1 - z_2^{\mathrm{T}} K_2 z_2 + z_2^{\mathrm{T}} G z_3 + z_2^{\mathrm{T}} F_2(x_3) + \frac{1}{2}\|\varepsilon\|^2 + \frac{1}{2}\|z_2\|^2 - \frac{\sigma}{2}\|\tilde{\Theta}\|^2 + \frac{\sigma}{2}\|\Theta^*\|^2 \quad (44)$$

Step 3: Differentiating z_3 with respect to time yields

$$\dot{z}_3 = \dot{x}_3 - \dot{\alpha}_2 = -\lambda(x_3 - u) - \dot{\alpha}_2 \quad (45)$$

Consider the Lyapunov function candidate

$$V_3 = V_2 + \frac{1}{2} z_3^{\mathrm{T}} z_3 \quad (46)$$

Considering (45), the time derivative of V_3 is given by

$$\begin{aligned}\dot{V}_3 \leq & -z_1^{\mathrm{T}} K_1 z_1 - z_2^{\mathrm{T}} K_2 z_2 + z_2^{\mathrm{T}} G z_3 + z_2^{\mathrm{T}} F_2(x_3) \\ & + \frac{1}{2}\|\varepsilon\|^2 + \frac{1}{2}\|z_2\|^2 - \frac{\sigma}{2}\|\tilde{\Theta}\|^2 + \frac{\sigma}{2}\|\Theta^*\|^2 \\ & - \lambda z_3^{\mathrm{T}} x_3 + \lambda z_3^{\mathrm{T}} u - z_3^{\mathrm{T}} \dot{\alpha}_2 \end{aligned} \quad (47)$$

The control law u is proposed as follows (see (48))

where $K_3 = K_3^{\mathrm{T}} > 0$ and ε_3 are the design parameters.

The above design procedure can be summarised in the following theorem:

Theorem 2: Consider the helicopter attitude dynamics (13) satisfies the Assumptions 1–2. The robust attitude control is designed according to (48) using NNs and parameter updated law is chosen as (41). For bounded initial conditions, there exist design parameters $\sigma > 0$, $\Lambda = \Lambda^{\mathrm{T}} > 0$, $K_1 = K_1^{\mathrm{T}} > 0$, $K_2 = K_2^{\mathrm{T}} > 0$ and $K_3 = K_3^{\mathrm{T}} > 0$ such that the overall closed-loop control system is semi-globally stable in the sense that all of the closed-loop signals z_1, z_2, z_3 and $\tilde{\Theta}$ are bounded. Furthermore, the tracking error z_1 converges to the compact set $\Omega_{z_1} := \{ z_1 \in R^4 | \ \|z_1\| \leq \sqrt{D} \}$ where $D = 2(V_3(0) + (C/\kappa))$, C and κ are defined in (49).

Proof: When $\|z_3\| \geq \varepsilon_3$, substituting (48) into (47), we obtain

$$\begin{aligned}\dot{V}_3 \leq & -z_1^{\mathrm{T}} K_1 z_1 - z_2^{\mathrm{T}} \left(K_2 - \frac{1}{2} I_{3\times 3}\right) z_2 \\ & - z_3^{\mathrm{T}} K_3 z_3 + \frac{1}{2}\|\varepsilon\|^2 - \frac{\sigma}{2}\|\tilde{\Theta}\|^2 + \frac{\sigma}{2}\|\Theta^*\|^2 \\ \leq & -\kappa V_3 + C \end{aligned} \quad (49)$$

where

$$\begin{aligned}\kappa : &= \min\Bigg(2\lambda_{\min}(K_1), 2\lambda_{\min}\left(K_2 - \frac{1}{2} I_{3\times 3}\right), \\ & 2\lambda_{\min}(K_3), \frac{2\sigma}{\lambda_{\max}(\Lambda^{-1})}\Bigg) \\ C : &= \frac{1}{2}\|\bar{\varepsilon}\|^2 + \frac{\sigma}{2}\|\Theta^*\|^2 \end{aligned} \quad (50)$$

To ensure that $\kappa > 0$, the design parameter K_2 must make $K_2 - (1/2) I_{3\times 3} > 0$.

Multiplying (49) by $e^{\kappa t}$ yields

$$\frac{\mathrm{d}}{\mathrm{d}t}(V_3(t) e^{\kappa t}) \leq e^{\kappa t} C \quad (51)$$

Integrating (51) over $[0, t]$, we obtain

$$0 \leq V_3(t) \leq \frac{C}{\kappa} + \left[V_3(0) - \frac{C}{\kappa}\right] e^{-\kappa t} \quad (52)$$

According to (51) and (52), we can prove that the bounded stability of the closed-loop system when $\|z_3\| \geq \varepsilon_3$. When $\|z_3\| < \varepsilon_3$, we can also conclude that all signals of the closed-loop system are bounded based on Lemma 1 if only appropriate design parameters K_1 and K_2 are chosen according to the bounded $F_2(x_3)$. Therefore all signals of the closed-loop system, that is, z_1, z_2, z_3 and $\tilde{\Theta}$, are uniformly ultimately bounded. From (46), we know that for any given design parameters σ, Λ, K_1, K_2 and K_3 can be used to adjust the closed-loop system performance. This concludes the proof. □

5 Approximation-based attitude control of helicopters

In this section, we consider the case where all moment coefficients and the helicopter mass are unknown which make the attitude control design of (13) more complicated. The approximation-based control in combination with the backstepping technique is employed to keep the desired attitude of helicopter system (13). Define an auxiliary design variable $\xi = J(x_1) x_2$ and the error variables $z_1 = x_1 - x_{1d}$, $z_2 = \xi - \alpha_1$ and $z_3 = x_3 - \alpha_2$, where $\alpha_1 \in \mathcal{R}^4$ and $\alpha_2 \in \mathcal{R}^4$ are virtual control laws. It is apparent that $x_2 \to 0$ if $\xi \to 0$ because of the non-singularity of $J(x_1)$.

$$u = \begin{cases} x_3 - \lambda^{-1}\left(K_3 z_3 - \dot{\alpha}_2 + G^{\mathrm{T}} z_2 + \dfrac{z_3 z_2^{\mathrm{T}} F_2(x_3)}{\|z_3\|^2}\right), & \|z_3\| \geq \varepsilon_3 \\ 0, & \|z_3\| < \varepsilon_3 \end{cases} \quad (48)$$

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Step 1: Differentiating z_1 in (13) with respect to time yields

$$\dot{z}_1 = \dot{x}_1 - \dot{x}_{1d} = \xi - \dot{x}_{1d} = z_2 + \alpha_1 - \dot{x}_{1d} \tag{53}$$

The virtual control law α_1 is chosen as

$$\alpha_1 = -K_1 z_1 + \dot{x}_{1d} \tag{54}$$

where $K_1 = K_1^{\mathrm{T}} > 0$.

Substituting (54) into (53), we obtain

$$\dot{z}_1 = -K_1 z_1 + z_2 \tag{55}$$

Consider the Lyapunov function candidate $V_1 = (1/2)z_1^{\mathrm{T}} z_1$. The time derivative of V_1 is

$$\dot{V}_1 = -z_1^{\mathrm{T}} K_1 z_1 + z_1^{\mathrm{T}} z_2 \tag{56}$$

Step 2: Differentiating z_2 with respect to time yields

$$\begin{aligned}\dot{z}_2 &= \dot{\xi} - \dot{\alpha}_1 = \dot{J}(x_1)x_2 + J(x_1)\dot{x}_2 - \dot{\alpha}_1 \\ &= \dot{J}(x_1)x_2 + J(x_1)F(x_1, x_2) \\ &\quad + J(x_1)Gx_3 + J(x_1)\Delta F_1(x_1, x_2) \\ &\quad + J(x_1)F_2(x_3) + J(x_1)D(x_1, x_2, t) - \dot{\alpha}_1\end{aligned} \tag{57}$$

where $\dot{\alpha}_1 = \ddot{x}_{1d} - K_1\dot{z}_1$.

Consider the Lyapunov function candidate

$$V_2^* = V_1 + \frac{1}{2} z_2^{\mathrm{T}} z_2 \tag{58}$$

Owing to (57) and Assumption 1, the time derivative of V_2^* is given by

$$\begin{aligned}\dot{V}_2^* &\le -z_1^{\mathrm{T}} K_1 z_1 + z_1^{\mathrm{T}} z_2 + z_2^{\mathrm{T}} J(x_1)F_2(x_3) \\ &\quad - z_2^{\mathrm{T}}\rho_1(Z_1) - z_2^{\mathrm{T}}\rho_2(Z_2)\alpha_2 - z_2^{\mathrm{T}}\rho_2(Z_2)z_3 - z_2^{\mathrm{T}}\dot{\alpha}_1\end{aligned} \tag{59}$$

where $\rho_1(Z_1) = -\dot{J}(x_1)x_2 - J(x_1)F(x_1, x_2) - J(x_1)\Delta F_1(x_1, x_2) - J(x_1)\ \mathrm{Sgn}(\zeta_2)(d^f(x_1, x_2) + \bar{d})$, $\rho_2(Z_2) = -J(x_1)G$, $Z_1 = [x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \alpha_1^{\mathrm{T}}]$, $Z_2 = x_1$, $\mathrm{Sgn}(\zeta_2) := \mathrm{diag}\{\mathrm{sgn}(\zeta_{2j})\}$, $\zeta_2 = z_2^{\mathrm{T}} J(x_1)$, $d^f(x_1, x_2) := [d_1^f(x_1, x_2), d_2^f(x_1, x_2), d_3^f(x_1, x_2), d_4^f(x_1, x_2)]^{\mathrm{T}}$ and $\bar{d} = [\bar{d}_1, \bar{d}_2, \bar{d}_3, \bar{d}_4]^{\mathrm{T}}$. Since $F(x_1, x_2)$ and G are all unknown, the previous proposed robust attitude control cannot be implemented. To overcome this problem, we utilise RBFNNs in [36] to approximate the unknown terms $\rho_1(Z_1)$ and $\rho_2(Z_2)$ as

$$\hat{\rho}_1(Z_1) = \hat{\Theta}_1^{\mathrm{T}} S_1(Z_1) \tag{60}$$

$$\hat{\rho}_2(Z_2) = \hat{\Theta}_2^{\mathrm{T}} S_2(Z_2) \tag{61}$$

where $\hat{\Theta}_i$ are the approximation parameters and $S_i(Z_i)$ represents the basis functions, $i = 1, 2$. The optimal approximation $\Theta_i^{*\mathrm{T}} S_i(Z_i)$ given by

$$\Theta_1^{*\mathrm{T}} S_1(Z_1) + \varepsilon_1 = \rho_1(Z_1) \tag{62}$$

$$\Theta_2^{*\mathrm{T}} S_2(Z_2) + \varepsilon_2 = \rho_2(Z_2) \tag{63}$$

where Θ_i^* are optimal weight values of RBFNNs and ε_i is the approximation error satisfying $\|\varepsilon_i\| \le \|\bar{\varepsilon}_i\|$, $i = 1, 2$.

Substituting (62) and (63) into (59), we obtain

$$\begin{aligned}\dot{V}_2^* &\le -z_1^{\mathrm{T}} K_1 z_1 + z_1^{\mathrm{T}} z_2 + z_2^{\mathrm{T}} J(x_1)F_2(x_3) \\ &\quad + z_2^{\mathrm{T}}\tilde{\Theta}_1^{\mathrm{T}} S_1(Z_1) - z_2^{\mathrm{T}}\varepsilon_1 + z_2^{\mathrm{T}}\tilde{\Theta}_2^{\mathrm{T}} S_2(Z_2)\alpha_2 - z_2^{\mathrm{T}}\varepsilon_2\alpha_2 \\ &\quad - z_2^{\mathrm{T}}\hat{\Theta}_1^{\mathrm{T}} S_1(Z_1) - z_2^{\mathrm{T}}\hat{\Theta}_2^{\mathrm{T}} S_2(Z_1)\alpha_2 - z_2^{\mathrm{T}}\rho_2(Z_1)z_3 - z_2^{\mathrm{T}}\dot{\alpha}_1\end{aligned} \tag{64}$$

where $\tilde{\Theta}_1 = \hat{\Theta}_1 - \Theta_1^*$ and $\tilde{\Theta}_2 = \hat{\Theta}_2 - \Theta_2^*$.

The virtual control law α_2 is proposed based on the NNs as follows

$$\alpha_2 = \hat{\rho}_2^{+}(Z_2)\alpha_{20} \tag{65}$$

where

$$\alpha_{20} = K_2 z_2 - \dot{\alpha}_1 - \hat{\rho}_1(Z_1) + z_1 \tag{66}$$

$$\hat{\rho}_2^{+}(Z_2) = \hat{\rho}_2^{\mathrm{T}}(Z_2)[\delta I_{4\times 4} + \hat{\rho}_2(Z_2)\hat{\rho}_2^{\mathrm{T}}(Z_2)]^{-1} \tag{67}$$

herein $K_2 = K_2^{\mathrm{T}} > 0$ and $\delta > 0$ are design parameters.

It is clear that we have

$$\begin{aligned}&\hat{\rho}_2(Z_2)\hat{\rho}_2^{\mathrm{T}}(Z_2)[\delta I_{4\times 4} + \hat{\rho}_2(Z_2)\hat{\rho}_2^{\mathrm{T}}(Z_2)]^{-1} \\ &\quad = I - \delta[\delta I_{4\times 4} + \hat{\rho}_2(Z_2)\hat{\rho}_2^{\mathrm{T}}(Z_2)]^{-1}\end{aligned} \tag{68}$$

Substituting (65) and (68) into (64), we obtain

$$\begin{aligned}\dot{V}_2^* &\le -z_1^{\mathrm{T}} K_1 z_1 - z_2^{\mathrm{T}} K_2 z_2 + z_2^{\mathrm{T}}\tilde{\Theta}_1^{\mathrm{T}} S_1(Z_1) \\ &\quad - z_2^{\mathrm{T}}\varepsilon_1 + z_2^{\mathrm{T}}\tilde{\Theta}_2^{\mathrm{T}} S_2(Z_2)\alpha_2 - z_2^{\mathrm{T}}\varepsilon_2\alpha_2 \\ &\quad + \delta z_2^{\mathrm{T}}[\delta I_{4\times 4} + \hat{\rho}_2(Z_2)\hat{\rho}_2^{\mathrm{T}}(Z_2)]^{-1}\alpha_{20} \\ &\quad - z_2^{\mathrm{T}}\rho_2(Z_2)z_3 + z_2^{\mathrm{T}} J(x_1)F_2(x_3)\end{aligned} \tag{69}$$

Considering the stability of error signals $\tilde{\Theta}_1$ and $\tilde{\Theta}_2$, the augmented Lyapunov function candidate can be written as

$$V_2 = V_2^* + \frac{1}{2}\mathrm{tr}(\tilde{\Theta}_1^{\mathrm{T}}\Lambda_1^{-1}\tilde{\Theta}_1) + \frac{1}{2}\mathrm{tr}(\tilde{\Theta}_2^{\mathrm{T}}\Lambda_2^{-1}\tilde{\Theta}_2) \tag{70}$$

where $\Lambda_1 = \Lambda_1^{\mathrm{T}} > 0$ and $\Lambda_2 = \Lambda_2^{\mathrm{T}} > 0$.

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The time derivative of V_2 along (69) is

$$\begin{aligned}\dot{V}_2 \le & -z_1^{\mathrm{T}}K_1z_1 - z_2^{\mathrm{T}}K_2z_2 + z_2^{\mathrm{T}}J(x_1)F_2(x_3) \\ & + z_2^{\mathrm{T}}\tilde{\Theta}_1^{\mathrm{T}}S_1(Z_1) - z_2^{\mathrm{T}}\varepsilon_1 + z_2^{\mathrm{T}}\tilde{\Theta}_2^{\mathrm{T}}S_2(Z_2)\alpha_2 - z_2^{\mathrm{T}}\varepsilon_2\alpha_2 \\ & + \delta z_2^{\mathrm{T}}[\delta I_{4\times 4} + \hat{\rho}_2(Z_2)\hat{\rho}_2^{\mathrm{T}}(Z_2)]^{-1}\alpha_{20} \\ & - z_2^{\mathrm{T}}\rho_2(Z_2)z_3 + \mathrm{tr}(\tilde{\Theta}_1^{\mathrm{T}}\Lambda_1^{-1}\dot{\hat{\Theta}}_1) + \mathrm{tr}(\tilde{\Theta}_2^{\mathrm{T}}\Lambda_2^{-1}\dot{\hat{\Theta}}_2)\end{aligned} \tag{71}$$

Consider the adaptive laws for $\hat{\Theta}_1$ and $\hat{\Theta}_2$ as

$$\dot{\hat{\Theta}}_1 = -\Lambda_1(S_1(Z_1)z_2^{\mathrm{T}} + \sigma_1\hat{\Theta}_1) \tag{72}$$

$$\dot{\hat{\Theta}}_2 = -\Lambda_2(S_2(Z_1)\alpha_2 z_2^{\mathrm{T}} + \sigma_2\hat{\Theta}_2) \tag{73}$$

where $\sigma_1 > 0$ and $\sigma_2 > 0$.

Noting the following facts

$$-z_2^{\mathrm{T}}\varepsilon_1 \le \frac{1}{2}\|\varepsilon_1\|^2 + \frac{1}{2}\|z_2\|^2 \tag{74}$$

$$-z_2^{\mathrm{T}}\varepsilon_2\alpha_2 \le \frac{1}{2}\|\varepsilon_2\|^2 + \frac{1}{2}\|z_2\|^2\|\alpha_2\|^2 \tag{75}$$

$$\begin{aligned}2\tilde{\Theta}_1^{\mathrm{T}}\hat{\Theta}_1 &= \|\tilde{\Theta}_1\|^2 + \|\hat{\Theta}_1\|^2 - \|\Theta_1^*\|^2 \\ &\ge \|\tilde{\Theta}_1\|^2 - \|\Theta_1^*\|^2\end{aligned} \tag{76}$$

$$\begin{aligned}2\tilde{\Theta}_2^{\mathrm{T}}\hat{\Theta}_2 &= \|\tilde{\Theta}_2\|^2 + \|\hat{\Theta}_2\|^2 - \|\Theta_2^*\|^2 \\ &\ge \|\tilde{\Theta}_2\|^2 - \|\Theta_2^*\|^2\end{aligned} \tag{77}$$

and considering (72) and (73), we have

$$\begin{aligned}\dot{V}_2 \le & -z_1^{\mathrm{T}}K_1z_1 - z_2^{\mathrm{T}}K_2z_2 + z_2^{\mathrm{T}}J(x_1)F_2(x_3) + \frac{1}{2}\|\varepsilon_1\|^2 \\ & + \frac{1}{2}\|z_2\|^2 + \frac{1}{2}\|\varepsilon_2\|^2 + \frac{1}{2}\|z_2\|^2\|\alpha_2\|^2 \\ & + \delta z_2^{\mathrm{T}}[\delta I_{4\times 4} + \hat{\rho}_2(Z_2)\hat{\rho}_2^{\mathrm{T}}(Z_2)]^{-1}\alpha_{20} - z_2^{\mathrm{T}}\rho_2(Z_2)z_3 \\ & - \frac{\sigma_1}{2}\|\tilde{\Theta}_1\|^2 + \frac{\sigma_1}{2}\|\Theta_1^*\|^2 - \frac{\sigma_2}{2}\|\tilde{\Theta}_2\|^2 + \frac{\sigma_2}{2}\|\Theta_2^*\|^2\end{aligned} \tag{78}$$

Step 3: Differentiating z_3 with respect to time yields

$$\dot{z}_3 = \dot{x}_3 - \dot{\alpha}_2 = -\lambda(x_3 - u) - \dot{\alpha}_2 \tag{79}$$

Consider the Lyapunov function candidate

$$V_3^* = V_2 + \frac{1}{2}z_3^{\mathrm{T}}z_3 \tag{80}$$

Considering (79), the time derivative of V_3^* is

$$\begin{aligned}\dot{V}_3^* \le & -z_1^{\mathrm{T}}K_1z_1 - z_2^{\mathrm{T}}K_2z_2 + z_2^{\mathrm{T}}J(x_1)F_2(x_3) \\ & + \frac{1}{2}\|\varepsilon_1\|^2 + \frac{1}{2}\|z_2\|^2 + \frac{1}{2}\|\varepsilon_2\|^2 + \frac{1}{2}\|z_2\|^2\|\alpha_2\|^2 \\ & + \delta z_2^{\mathrm{T}}[\delta I_{4\times 4} + \hat{\rho}_2(Z_1)\hat{\rho}_2^{\mathrm{T}}(Z_1)]^{-1}\alpha_{20} \\ & - z_2^{\mathrm{T}}\rho_2(Z_1)z_3 - \frac{\sigma_1}{2}\|\tilde{\Theta}_1\|^2 \\ & + \frac{\sigma_1}{2}\|\Theta_1^*\|^2 - \frac{\sigma_2}{2}\|\tilde{\Theta}_2\|^2 + \frac{\sigma_2}{2}\|\Theta_2^*\|^2 \\ & - \lambda z_3^{\mathrm{T}}x_3 + \lambda z_3^{\mathrm{T}}u - z_3^{\mathrm{T}}\dot{\alpha}_2\end{aligned} \tag{81}$$

From (14), we know that $F_2(x_3)$ is unknown because of the unknown moment coefficients. Thus, it cannot be used to design the attitude control. To conveniently develop the approximation-based attitude control, the following variables are given

$$\bar{\zeta}_2 = [\zeta_{22}, \zeta_{23}, \zeta_{24}]^{\mathrm{T}} \tag{82}$$

$$\bar{F}_2(x_3) = [f_{22}(x_3), f_{23}(x_3), f_{24}(x_3)]^{\mathrm{T}} \tag{83}$$

where ζ_{2i} is the ith row of vector $z_2^{\mathrm{T}}J(x_1)$.

Considering (82) and (83), (81) can be written as

$$\begin{aligned}\dot{V}_3^* \le & -z_1^{\mathrm{T}}K_1z_1 - z_2^{\mathrm{T}}K_2z_2 + \zeta_{21}f_{31}(x_3) + \varsigma_2^{\mathrm{T}}\varphi + \frac{1}{2}\|\varepsilon_1\|^2 \\ & + \frac{1}{2}\|z_2\|^2 + \frac{1}{2}\|\varepsilon_2\|^2 + \frac{1}{2}\|z_2\|^2\|\alpha_2\|^2 \\ & + \delta z_2^{\mathrm{T}}[\delta I_{4\times 4} + \hat{\rho}_2(Z_1)\hat{\rho}_2^{\mathrm{T}}(Z_1)]^{-1}\alpha_{20} \\ & - z_2^{\mathrm{T}}\rho_2(Z_1)z_3 - \frac{\sigma_1}{2}\|\tilde{\Theta}_1\|^2 \\ & + \frac{\sigma_1}{2}\|\Theta_1^*\|^2 - \frac{\sigma_2}{2}\|\tilde{\Theta}_2\|^2 + \frac{\sigma_2}{2}\|\Theta_2^*\|^2 \\ & - \lambda z_3^{\mathrm{T}}x_3 + \lambda z_3^{\mathrm{T}}u - z_3^{\mathrm{T}}\dot{\alpha}_2\end{aligned} \tag{84}$$

where $\varsigma = [\zeta_{22}f_{32}, \zeta_{22}f_{34} + \zeta_{24}f_{32}, \zeta_{23}f_{34}, \zeta_{24}f_{34}]^{\mathrm{T}}$ and $\varphi = [c_3, c_4, c_7, c_9]^{\mathrm{T}}$.

The approximation-based attitude control u is proposed based on the NNs as follows

$$u = \begin{cases} u_0, & \|z_3\| \ge \varepsilon_3 \\ 0, & \|z_3\| < \varepsilon_3 \end{cases} \tag{85}$$

where $u_0 = x_3 - \lambda^{-1}(\delta z_2^{\mathrm{T}}[\delta I_{4\times 4} + \hat{\rho}_2(Z_1)\hat{\rho}_2^{\mathrm{T}}(Z_1)]^{-1})\alpha_{20} + \zeta_{21}f_{31}(x_3) + \varsigma_2^{\mathrm{T}}\hat{\varphi} + \|z_2\|^2 + (1/2)\|z_2\|^2\|\alpha_2\|^2)/\|z_3\|^2 z_3 - \lambda^{-1}(K_3z_3 + 2\gamma\|z_2\|^2 z_3 + (1/2)\|z_2\|^2 z_3 + \dot{\alpha}_2 + \hat{\rho}_2^{\mathrm{T}}(Z_1)z_2)$, $K_3 = K_3^{\mathrm{T}} > 0$, $\varepsilon_3 > 0$ and $\gamma > 0$ are design parameters.

Substituting (85) into (84), we obtain

$$\begin{aligned}\dot{V}_3^* \le & -z_1^{\mathrm{T}}K_1z_1 - z_2^{\mathrm{T}}K_2z_2 - z_3^{\mathrm{T}}K_3z_3 - \varsigma_2^{\mathrm{T}}\tilde{\varphi} \\ & + \frac{1}{2}\|\varepsilon_1\|^2 + \frac{1}{2}\|\varepsilon_2\|^2 - 2\gamma\|z_2\|^2\|z_3\|^2 - \frac{1}{2}\|z_2\|^2\|z_3\|^2 \\ & - z_2^{\mathrm{T}}\tilde{\Theta}_2^{\mathrm{T}}S_2(Z_1)z_3 - z_2^{\mathrm{T}}\varepsilon_2 z_3 - \frac{1}{2}\|z_2\|^2 \\ & - \frac{\sigma_1}{2}\|\tilde{\Theta}_1\|^2 + \frac{\sigma_1}{2}\|\Theta_1^*\|^2 - \frac{\sigma_2}{2}\|\tilde{\Theta}_2\|^2 + \frac{\sigma_2}{2}\|\Theta_2^*\|^2\end{aligned} \quad (86)$$

where $\tilde{\varphi} = \hat{\varphi} - \varphi$.

To achieve the stability of error signals $\tilde{\varphi}$, the augmented Lyapunov function candidate can be chosen as

$$V_3 = V_3^* + \frac{1}{2}\tilde{\varphi}^{\mathrm{T}}\Lambda_3^{-1}\tilde{\varphi} \quad (87)$$

where $\Lambda_3 = \Lambda_3^{\mathrm{T}} > 0$.

Choose the adaptive law for $\hat{\varphi}$ as

$$\dot{\hat{\varphi}} = \Lambda_3(\varsigma_2 - \sigma_3\hat{\varphi}) \quad (88)$$

where $\sigma_3 > 0$

Considering (88), the time derivative of V_3 along (86) is

$$\begin{aligned}\dot{V}_3^* \le & -z_1^{\mathrm{T}}K_1z_1 - z_2^{\mathrm{T}}K_2z_2 - z_3^{\mathrm{T}}K_3z_3 + \frac{1}{2}\|\varepsilon_1\|^2 + \frac{1}{2}\|\varepsilon_2\|^2 \\ & - 2\gamma\|z_2\|^2\|z_3\|^2 - \frac{1}{2}\|z_2\|^2\|z_3\|^2 - z_2^{\mathrm{T}}\tilde{\Theta}_2^{\mathrm{T}}S_2(Z_1)z_3 \\ & - z_2^{\mathrm{T}}\varepsilon_2 z_3 - \frac{1}{2}\|z_2\|^2 - \frac{\sigma_1}{2}\|\tilde{\Theta}_1\|^2 + \frac{\sigma_1}{2}\|\Theta_1^*\|^2 - \frac{\sigma_2}{2}\|\tilde{\Theta}_2\|^2 \\ & + \frac{\sigma_2}{2}\|\Theta_2^*\|^2 - \frac{\sigma_3}{2}\|\tilde{\varphi}\|^2 + \frac{\sigma_3}{2}\|\varphi\|^2\end{aligned} \quad (89)$$

It is apparent that there are the following facts

$$\begin{aligned}-z_2^{\mathrm{T}}\tilde{\Theta}_2^{\mathrm{T}}S_2(Z_1)z_3 & \le \gamma\|z_2\|\|\tilde{\Theta}_2\|\|z_3\| \\ & \le 2\gamma\|z_2\|^2\|z_3\|^2 + \frac{\gamma\|\tilde{\Theta}_2\|^2}{2}\end{aligned} \quad (90)$$

$$-z_2^{\mathrm{T}}\varepsilon_2 z_3 \le \|z_2\|\|\varepsilon_2\|\|z_3\| \le \frac{1}{2}\|\varepsilon_2\|^2 + \frac{1}{2}\|z_2\|^2\|z_3\|^2 \quad (91)$$

where $\gamma > 0$.

Substituting (90) and (91) into (89) yields

$$\begin{aligned}\dot{V}_3 \le & -z_1^{\mathrm{T}}K_1z_1 - z_2^{\mathrm{T}}K_2z_2 - z_3^{\mathrm{T}}K_3z_3 + \frac{1}{2}\|\varepsilon_1\|^2 + \|\varepsilon_2\|^2 \\ & - \frac{\sigma_1}{2}\|\tilde{\Theta}_1\|^2 + \frac{\sigma_1}{2}\|\Theta_1^*\|^2 - \frac{\sigma_2 - \gamma}{2}\|\tilde{\Theta}_2\|^2 \\ & + \frac{\sigma_2}{2}\|\Theta_2^*\|^2 - \frac{\sigma_3}{2}\|\tilde{\varphi}\|^2 + \frac{\sigma_3}{2}\|\varphi\|^2 \\ \le & -\kappa V_3 + C\end{aligned} \quad (92)$$

where

$$\begin{aligned}\kappa :=\ & \min\Big(2\lambda_{\min}(K_1), 2\lambda_{\min}(K_2), 2\lambda_{\min}(K_3), \\ & \frac{2\sigma_1}{\lambda_{\max}(\Lambda_1^{-1})}, \frac{\sigma_2 - \gamma}{\lambda_{\max}(\Lambda_2^{-1})}, \frac{2\sigma_3}{\lambda_{\max}(\Lambda_3^{-1})}\Big) \\ C :=\ & \frac{1}{2}\|\bar{\varepsilon}_1\|^2 + \|\bar{\varepsilon}_2\|^2 + \frac{\sigma_1}{2}\|\Theta_1^*\|^2 + \frac{\sigma_2}{2}\|\Theta_2^*\|^2 + \frac{\sigma_3}{2}\|\varphi\|^2\end{aligned} \quad (93)$$

To ensure that $\kappa > 0$, the design parameter σ_2 and γ must make $\sigma_2 - \gamma > 0$. The above design procedure can be summarised in the following theorem.

Theorem 3: Considering the helicopter attitude dynamics (13) with unknown moment coefficients and mass, model uncertainty and disturbance satisfy Assumptions 1–2. The approximation-based flight control is proposed according to (85) using NNs and parameter updated laws are chosen as (72), (73) and (88). For bounded initial conditions, there exist design parameters $\sigma_i > 0$, $i = 1, 2, 3$, γ, $\Lambda_1 = \Lambda_1^{\mathrm{T}} > 0$, $\Lambda_2 = \Lambda_2^{\mathrm{T}} > 0$, $\Lambda_3 = \Lambda_3^{\mathrm{T}} > 0$, $K_1 = K_1^{\mathrm{T}} > 0$, $K_2 = K_2^{\mathrm{T}} > 0$ and $K_3 = K_3^{\mathrm{T}} > 0$, such that the overall closed-loop control system is semi-globally stable in the sense that all of the closed-loop signals z_1, z_2, z_3, $\tilde{\Theta}_1$, $\tilde{\Theta}_2$ and $\tilde{\varphi}$ are bounded.

Proof: The proof is similar to that of Theorem 2 and omitted here because of the limited space. □

6 Simulation results

In this section, extensive simulations are given to demonstrate the effectiveness of the proposed helicopter attitude control techniques. The APID MK-III helicopter model is used in our simulation, which is described by [22, 37]

$$\begin{aligned}\ddot{z} &= \frac{1}{m}(Z_g - K_{\mathrm{M}}\Omega^2\theta_{\mathrm{M}}\cos\phi\cos\theta) \\ \ddot{\phi} &= -a\dot{\phi} + dK_{\mathrm{M}}\Omega^2 b_{1_s}\theta_{\mathrm{M}} \\ \ddot{\theta} &= -b\dot{\theta} - eK_{\mathrm{M}}\Omega^2 a_{1_s}\theta_{\mathrm{M}} \\ \ddot{\psi} &= -c\dot{\psi} + f(\theta_{\mathrm{T}} + \psi_{\mathrm{T}}) \\ \dot{b}_{1_s} &= -\lambda(b_{1_s} - u_{b_{1_s}}) \\ \dot{a}_{1_s} &= -\lambda(a_{1_s} - u_{a_{1_s}})\end{aligned}$$

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Table 1 Parameters of the helicopter

$m = 50$ kg, $a = 8.7072$, $b = 10.1815$, $c = 0.434$, $K_M\Omega^2 = 1703.4$, $dK_M\Omega^2 = 223.5824$, $eK_M\Omega^2 -58.3528$, $f = 31.9065$, $\lambda = 300$

Table 2 Design parameters of the model-based attitude control

$K_1 = \text{diag}\{2, 2, 2, 2\}$, $K_2 = \text{diag}\{100, 100, 100, 100\}$, $K_3 = \text{diag}\{300, 300, 300, 300\}$

$$\dot{\theta}_M = -\lambda(\theta_M - u_{\theta_M})$$
$$\dot{\theta}_T = \lambda(\theta_T - u_{\theta_T}) \tag{94}$$

It is apparent that the dynamics of APID MK-III helicopter as shown in (94) has the same form with the model (1) if we neglect the flapping dynamics and engine dynamics. The helicopter's nonminal parameters are shown in Table 1.

In this simulation, the control objective is to keep a certain desired altitude/attitude of the helicopter. In Subsections 6.1 and 6.2, we test the proposed flight control on the desired maneuver requiring aggressive attitude configurations and suppose that the desired altitude/attitude is $x_{1d} = [20, 0.2\sin(1.5t) + 0.5\cos(0.5t), 0.4(\sin(t) + 0.5\sin(0.5t)), 0.1\sin(1.5t) + 0.4\cos(0.5t)]^T$. Initial states $z = 15.0$, $\phi = 0.2$, $\theta = -0.1$, $\psi = 0.1$, $b_{1_s} = 0.0$, $a_{1_s} = 0.0$, $\theta_M = 0.0$ and $\theta_T = 0.0$. In Subsection 6.3, the hovering flight is used to illustrate the effectiveness of the proposed approximation-based attitude control. In all cases, the saturation values of the command control signals are chosen as $|u_{b_{1s}}| = |u_{a_{1s}}| = 1.8$ and $|u_{\theta_M}| = |u_{\theta_T}| = 1.0$. The control gain matrix G and the design matrix W are chosen as follows

$$G = \begin{bmatrix} 100 & 100 & -1703.4/m & 0 \\ 223.5824 & 0 & 223.5824 & 0 \\ 0 & -58.3258 & -58.3258 & 0 \\ 0 & 0 & 0 & 31.9065 \end{bmatrix}$$

$$W = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6.1 Model-based attitude control for nominal plant

In this subsection, the model-based attitude control is designed according to (29). The control design parameters are shown in Table 2.

Under the proposed model-based attitude control, it can be observed from Fig. 1 that the maneuver altitude/attitude of the helicopter can be maintained within a small envelop

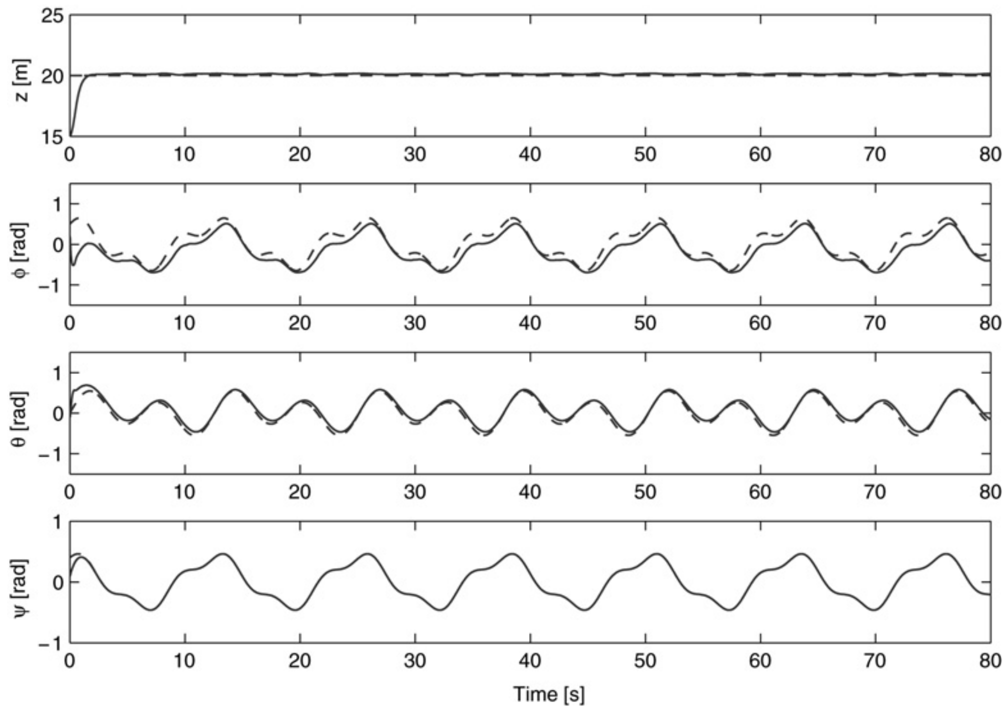

Figure 1 *Altitude/attitude (solid line) follows the desired altitude/attitude (dashed line) under the model-based control*

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of the desired altitude/attitude. From Fig. 2, we know that the velocity along the z-axis and angular velocity under the model-based control converge to a small neighbourhood of the origin. Fig. 3 shows that the command control signals are saturated within the limits of the actuators. There exists chattering phenomenon in Figs. 2 and 3 which are caused by the discontinuous control input and the maneuver flight.

6.2 Robust attitude control of helicopters using NNs

In general, the model-based control is sensitive to disturbance and system uncertainty. When there exist the disturbance and system uncertainty in helicopter dynamics (94), the closed-loop system control performance will be degraded, and even will lead to the closed-loop system unstable. To improve the robustness of the attitude control, the robust attitude control of helicopters using NNs is designed according to (48) and the adaptation law is presented as (41) in this subsection. In this simulation, we consider the parameter uncertainties, function uncertainties and external disturbance in helicopter dynamics (94). Consider that the helicopter has 10% mass (m) uncertainty and 20% system parameter (a, b, c) uncertainties. At the same time, the function uncertainties and external disturbance are give by

$$\begin{aligned}\Delta F_1 &= 5.25(0.3\sin(0.6\dot{z}\dot{\phi}) + 0.04\cos(0.3t) \\ &\quad + 0.06\sin(1.5t)) + 0.5C_dA_cV_W^2 \\ \Delta F_2 &= 4.5(0.2\sin(0.5\dot{\phi}\dot{\theta}) + 0.05\sin(1.5t) + 0.05\sin(0.8t)) \\ \Delta F_3 &= 9.5(0.2\sin(0.6\dot{\psi}\dot{\theta}) + 0.04\sin(0.45t) + 0.06\sin(1.9t)) \\ \Delta F_4 &= 9.45(0.3\sin(0.5\dot{\psi}\dot{z}) + 0.06\sin(0.5t) + 0.04\sin(1.8t))\end{aligned} \tag{95}$$

where $A_c = 4\pi R_c^2$. A_c is the area of the cabin in each direction, C_d is a given drag coefficient and V_W is the wind speed. Here, we choose wind speed $V_W = 10\,\text{m/s}$. The robust control design parameters are chosen as in Table 2.

The simulation results under the robust attitude control (41) are given in Figs. 4–7. It can be observed from Fig. 4 that the altitude/attitude of the helicopter can also be maintained within a small envelop of the desired maneuver

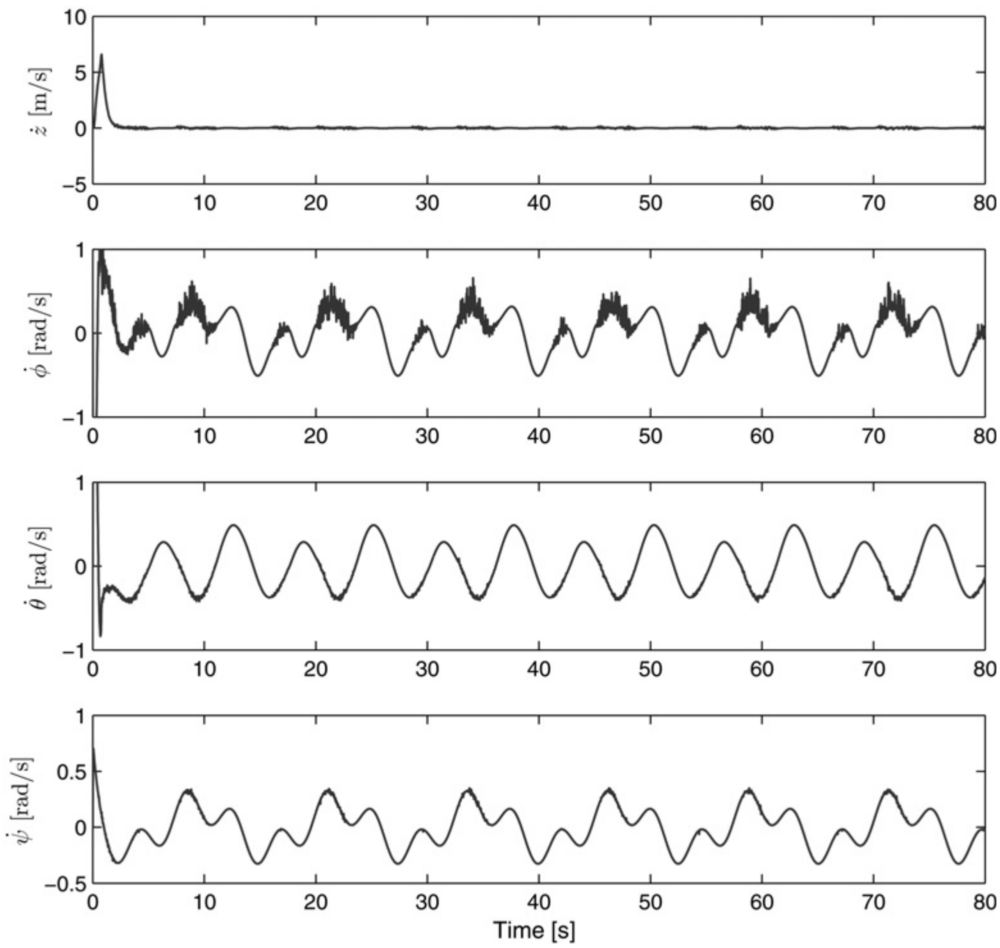

Figure 2 *Velocity along the z-axis and the angular velocity under the model-based control*

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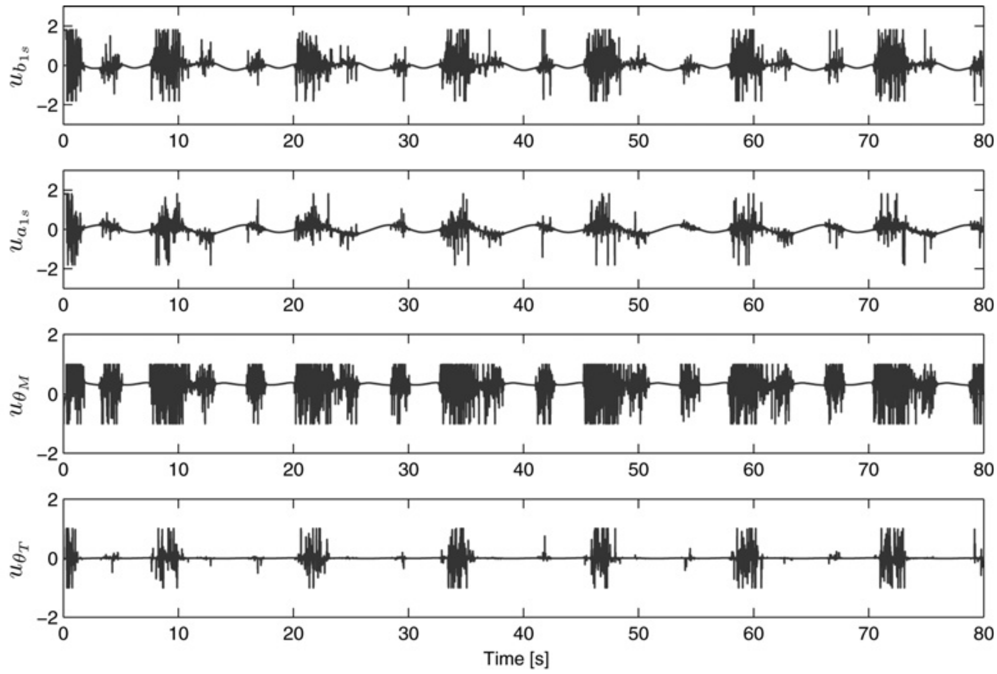

Figure 3 *Command control signal of the model-based control*

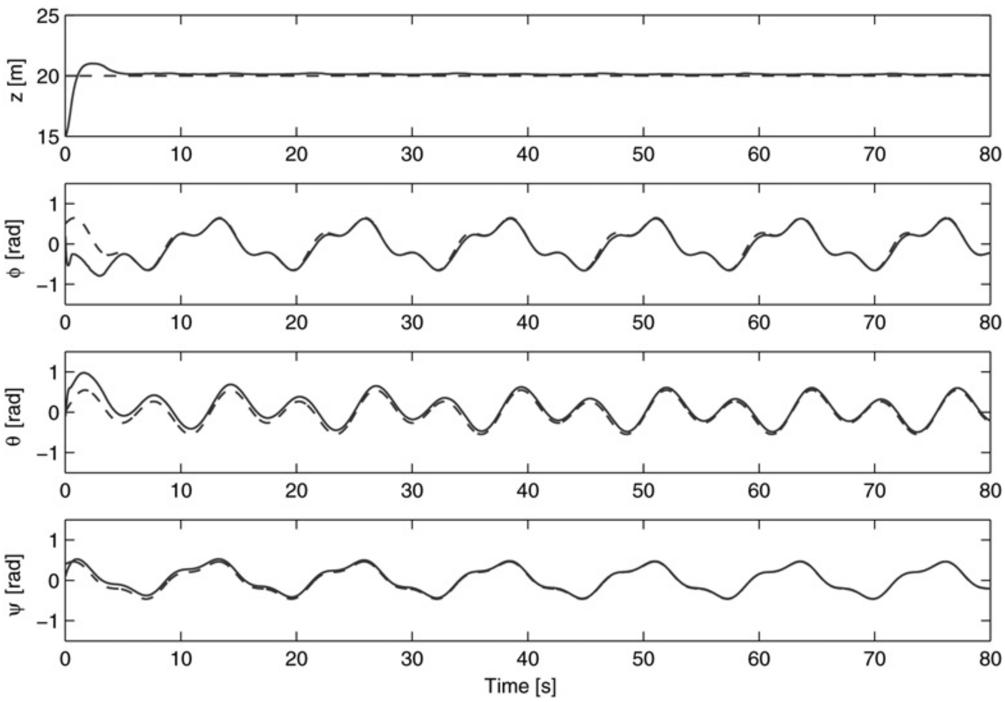

Figure 4 *Altitude/attitude (solid line) follows the desired altitude/attitude (dashed line) for the robust control with disturbance and uncertainty*

altitude/attitude with the uncertainties and disturbances. From Fig. 5, the velocity along the z-axis and the angular velocity with the uncertainties and disturbances can converge to a compact set. Fig. 6 shows that the command control signals are saturated within the limits of the actuators. The norms of approximation parameters adapted online with the uncertainties and disturbances are shown in Fig. 7 and they are bounded.

6.3 Approximation-based attitude control for helicopters

In this subsection, the approximation-based attitude control is designed according to (85). In the approximation-based attitude control, we would like to highlight that parametric uncertainties may exist in the helicopter model. All helicopter moment coefficients and helicopter mass are

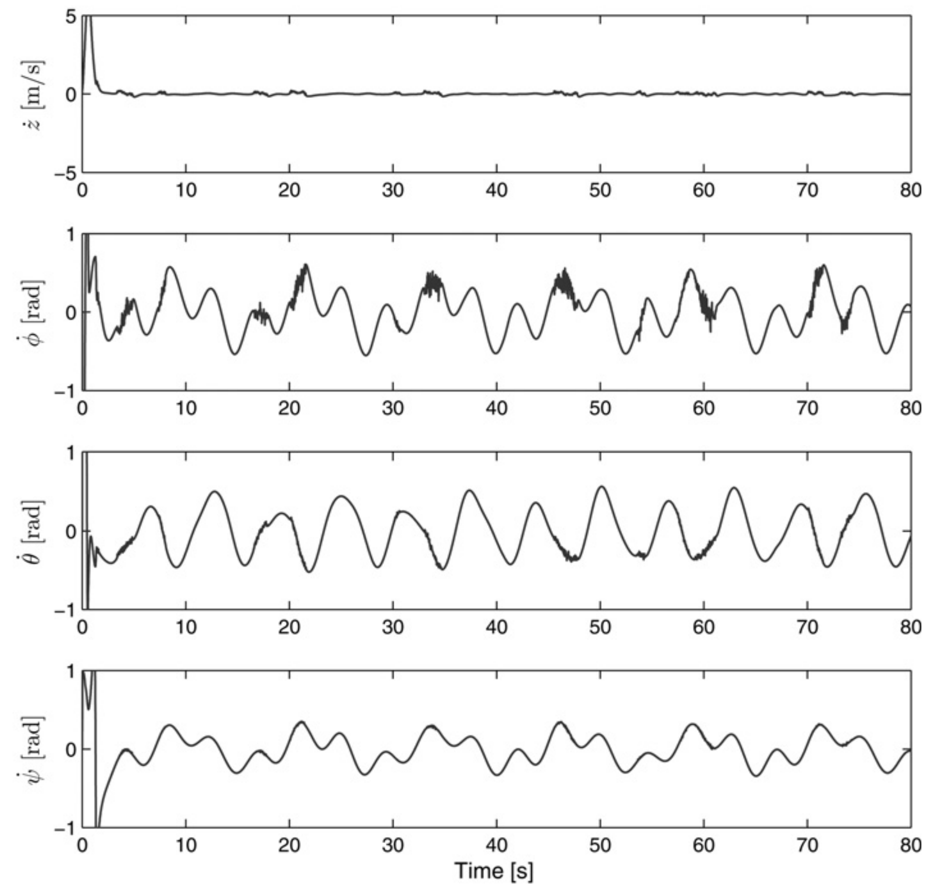

Figure 5 *Velocity along the z-axis and the angular velocity for the robust attitude control with disturbance and uncertainty*

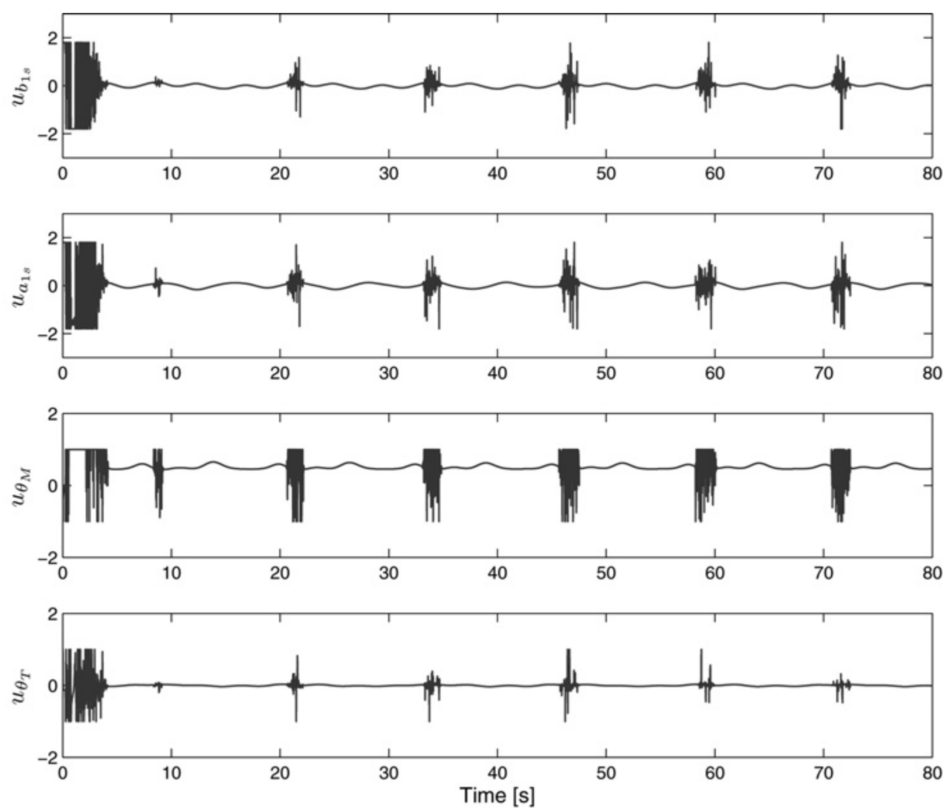

Figure 6 *Command control signal of the robust attitude control*

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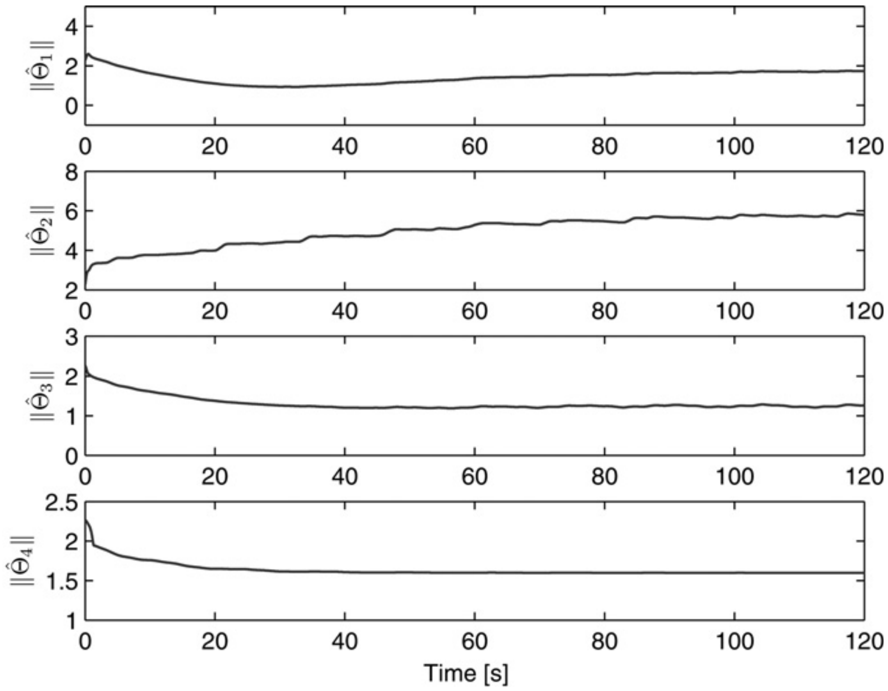

Figure 7 *Norms of approximation parameters adapted online of the robust attitude control*

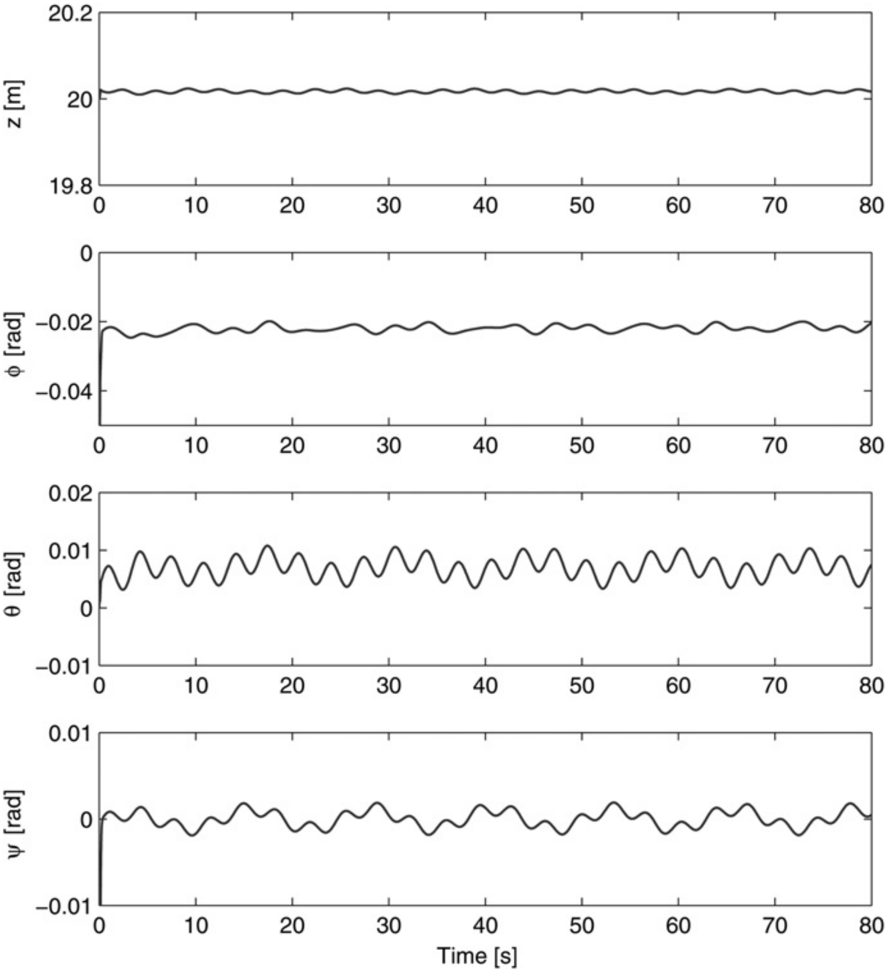

Figure 8 *Altitude/attitude of the hovering helicopter using the approximation-based attitude control*

doi: 10.1049/iet-cta.2009.0478

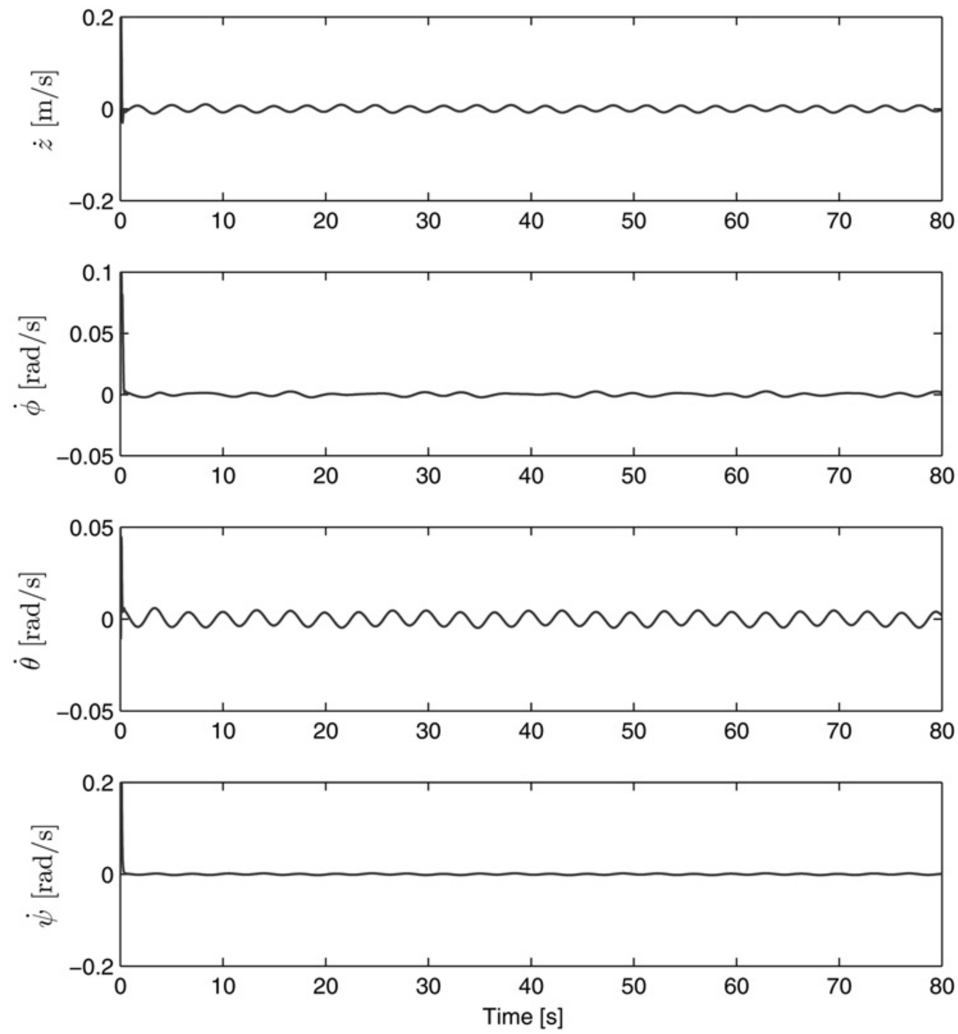

Figure 9 *Velocity along the z-axis and the angular velocity of the hovering helicopter using the approximation-based attitude control*

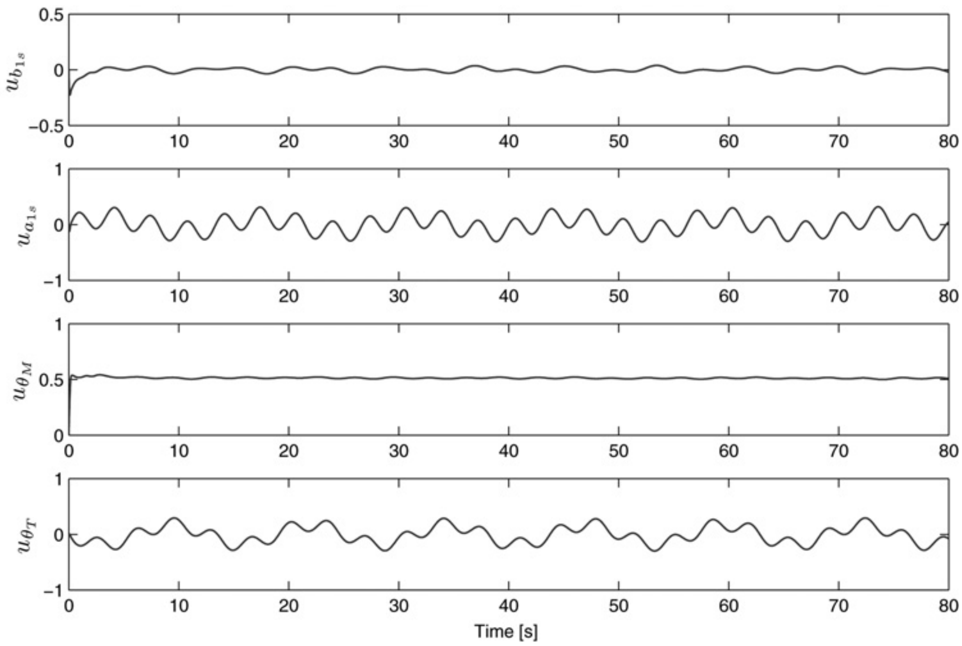

Figure 10 *Command control signal of the hovering helicopter using the approximation-based attitude control*

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completely unknown. At the same time, the function uncertainties and external disturbance described in (95) are included. Here, we assume that the helicopter is hovering flight. Thus, the desired altitude/attitude is given by $x_{1d} = [20, 0, 0, 0]^T$. Initial states $z = 15.0$, $\phi = 0.01$, $\theta = 0$, $\psi = 0.04$, $b_{1s} = 0.0$, $a_{1s} = 0.0$, $\theta_M = 0.0$ and $\theta_T = 0.0$. The saturation values of the command control signals are the same as aforementioned.

In practice, the selection of the centres and widths of RBF has a great influence on the performance of the designed controller. According to [36], Gaussian RBFNNs arranged on a regular lattice on R^n can uniformly approximate sufficiently smooth functions on closed, bounded subsets. Accordingly, in the following simulation studies, the centres and widths are chosen on a regular lattice in the respective compact sets. Specifically, we employ eight nodes for each input dimension of $\hat{W}_1^T S(Z_1)$ and four nodes for each input dimension of $\hat{W}_2^T S(Z_2)$; thus, we end up with 512 nodes (i.e. $l_1 = 512$) with centres μ_i $(i = 1, 2, \ldots, l_1)$ evenly spaced in $[-1.0, +1.0]$ and widths $\eta_i = 2000.0$ for NN $\hat{W}_1^T S(Z_1)$; and 64 nodes (i.e. $l_2 = 64$) with centres μ_j $(j = 1, 2, \ldots, l_2)$ evenly spaced in $[-1.0, +1.0]$ and widths $\eta_j = 1000.0$ for NN $\hat{W}_2^T S(Z_2)$.

Under the proposed approximation-based attitude control (85), we observe that the hovering flight is stable, that is, the altitude/attitude of the helicopter can be maintained within a small envelop of the desired altitude/attitude in Fig. 8. From Fig. 9, it can be observed that the line velocity along the z-axis and the angular velocity of the hovering flight converge to a small neighbourhood of the origin. The command control signals are shown in Fig. 10.

7 Conclusion

In this paper, the model-based control has been presented for the nominal attitude dynamics, followed by the robust attitude control in the presence of parametric uncertainty, function uncertainty and unknown disturbance. Considering the unknown moment coefficients and helicopter mass, the approximation-based attitude control has been investigated for helicopters. In the proposed attitude control techniques, the MIMO non-linear dynamics have been considered and the semi-globally uniform boundedness of the closed-loop signals have been guaranteed via Lyapunov analysis. Finally, simulation studies have been provided to illustrate the effectiveness of the proposed attitude control.

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