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Adaptive tracking control of uncertain MIMO nonlinear systems with input constraints☆

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ABSTRACT

In this paper, adaptive tracking control is proposed for a class of uncertain multi-input and multi-output nonlinear systems with non-symmetric input constraints. The auxiliary design system is introduced to analyze the effect of input constraints, and its states are used to adaptive tracking control design. The spectral radius of the control coefficient matrix is used to relax the nonsingular assumption of the control coefficient matrix. Subsequently, the constrained adaptive control is presented, where command filters are adopted to implement the emulate of actuator physical constraints on the control law and virtual control laws and avoid the tedious analytic computations of time derivatives of virtual control laws in the backstepping procedure. Under the proposed control techniques, the closed-loop semi-global uniformly ultimate bounded stability is achieved via Lyapunov synthesis. Finally, simulation studies are presented to illustrate the effectiveness of the proposed adaptive tracking control.

1. Introduction

During the past several decades, adaptive control of nonlinear systems has received much attention for establishing the globally asymptotical stability of the closed-loop system (Ge, 1996a,b; Ge & Wang, 2003; Hung, Tuan, Narikiyo, & Apkarian, 2008; Krstić & Kokotović, 1995; Luo, Chu, & Ling, 2005; Makoudi & Radouane, 2000; Mirkin & Gutman, 2005; Skjetnea, Fossen, & Kokotović, 2000; Tang, Tao, & Joshi, 2007; Yao & Tomizuka, 2001; Yu & Sun, 2001). In practice, most control plants are nonlinear, uncertain and multivariable in character. It is important to investigate effective adaptive control techniques for uncertain multi-input and multi-output (MIMO) nonlinear systems. In Tang et al. (2007), direct adaptive control was developed for a class of MIMO nonlinear systems in the presence of uncertain failures of redundant actuators. In Yao and Tomizuka (2001), adaptive robust control was proposed for MIMO nonlinear systems in semi-strict feedback forms. Robust adaptive tracking control was developed for the time varying uncertain nonlinear systems with unknown control coefficients (Ge & Wang, 2003). As an effective control technology, adaptive control has been successively used in a variety of practical control systems. In Ge (1996a), adaptive control was proposed for robots with both dynamic parameter uncertainties and unknown input scalings. Adaptive control for flexible joint robots was presented based on singular perturbation theory and position information in Ge (1996b). Adaptive recursive design was developed for a parametric uncertain nonlinear plant describing the dynamics of a ship (Skjetnea et al., 2000). In Luo et al. (2005), inverse optimal adaptive control was presented for the attitude tracking of spacecraft. Adaptive control was studied for nonlinearly parameterized uncertainties in robot manipulators (Hung et al., 2008). In the adaptive control of uncertain MIMO nonlinear systems, one main challenge is the possible singularity of the control coefficient matrix which makes the control design become more complicated. Existing research results of adaptive control techniques for the MIMO nonlinear system mostly assume that the control coefficient matrix is known and nonsingular (Kwan & Lewis, 2000). In this paper, the spectral radius of the control coefficient matrix is introduced in the control design to relax the nonsingular assumption of the control coefficient matrix.

Since actuator physical constraints can severely degrade the closed-loop system performance, control design for uncertain MIMO nonlinear systems with actuator constraints presents a tremendous challenge. During the past decades, there has extensive research on the control of mechanical systems with various

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constraints. Analysis and design of control systems with input saturation constraints have been studied in Cao and Lin (2003), Chen, Ge, and Choo (2009), Chen, Ge, and How (2010), Hu, Ma, and Xie (2008), Gao and Selmic (2006) and Zhong (2005). To handle the physical limitation, constrained adaptive backstepping control was proposed in which command filters were used to implement the emulate of constraints on the control command and the virtual control laws (Farrell, Polycarpou, & Sharma, 2003; Polycarpou, Farrell, & Sharma, 2004, 2003; Sonneveldt, Chu, & Mulder, 2007). In Polycarpou et al. (2004), nonlinear approximation based backstepping control was presented for nonlinear dynamical systems subject to magnitude, rate, and bandwidth constraints. The control input saturation was investigated via on-line approximation based control for uncertain nonlinear systems (Polycarpou et al., 2003). Constrained adaptive backstepping control was presented for fighter aircraft in Sonneveldt et al. (2007). In the constrained adaptive control, the key problem is how to analyze the constraint effect of the actuator's physical constraints. To this end, we introduce an auxiliary design system to analyze the constraint effect in this paper. Based on the states of the auxiliary design system, constrained adaptive control is investigated for a class of uncertain MIMO nonlinear systems with input constraints using backstepping technique.

Backstepping control has became one of the most popular robust adaptive control design techniques for some special classes of nonlinear systems (Gong & Yao, 2001; Wang & Huang, 2005; Zhang, Ge, & Hang, 2000). In recent years, the universal approximation ability of neural network (NN) or fuzzy logical system (FLS) has been employed to design robust adaptive control combing with backstepping technique for the uncertain MIMO nonlinear systems, and various robust adaptive control strategies have been proposed (Chang, 2000, 2001; Chang & Yen, 2005; Ge, 1998; Ge & Wang, 2004; Ge & Tee, 2007; Ge, Li, Zhang, & Lee, 2004; Ge, Zhang, & Lee, 2004; Lee & Lee, 2004; Zhang, Ge, & Lee, 2005). The proposed robust adaptive control based on NN or FLS is an efficient control approach of MIMO nonlinear systems, but the model-based adaptive control should be widely developed due to the relatively easy realization (Narendra & Annaswamy, 1989; Qu, Dorsey, & Dawson, 1994). Furthermore, the adaptive backstepping control of uncertain MIMO nonlinear systems with non-symmetric input constraints need to be further investigated.

In this paper, adaptive tracking control is proposed to handle the input saturation and actuator physical constraints for uncertain MIMO nonlinear systems. The main contributions of the paper are as follows:

(i) To the best of our knowledge, it is the first time in the literature that the non-symmetric nonlinear input saturation constraint is considered for the adaptive tracking control of uncertain MIMO nonlinear systems.
(ii) The spectral radius of the control coefficient matrix is employed in the control design to relax the nonsingular assumption of the control coefficient matrix.
(iii) To handle the non-symmetric input saturation constraint, the auxiliary design system is introduced to analyze the effect of input constraints, and the states of auxiliary design system are used to develop adaptive tracking control.
(iv) command filters are introduced to implement the emulate of actuator physical constraints on the control command and virtual control laws, and avoid the tedious analytic computations of time derivatives of virtual control laws in the backstepping procedure.

The rest of the paper is organized in the following manner. Section 2 presents the problem formulation and preliminaries. Adaptive tracking control is investigated for uncertain MIMO nonlinear systems with input saturation in Section 3, followed by the constrained adaptive control considering actuator physical constraints in Section 4. The simulation results are presented to demonstrate the effectiveness of proposed adaptive control in Section 5. Section 6 contains the conclusion.

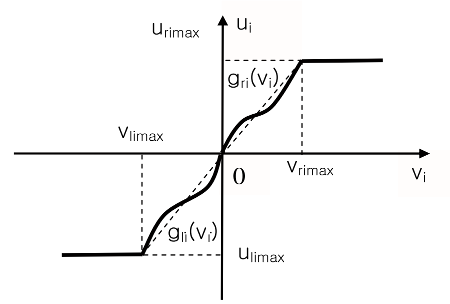

Fig. 1. Non-symmetric input saturation constraint.

Notations: $\|\cdot\|$ denotes for Frobenius norm of matrices and Euclidean norm of vectors, i.e., given a matrix B and a vector ξ, the Frobenius norm and Euclidean norm are given by $\|B\|^2 = \mathrm{tr}(B^TB) = \sum_{i,j} b_{ij}^2$ and $\|\xi\|^2 = \sum_i \xi_i^2$. $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T \in R^{i\times m}$ stands the vector of partial state variables in the nonlinear system. For integer indices i and j, we define $\mathrm{Tanh}(z_i) := \mathrm{diag}\left\{\tanh\left(\frac{z_{ij}}{\varepsilon_{ij}}\right)\right\}$, $\varepsilon_{ij} > 0$, $\Psi_i = [k\varepsilon_{i1}, k\varepsilon_{i2}, \ldots, k\varepsilon_{im}]^T$, $k = 0.2758$, $\rho_i(\bar{x}_i) := \mathrm{diag}\{\rho_{ij}(\bar{x}_i)\}$ and $\Theta_i = [\Theta_{i1}, \Theta_{i2}, \ldots, \Theta_{im}]^T$. $\hat{\theta}_i$ and $\hat{\Theta}_i$ denote the estimates of uncertain parameter vectors θ_i and Θ_i, respectively, and the estimate errors are defined as $\tilde{\theta}_i := \hat{\theta}_i - \theta_i$ and $\tilde{\Theta}_i := \hat{\Theta}_i - \Theta_i$, $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$.

2. Problem formulation

Consider a class of uncertain MIMO nonlinear systems in the form of

$$\begin{aligned}
\dot{x}_i &= F_i(\bar{x}_i)\theta_i + (G_i(\bar{x}_i) + \Delta G_i(\bar{x}_i))x_{i+1} \\
&\quad + D_i(\bar{x}_i, t), \quad i = 1, 2, \ldots, n-1 \\
&\quad \cdots \\
\dot{x}_n &= F_n(\bar{x}_n)\theta_n + (G_n(\bar{x}_n) + \Delta G_n(\bar{x}_n))u + D_n(\bar{x}_n, t) \\
y &= x_1
\end{aligned} \tag{1}$$

where $x_i \in R^m$, $i = 1, 2, \ldots, n$ are the state vectors; $\theta_i \in R^{q_i}$, $i = 1, 2, \ldots, n$ are the uncertain parameter vectors; $F_i \in R^{m\times q_i}$, $i = 1, 2, \ldots, n$ are known nonlinear functions; $G_i \in R^{m\times m}$, $i = 1, 2, \ldots, n$ are known control coefficient matrices; $D_i \in R^m$, $i = 1, 2, \ldots, n$ are unknown time-varying disturbances; $u \in R^m$ is the control input vector; $y \in R^m$ is the system output vector; q_i are positive integers and $\Delta G_i \in R^{m\times m}$, $i = 1, 2, \ldots, n$ are unknown bounded perturbations of control coefficient matrices.

Considering actuator non-symmetric input constraints as shown in Fig. 1, the control input $u = [u_1, \ldots, u_m]^T$ is defined by

$$u_i = \begin{cases} u_{rimax}, & \text{if } v_i > v_{rimax} \\ g_{ri}(v_i), & \text{if } 0 \le v_i \le v_{rimax} \\ g_{li}(v_i), & \text{if } v_{limax} \le v_i < 0 \\ u_{limax}, & \text{if } v_i < v_{limax} \end{cases} \tag{2}$$

where v_i is ith element of the designed control law $v = [v_1, v_2 \ldots, v_m]^T$, $v_{limax} < 0$, and $v_{rimax} > 0$ are known constants; and $g_{ri}(v_i)$ and $g_{li}(v_i)$ are smooth continuous known nonlinear functions.

To facilitate control system design, the following assumptions and lemmas are presented and will be used in the subsequent developments.

Assumption 1 (*Zhang & Ge, 2007, 2008*)**.** There exists positive constants k_{li0}, k_{li1}, k_{ri0} and k_{ri1} such that

$$0 < k_{ri0} \le g'_{ri}(v_i) \le k_{ri1}, \quad v_i \in [0, v_{rimax}] \tag{3}$$

$$0 < k_{li0} \le g'_{li}(v_i) \le k_{li1}, \quad v_i \in [v_{limax}, 0). \tag{4}$$

Assumption 2 (*Tee & Ge, 2006*)**.** For the disturbance terms $\forall(\bar{x}_i, t) \in R^{i\times m} \times R^+$, $D_{ij}(\bar{x}_i, t)$, $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$, there exist known smooth functions $\rho_{ij}(\bar{x}_i) \in R^+$, $\forall t > t_0$ and unknown bounded constants Θ_{ij} such that

$$|D_{ij}(\bar{x}_i, t)| \le \rho_{ij}(\bar{x}_i)\Theta_{ij}. \tag{5}$$

Assumption 3 (*Kim & Ha, 2000*)**.** For all known control coefficient matrices $G_i(\bar{x}_i)$, $i = 1, 2, \ldots, n$ of the uncertain nonlinear system (1), there exist known positive constants $\zeta_i > 0$ such that $\|G_i(\bar{x}_i)\| \le \zeta_i$, $\forall \bar{x}_i \in \Omega_i \subset R^{i\times m}$ with compact subset Ω_i containing the origin.

Assumption 4. For $1 \le i \le n$, there exist known constants $\xi_{i1} \ge 0$ such that $\|\Delta G_i(\bar{x}_i)\| \le \xi_{i1}$.

Lemma 1 (*Polycarpou & Ioannou, 1996*)**.** *The following inequality holds for any $\varepsilon > 0$ and for any $\eta \in R$*

$$0 \le |\eta| - \eta \tanh\left(\frac{\eta}{\varepsilon}\right) \le k_p\varepsilon \tag{6}$$

where k_p is a constant that satisfies $k_p = e^{-(k_p+1)}$, i.e, $k_p = 0.2758$.

Lemma 2 (*Usmani, 1987; Chen et al., 2010*)**.** *No eigenvalue of a matrix $A \in R^{m\times m}$ exceeds any of its norm in absolute value, that is*

$$|\lambda_i| \le \|A\|, \quad i = 1, 2, \ldots, m \tag{7}$$

where λ_i is a eigenvalue of matrix A.

Lemma 3 (*Chen et al., 2010*)**.** *Considering a matrix $B \in R^{m\times m}$ with spectral radius $\varrho(B)$, there exists a positive constant $\Delta > 0$ which makes matrix $B + (\varrho(B) + \Delta)I_{m\times m}$ nonsingular.*

The control objective is to make x_1 follow a certain desired trajectory x_{1d} to a compact set in the presence of system uncertainties and disturbances under the designed adaptive control law v.

Assumption 5. There exist ε_{0i} such that for all t, $\|x_{1d}^{(i)}(t)\| \le \varepsilon_{0i}$, $i = 1, 2, \ldots, n$.

Remark 1. Assumption 1 means that the nonlinear functions $g_{ri}(v_i)$ and $g_{li}(v_i)$ of the non-symmetric input saturation are strictly monotonous. Assumption 2 is reasonable since the time-dependent component of the disturbance with finite energy is always bounded (Tee & Ge, 2006). Assumption 3 is similar to the Assumption A2 in Kim and Ha (2000). Assumption 4 means that perturbations $\Delta G_i(\bar{x}_i)$ of control coefficient matrices $G_i(\bar{x}_i)$, $i = 1, 2, \ldots, n$ are bounded. There are many practical systems can be expressed as the nonlinear system form as shown in (1). For example, rigid robots and motors (Dawson, Carroll, & Schneider, 1994), ships (Tee & Ge, 2006) and aircraft (Tang et al., 2007; Tee, Ge, & Tay, 2008).

Remark 2. In this paper, the matrix spectral radius is employed to design adaptive control for uncertain MIMO nonlinear systems (1). We do not assume that all control coefficient matrices $G_i(\bar{x}_i)$, $i = 1, 2, \ldots, n$ are invertible, but only require that the norm of control coefficient matrix is bounded. This point is always valid for a practical control plant. Considering Assumption 3 and Lemma 2, the spectral radius $\varrho(G_i)$ of $G_i(\bar{x}_i)$ satisfies $\varrho(G_i) \le \zeta_i$ (Chen et al., 2010). According to Lemma 3, we know that $G_i(\bar{x}_i) + (\zeta_i + \tau_i)I_{m\times m}$ are nonsingular with $\tau_i > 0$, $i = 1, 2, \ldots, n$.

3. Adaptive control design and stability analysis

In this section, adaptive control is proposed for the uncertain nonlinear system with control input saturation. The auxiliary design system is adopted to analyze the input saturation constraints. The spectral radius of the control coefficient matrix is introduced to design adaptive control and the bounded stability of all signals in the closed-loop system is achieved.

Step 1: Define error variables $z_1 = x_1 - x_{1d}$, and $z_2 = x_2 - \alpha_1$, where $\alpha_1 \in R^m$ will be defined. Considering (1) and differentiating z_1 with respect to time, we obtain

$$\begin{aligned}\dot{z}_1 &= F_1(x_1)\theta_1 + G_1(x_1)(z_2 + \alpha_1)\\ &\quad + \Delta G_1(x_1)x_2 + D_1(x_1, t) - \dot{x}_{1d}.\end{aligned} \tag{8}$$

Consider the Lyapunov function candidate

$$V_1^* = \frac{1}{2}z_1^T z_1. \tag{9}$$

Its derivative is given by

$$\begin{aligned}\dot{V}_1^* &= z_1^T F_1(x_1)\theta_1 + z_1^T G_1(x_1)(z_2 + \alpha_1) + z_1^T \Delta G_1(x_1)x_2\\ &\quad + z_1^T D_1(x_1, t) - z_1^T \dot{x}_{1d}\\ &\le z_1^T F_1(x_1)\theta_1 + z_1^T G_1(x_1)(z_2 + \alpha_1) + \xi_{11}\|z_1\|\|x_2\|\\ &\quad + \sum_{j=1}^{m} |z_{1j}|\rho_{1j}(x_1)\Theta_{1j} - z_1^T \dot{x}_{1d}.\end{aligned} \tag{10}$$

Invoking Lemma 1, we have

$$\begin{aligned}&\sum_{j=1}^{m} |z_{1j}|\rho_{1j}(x_1)\Theta_{1j}\\ &\quad \le \sum_{j=1}^{m}\left(k\varepsilon_{1j} + z_{1j}\tanh\left(\frac{z_{1j}}{\varepsilon_{1j}}\right)\right)\rho_{1j}(x_1)\Theta_{1j}\\ &\quad = \Psi_1^T \rho_1(x_1)\Theta_1 + z_1^T \mathrm{Tanh}(z_1)\rho_1(x_1)\Theta_1\\ &\quad \le z_1^T \mathrm{Tanh}(z_1)\rho_1(x_1)\Theta_1 + \frac{\|\Psi_1^T\rho_1(x_1)\|^2}{2} + \frac{\|\Theta_1\|^2}{2}.\end{aligned} \tag{11}$$

Substituting (11) into (10), we have

$$\begin{aligned}\dot{V}_1^* &\le z_1^T F_1(x_1)\theta_1 + z_1^T G_1(x_1)(z_2 + \alpha_1)\\ &\quad + \xi_{11}\|z_1\|\|x_2\| + z_1^T \mathrm{Tanh}(z_1)\rho_1(x_1)\Theta_1\\ &\quad + \frac{\|\Psi_1^T\rho_1(x_1)\|^2}{2} + \frac{\|\Theta_1\|^2}{2} - z_1^T\dot{x}_{1d}.\end{aligned} \tag{12}$$

Invoking Lemma 3, choose the following virtual control law:

$$\begin{aligned}\alpha_1 &= (G_1(x_1) + \gamma_1 I_{m\times m})^{-1}(-K_1 z_1 - F_1(x_1)\hat{\theta}_1\\ &\quad - \mathrm{Tanh}(z_1)\rho_1(x_1)\hat{\Theta}_1 + \dot{x}_{1d})\end{aligned} \tag{13}$$

where $K_1 = K_1^T > 0$ and $\gamma_1 = \zeta_1 + \tau_1$.

Substituting (13) into (12) yields

$$\begin{aligned}\dot{V}_1^* &\le -z_1^T K_1 z_1 + z_1^T G_1(x_1)z_2 + z_1^T F_1(x_1)\theta_1\\ &\quad - z_1^T F_1(x_1)\hat{\theta}_1 + z_1^T \mathrm{Tanh}(z_1)\rho_1(x_1)\Theta_1\\ &\quad - z_1^T \mathrm{Tanh}(z_1)\rho_1(x_1)\hat{\Theta}_1 + \xi_{11}\|z_1\|\|x_2\|\\ &\quad - \gamma_1 z_1^T\alpha_1 + \frac{\|\Psi_1^T\rho_1(x_1)\|^2}{2} + \frac{\|\Theta_1\|^2}{2}\\ &= -z_1^T K_1 z_1 + z_1^T G_1(x_1)z_2 - z_1^T F_1(x_1)\tilde{\theta}_1\\ &\quad - z_1^T \mathrm{Tanh}(z_1)\rho_1(x_1)\tilde{\Theta}_1 + \xi_{11}\|z_1\|\|x_2\|\\ &\quad - \gamma_1 z_1^T\alpha_1 + \frac{\|\Psi_1^T\rho_1(x_1)\|^2}{2} + \frac{\|\Theta_1\|^2}{2}.\end{aligned} \tag{14}$$

Considering the error signals $\tilde{\theta}_1$ and $\tilde{\Theta}_1$, the augmented Lyapunov function candidate is written as

$$V_1 = V_1^* + \frac{1}{2}\tilde{\theta}_1^T \Lambda_{11}^{-1}\tilde{\theta}_1 + \frac{1}{2}\tilde{\Theta}_1^T \Lambda_{12}^{-1}\tilde{\Theta}_1 \tag{15}$$

where $\Lambda_{11} = \Lambda_{11}^T > 0$ and $\Lambda_{12} = \Lambda_{12}^T > 0$.

The time derivative of V_1 is given by

$$\begin{aligned}\dot{V}_1 \le& -z_1^TK_1z_1 + z_1^TG_1(x_1)z_2 - z_1^TF_1(x_1)\tilde{\theta}_1\\ &- z_1^T\mathrm{Tanh}(z_1)\rho_1(x_1)\tilde{\Theta}_1 + \xi_{11}\|z_1\|\|x_2\|\\ &- \gamma_1z_1^T\alpha_1 + \frac{\|\Psi_1^T\rho_1(x_1)\|^2}{2} + \frac{\|\Theta_1\|^2}{2}\\ &+ \tilde{\theta}_1^T\Lambda_{11}^{-1}\dot{\hat{\theta}}_1 + \tilde{\Theta}_1^T\Lambda_{12}^{-1}\dot{\hat{\Theta}}_1. \end{aligned} \tag{16}$$

Consider the adaptive laws for $\hat{\theta}_1$ and $\hat{\Theta}_1$ as

$$\dot{\hat{\theta}}_1 = \Lambda_{11}(F_1^T(x_1)z_1 - \beta_{11}\hat{\theta}_1) \tag{17}$$

$$\dot{\hat{\Theta}}_1 = \Lambda_{12}(\rho_1(x_1)\mathrm{Tanh}(z_1)z_1 - \beta_{12}\hat{\Theta}_1) \tag{18}$$

where $\beta_{11} > 0$ and $\beta_{12} > 0$.

Substituting (17) and (18) into (16), and considering the following facts by completion of squares:

$$-\tilde{\theta}_1^T\hat{\theta}_1 \le -\frac{\|\tilde{\theta}_1\|^2}{2} + \frac{\|\theta_1\|^2}{2} \tag{19}$$

$$-\tilde{\Theta}_1^T\hat{\Theta}_1 \le -\frac{\|\tilde{\Theta}_1\|^2}{2} + \frac{\|\Theta_1\|^2}{2} \tag{20}$$

we have

$$\begin{aligned}\dot{V}_1 \le& -z_1^TK_1z_1 + z_1^TG_1(x_1)z_2 + \xi_{11}\|z_1\|\|x_2\| - \gamma_1z_1^T\alpha_1\\ &+ \frac{\|\Psi_1^T\rho_1(x_1)\|^2}{2} - \frac{\beta_{11}}{2}\|\tilde{\theta}_1\|^2 + \frac{\beta_{11}}{2}\|\theta_1\|^2\\ &- \frac{\beta_{12}}{2}\|\tilde{\Theta}_1\|^2 + \frac{\beta_{12}}{2}\|\Theta_1\|^2 + \frac{\|\Theta_1\|^2}{2}. \end{aligned} \tag{21}$$

The first term on the right-hand side is negative, and the second term will be canceled in the next step. The other terms will be considered in stability analysis of the closed-loop system.

Step 2: Define the error variable $z_3 = x_3 - \alpha_2$. Considering (1) and differentiating z_2 with respect to time, we obtain

$$\dot{z}_2 = F_2(\bar{x}_2)\theta_2 + G_2(\bar{x}_2)x_3 + \Delta G_2(\bar{x}_2)x_3 + D_2(\bar{x}_2, t) - \dot{\alpha}_1. \tag{22}$$

Consider the Lyapunov function candidate

$$V_2^* = \frac{1}{2}z_2^Tz_2. \tag{23}$$

Considering Lemma 1, the derivative of V_2^* is

$$\begin{aligned}\dot{V}_2^* =& z_2^TF_2(\bar{x}_2)\theta_2 + z_2^TG_2(\bar{x}_2)(z_3+\alpha_2)\\ &+ z_2^T\Delta G_2(\bar{x}_2)x_3 + z_2^TD_2(\bar{x}_2,t) - z_2^T\dot{\alpha}_1\\ \le& z_2^TF_2(\bar{x}_2)\theta_2 + z_2^TG_2(\bar{x}_2)(z_3+\alpha_2)\\ &+ \xi_{21}\|z_2\|\|x_3\| + \sum_{j=1}^{m}|z_{2j}|\rho_{2j}(\bar{x}_2)\Theta_{2j} - z_2^T\dot{\alpha}_1\\ \le& z_2^TF_2(\bar{x}_2)\theta_2 + z_2^TG_2(\bar{x}_2)(z_3+\alpha_2)\\ &+ \xi_{21}\|z_2\|\|x_3\| + z_2^T\mathrm{Tanh}(z_2)\rho_2(\bar{x}_2)\Theta_2\\ &+ \frac{\|\Psi_2^T\rho_2(\bar{x}_2)\|^2}{2} + \frac{\|\Theta_2\|^2}{2} - z_2^T\dot{\alpha}_1. \end{aligned} \tag{24}$$

Invoking Lemma 3, choose the virtual control law as

$$\begin{aligned}\alpha_2 =& (G_2(\bar{x}_2) + \gamma_2I_{m\times m})^{-1}(-G_1^T(x_1)z_1 - K_2z_2\\ &- F_2(\bar{x}_2)\hat{\theta}_2 - \mathrm{Tanh}(z_2)\rho_2(\bar{x}_2)\hat{\Theta}_2 + \dot{\alpha}_1) \end{aligned} \tag{25}$$

where $K_2 = K_2^T > 0$ and $\gamma_2 = \zeta_2 + \tau_2$.

Substituting (25) into (24), we obtain

$$\begin{aligned}\dot{V}_2^* \le& -z_2^TK_2z_2 + z_2^TG_2(\bar{x}_2)z_3 + z_2^TF_2(\bar{x}_2)\theta_2\\ &- z_2^TF_2(\bar{x}_2)\hat{\theta}_2 + z_2^T\mathrm{Tanh}(z_2)\rho_2(\bar{x}_2)\Theta_2\\ &- z_2^T\mathrm{Tanh}(z_2)\rho_2(\bar{x}_2)\hat{\Theta}_2 - z_2^TG_1^T(x_1)z_1 - \gamma_2z_2^T\alpha_2\\ &+ \xi_{21}\|z_2\|\|x_3\| + \frac{\|\Psi_2^T\rho_2(\bar{x}_2)\|^2}{2} + \frac{\|\Theta_2\|^2}{2}\\ =& -z_2^TK_2z_2 + z_2^TG_2(\bar{x}_2)z_3 - z_2^TF_2(\bar{x}_2)\tilde{\theta}_2\\ &- z_2^T\mathrm{Tanh}(z_2)\rho_2(\bar{x}_2)\tilde{\Theta}_2 - z_1^TG_1(x_1)z_2 - \gamma_2z_2^T\alpha_2\\ &+ \xi_{21}\|z_2\|\|x_3\| + \frac{\|\Psi_2^T\rho_2(\bar{x}_2)\|^2}{2} + \frac{\|\Theta_2\|^2}{2}. \end{aligned} \tag{26}$$

Considering the error signal $\tilde{\theta}_2$ and $\tilde{\Theta}_2$, the augmented Lyapunov function candidate can be written as

$$V_2 = V_1 + V_2^* + \frac{1}{2}\tilde{\theta}_2^T\Lambda_{21}^{-1}\tilde{\theta}_2 + \frac{1}{2}\tilde{\Theta}_2^T\Lambda_{22}^{-1}\tilde{\Theta}_2 \tag{27}$$

where $\Lambda_{21} = \Lambda_{21}^T > 0$ and $\Lambda_{22} = \Lambda_{22}^T > 0$.

Invoking (21) and (26), the time derivative of V_2 is

$$\begin{aligned}\dot{V}_2 \le& -\sum_{j=1}^{2}z_j^TK_jz_j - \sum_{j=1}^{2}\gamma_jz_j^T\alpha_j + \sum_{j=1}^{2}\xi_{j1}\|z_j\|\|x_{j+1}\|\\ &+ z_2^TG_2(\bar{x}_2)z_3 - z_2^TF_2(\bar{x}_2)\tilde{\theta}_2 - z_2^T\mathrm{Tanh}(z_2)\rho_2(\bar{x}_2)\tilde{\Theta}_2\\ &+ \tilde{\theta}_2^T\Lambda_{21}^{-1}\dot{\hat{\theta}}_2 + \tilde{\Theta}_2^T\Lambda_{22}^{-1}\dot{\hat{\Theta}}_2 + \sum_{j=1}^{2}\frac{\|\Psi_j^T\rho_j(\bar{x}_j)\|^2}{2}\\ &- \frac{\beta_{11}}{2}\|\tilde{\theta}_1\|^2 + \frac{\beta_{11}}{2}\|\theta_1\|^2 - \frac{\beta_{12}}{2}\|\tilde{\Theta}_1\|^2\\ &+ \frac{\beta_{12}}{2}\|\Theta_1\|^2 + \sum_{j=1}^{2}\frac{\|\Theta_j\|^2}{2}. \end{aligned} \tag{28}$$

Consider the adaptive laws for $\hat{\theta}_2$ and $\hat{\Theta}_2$ as

$$\dot{\hat{\theta}}_2 = \Lambda_{21}(F_2^T(\bar{x}_2)z_2 - \beta_{21}\hat{\theta}_2) \tag{29}$$

$$\dot{\hat{\Theta}}_2 = \Lambda_{22}(\rho_2(\bar{x}_2)\mathrm{Tanh}(z_2)z_2 - \beta_{22}\hat{\Theta}_2) \tag{30}$$

where $\beta_{21} > 0$ and $\beta_{22} > 0$.

Substituting (29) and (30) into (28), similar with (19) and (20) we have

$$\begin{aligned}\dot{V}_2 \le& -\sum_{j=1}^{2}z_j^TK_jz_j + z_2^TG_2(\bar{x}_2)z_3 - \sum_{j=1}^{2}\gamma_jz_j^T\alpha_j\\ &+ \sum_{j=1}^{2}\xi_{j1}\|z_j\|\|x_{j+1}\| + \sum_{j=1}^{2}\frac{\|\Psi_j^T\rho_j(\bar{x}_j)\|^2}{2}\\ &- \sum_{j=1}^{2}\frac{\beta_{j1}}{2}\|\tilde{\theta}_j\|^2 + \sum_{j=1}^{2}\frac{\beta_{j1}}{2}\|\theta_j\|^2 - \sum_{j=1}^{2}\frac{\beta_{j2}}{2}\|\tilde{\Theta}_j\|^2\\ &+ \sum_{j=1}^{2}\frac{\beta_{j2}}{2}\|\Theta_j\|^2 + \sum_{j=1}^{2}\frac{\|\Theta_j\|^2}{2}. \end{aligned} \tag{31}$$

The first term on the right-hand side is negative, and the second term will be canceled in the next step. The other terms will be considered in stability analysis of the closed-loop system.

Step i ($1 \le i \le n-1$): Define the error variable $z_{i+1} = x_{i+1} - \alpha_i$. Considering (1) and differentiating z_i with respect to time, we have

$$\begin{aligned}\dot{z}_i =& F_i(\bar{x}_i)\theta_i + G_i(\bar{x}_i)(z_{i+1}+\alpha_i) + \Delta G_i(\bar{x}_i)x_{i+1}\\ &+ D_i(\bar{x}_i,t) - \dot{\alpha}_{i-1}. \end{aligned} \tag{32}$$

Consider the Lyapunov function candidate

$$V_i^* = \frac{1}{2} z_i^T z_i. \tag{33}$$

Invoking Lemma 1, the derivative of V_i^* is

$$\begin{aligned}\dot{V}_i^* &= z_i^T F_i(\bar{x}_i)\theta_i + z_i^T G_i(\bar{x}_i)(z_{i+1} + \alpha_i) \\ &\quad + z_i^T \Delta G_i(\bar{x}_i) x_{i+1} + z_i^T D_i(\bar{x}_i, t) - z_i^T \dot{\alpha}_{i-1} \\ &\le z_i^T F_i(\bar{x}_i)\theta_i + z_i^T G_i(\bar{x}_i)(z_{i+1} + \alpha_i) \\ &\quad + \xi_{i1}\|z_i\|\|x_{i+1}\| + \sum_{j=1}^{m} |z_{ij}|\rho_{ij}(\bar{x}_i)\Theta_{ij} - z_i^T \dot{\alpha}_{i-1} \\ &\le z_i^T F_i(\bar{x}_i)\theta_i + z_i^T G_i(\bar{x}_i)(z_{i+1} + \alpha_i) \\ &\quad + \xi_{i1}\|z_i\|\|x_{i+1}\| + z_i^T \mathrm{Tanh}(z_i)\rho_i(\bar{x}_i)\Theta_i \\ &\quad + \frac{\|\Psi_i^T \rho_i(\bar{x}_i)\|^2}{2} + \frac{\|\Theta_i\|^2}{2} - z_i^T \dot{\alpha}_{i-1}. \end{aligned} \tag{34}$$

Considering Lemma 3, we choose the following virtual control law:

$$\begin{aligned}\alpha_i &= (G_i(\bar{x}_i) + \gamma_i I_{m\times m})^{-1}(-G_{i-1}^T(\bar{x}_{i-1})z_{i-1} - K_i z_i \\ &\quad - F_i(\bar{x}_i)\hat{\theta}_i - \mathrm{Tanh}(z_i)\rho_i(\bar{x}_i)\hat{\Theta}_i + \dot{\alpha}_{i-1})\end{aligned} \tag{35}$$

where $K_i = K_i^T > 0$ and $\gamma_i = \zeta_i + \tau_i$.

Substituting (35) into (34), we obtain

$$\begin{aligned}\dot{V}_i^* &\le -z_i^T K_i z_i + z_i^T G_i(\bar{x}_i) z_{i+1} + z_i^T F_i(\bar{x}_i)\theta_i \\ &\quad - z_i^T F_i(\bar{x}_i)\hat{\theta}_i + z_i^T \mathrm{Tanh}(z_i)\rho_i(\bar{x}_i)\Theta_2 \\ &\quad - z_i^T \mathrm{Tanh}(z_i)\rho_i(\bar{x}_i)\hat{\Theta}_i - z_i^T G_{i-1}^T(\bar{x}_{i-1})z_{i-1} \\ &\quad - \gamma_i z_i^T \alpha_i + \xi_{i1}\|z_i\|\|x_{i+1}\| + \frac{\|\Psi_i^T\rho_i(\bar{x}_i)\|^2}{2} + \frac{\|\Theta_i\|^2}{2} \\ &= -z_i^T K_i z_i + z_i^T G_i(\bar{x}_i) z_{i+1} - z_i^T F_i(\bar{x}_i)\tilde{\theta}_i \\ &\quad - z_i^T \mathrm{Tanh}(z_i)\rho_i(\bar{x}_i)\tilde{\Theta}_i - z_{i-1}^T G_{i-1}(\bar{x}_{i-1}) z_i - \gamma_i z_i^T \alpha_i \\ &\quad + \xi_{i1}\|z_i\|\|x_{i+1}\| + \frac{\|\Psi_i^T\rho_i(\bar{x}_i)\|^2}{2} + \frac{\|\Theta_i\|^2}{2}. \end{aligned} \tag{36}$$

Considering the error signals $\tilde{\theta}_i$ and $\tilde{\Theta}_i$, the augmented Lyapunov function candidate can be written as

$$V_i = V_{i-1} + V_i^* + \frac{1}{2}\tilde{\theta}_i^T \Lambda_{i1}^{-1}\tilde{\theta}_i + \frac{1}{2}\tilde{\Theta}_i^T \Lambda_{i2}^{-1}\tilde{\Theta}_i \tag{37}$$

where $\Lambda_{i1} = \Lambda_{i1}^T > 0$ and $\Lambda_{i2} = \Lambda_{i2}^T > 0$.

Invoking (31) and (36), the time derivative of V_i is given by

$$\begin{aligned}\dot{V}_i &\le -\sum_{j=1}^{i} z_j^T K_j z_j - \sum_{j=1}^{i}\gamma_j z_j^T \alpha_j + \sum_{j=1}^{i}\xi_{j1}\|z_j\|\|x_{j+1}\| \\ &\quad + z_i^T G_i(\bar{x}_i) z_{i+1} - z_i^T F_i(\bar{x}_i)\tilde{\theta}_i - z_i^T \mathrm{Tanh}(z_i)\rho_i(\bar{x}_i)\tilde{\Theta}_i \\ &\quad + \tilde{\theta}_i^T \Lambda_{i1}^{-1}\dot{\hat{\theta}}_i + \tilde{\Theta}_i^T \Lambda_{i2}^{-1}\dot{\hat{\Theta}}_i + \sum_{j=1}^{i}\frac{\|\Psi_j^T\rho_j(\bar{x}_j)\|^2}{2} \\ &\quad - \sum_{j=1}^{i-1}\frac{\beta_{j1}}{2}\|\tilde{\theta}_j\|^2 + \sum_{j=1}^{i-1}\frac{\beta_{j1}}{2}\|\theta_j\|^2 - \sum_{j=1}^{i-1}\frac{\beta_{j2}}{2}\|\tilde{\Theta}_j\|^2 \\ &\quad + \sum_{j=1}^{i-1}\frac{\beta_{j2}}{2}\|\Theta_j\|^2 + \sum_{j=1}^{i}\frac{\|\Theta_j\|^2}{2}. \end{aligned} \tag{38}$$

Consider the adaptive laws for $\hat{\theta}_i$ and $\hat{\Theta}_i$ as

$$\dot{\hat{\theta}}_i = \Lambda_{i1}(F_i^T(\bar{x}_i) z_i - \beta_{i1}\hat{\theta}_i) \tag{39}$$

$$\dot{\hat{\Theta}}_i = \Lambda_{i2}(\rho_i(\bar{x}_i)\mathrm{Tanh}(z_i) z_i - \beta_{i2}\hat{\Theta}_i) \tag{40}$$

where $\beta_{i1} > 0$ and $\beta_{i2} > 0$.

Substituting (39) and (40) into (38), similar with (19) and (20) we have

$$\begin{aligned}\dot{V}_i &\le -\sum_{j=1}^{i} z_j^T K_j z_j + z_i^T G_i(\bar{x}_i) z_{i+1} - \sum_{j=1}^{i}\gamma_j z_j^T \alpha_j \\ &\quad + \sum_{j=1}^{i}\xi_{j1}\|z_j\|\|x_{j+1}\| + \sum_{j=1}^{i}\frac{\|\Psi_j^T\rho_j(\bar{x}_j)\|^2}{2} \\ &\quad - \sum_{j=1}^{i}\frac{\beta_{j1}}{2}\|\tilde{\theta}_j\|^2 + \sum_{j=1}^{i}\frac{\beta_{j1}}{2}\|\theta_j\|^2 - \sum_{j=1}^{i}\frac{\beta_{j2}}{2}\|\tilde{\Theta}_j\|^2 \\ &\quad + \sum_{j=1}^{i}\frac{\beta_{j2}}{2}\|\Theta_j\|^2 + \sum_{j=1}^{i}\frac{\|\Theta_j\|^2}{2}. \end{aligned} \tag{41}$$

The first term on the right-hand side is negative, and the second term will be canceled in the next step. The other terms will be considered in stability analysis of the closed-loop system.

Step n: By differentiating $z_n = x_n - \alpha_{n-1}$ with respect to time yields

$$\begin{aligned}\dot{z}_n &= F_n(\bar{x}_n)\theta_n + G_n(\bar{x}_n)u + \Delta G_n(\bar{x}_n)u \\ &\quad + D_n(\bar{x}_n, t) - \dot{\alpha}_{n-1}. \end{aligned} \tag{42}$$

Consider the Lyapunov function candidate

$$V_n^* = \frac{1}{2} z_n^T z_n. \tag{43}$$

Note the fact $\|u\| \le U_{\max}$ with $U_{\max} > 0$. Invoking Lemma 1, the derivative of V_n^* is

$$\begin{aligned}\dot{V}_n^* &= z_n^T F_n(\bar{x}_n)\theta_n + z_n^T G_n(\bar{x}_n)u + z_n^T \Delta G_n u \\ &\quad + z_n^T D_n(\bar{x}_n, t) - z_n^T \dot{\alpha}_{n-1} \\ &\le z_n^T F_n(\bar{x}_n)\theta_n + z_n^T G_n(\bar{x}_n)u + \xi_{n1}\|z_n\|\|u\| \\ &\quad + \sum_{j=1}^{m} |z_{nj}|\rho_{nj}(\bar{x}_n)\Theta_{nj} - z_n^T \dot{\alpha}_{n-1} \\ &\le z_n^T F_n(\bar{x}_n)\theta_n + z_n^T G_n(\bar{x}_n)u + \xi_{n1}\|z_n\|\|u\| \\ &\quad + z_n^T \mathrm{Tanh}(z_n)\rho_n(\bar{x}_n)\Theta_n + \frac{\|\Psi_n^T\rho_n(\bar{x}_n)\|^2}{2} \\ &\quad + \frac{\|\Theta_n\|^2}{2} - z_n^T \dot{\alpha}_{n-1}. \end{aligned} \tag{44}$$

From (2), control inputs u have an upper limit and a lower limit. For convenience of input constraint effect analysis, the auxiliary design system is given by

$$\dot{e} = \begin{cases} -K_{n2}e - \dfrac{1}{\|e\|^2} f(u, \Delta u, z_n, \bar{x}_n)e & \\ \quad + (G_n(\bar{x}_n) + \gamma_n I_{m\times m})(v - u), & \|e\| \ge \sigma \\ 0, & \|e\| < \sigma \end{cases} \tag{45}$$

where $f(u, \Delta u, z_n, \bar{x}_n) = |z_n^T G_n(\bar{x}_n)\Delta u| + 0.5(\gamma_n + \zeta_n)^2 \Delta u^T \Delta u + |\gamma_n z_n^T u| + \xi_{n1}\|z_n\|\|u\|$, $\Delta u = u - v$, $K_{n2} = K_{n2}^T > 0$, $\gamma_n = \zeta_n + \tau_n$ and $e \in R^m$ is the state of auxiliary design system. The design parameter σ is a positive constant which should be chosen as an appropriate value in accordance with the requirement of the tracking performance.

Define

$$\begin{aligned}h(Z) &= \frac{1}{2} z_n^T K_n^T K_n z_n + \sum_{j=1}^{n-1}\gamma_j |z_j^T \alpha_j| \\ &\quad + \sum_{j=1}^{n-1}\xi_{j1}\|z_j\|\|x_{j+1}\| + \sum_{j=1}^{n}\frac{\|\Psi_j^T\rho_j(\bar{x}_j)\|^2}{2} \end{aligned} \tag{46}$$

where $K_n = K_n^T > 0$ and $Z = [\alpha_j, z_j, x_j]^T, j = 1, 2, \ldots, n$.

Invoking Lemma 3 and considering the input saturation effect, choose the following control law:

$$
\begin{aligned}
v &= (G_n(\bar{x}_n) + \gamma_n I_{m\times m})^{-1} v_0 \\
v_0 &= -G_{n-1}^T(\bar{x}_{n-1}) z_{n-1} - K_n(z_n - e) - F_n(\bar{x}_n)\hat{\theta}_n \\
&\quad - \mathrm{Tanh}(z_n)\rho_n(\bar{x}_n)\hat{\Theta}_n + \dot{\alpha}_{n-1} - \frac{z_n h(Z)}{\psi^2 + \|z_n\|^2} \qquad (47) \\
\dot{\psi} &= \begin{cases} -\dfrac{\psi h(Z)}{\psi^2 + \|z_n\|^2} - k_v \psi, & \|z_n\| \geq \ell \\ 0, & \|z_n\| < \ell \end{cases}
\end{aligned}
$$

where $k_v > 0$ and $\ell > 0$.

The above design procedure can be summarized in the following theorem, which contains the results of adaptive control for uncertain MIMO nonlinear systems (1).

Theorem 1. *Considering the strict-feedback nonlinear system (1) with known coefficient matrices satisfies Assumptions 1–5, and given that the full state information is available. Under the control law (47), parameter updated laws (17), (18), (29), (30), (39), (40), (53), (54), and for any bounded initial condition, there exist design parameters $\sigma > 0$, $K_i = K_i^T > 0$, $K_{n2} = K_{n2}^T > 0$, $\beta_{i1} > 0$, $\beta_{i2} > 0$ and $k_v > 0$ such that the overall closed-loop control system is semi-globally stable in the sense that all of the closed-loop signals e, z_i, ψ, $\tilde{\theta}_i$ and $\tilde{\Theta}_i$ are bounded, where $i = 1, 2, \ldots, n$. Furthermore, the tracking error signals z_1 remains within the compact sets Ω_{z1} defined by*

$$\Omega_{z_1} := \left\{ z_1 \in R^m \,|\, \|z_1\| \leq \sqrt{D} \right\}$$

where $D = 2(V_n(0) + \frac{C}{\kappa})$ with C and κ as defined in (55).

Proof. When $\|e\| \geq \sigma$, we consider the Lyapunov function candidate

$$V_n = V_{n-1} + V_n^* + \frac{1}{2} e^T e + \frac{1}{2}\tilde{\theta}_n^T \Lambda_{n1}^{-1}\tilde{\theta}_n + \frac{1}{2}\tilde{\Theta}_n^T \Lambda_{n2}^{-1}\tilde{\Theta}_n + \frac{1}{2}\psi^2 \qquad (48)$$

where $\Lambda_{n1} = \Lambda_{n1}^T > 0$ and $\Lambda_{n2} = \Lambda_{n2}^T > 0$.

Considering (41) and (44), the time derivative of V_n is

$$
\begin{aligned}
\dot{V}_n \leq& -\sum_{j=1}^{n-1} z_j^T K_j z_j + z_{n-1}^T G_{n-1}(\bar{x}_{n-1}) z_n - \sum_{j=1}^{n-1} \gamma_j z_j^T \alpha_j \\
&+ \sum_{j=1}^{n-1} \xi_{j1}\|z_j\|\|x_{j+1}\| + z_n^T F_n(\bar{x}_n)\theta_n + \xi_{n1}\|z_n\|\|u\| \\
&+ z_n^T G_n(\bar{x}_n)(v + \Delta u) + z_n^T \mathrm{Tanh}(z_n)\rho_n(\bar{x}_n)\Theta_n \\
&- z_n^T \dot{\alpha}_{n-1} + e^T\dot{e} + \tilde{\theta}_n^T \Lambda_{n1}^{-1}\dot{\hat{\theta}}_n + \tilde{\Theta}_n^T \Lambda_{n2}^{-1}\dot{\hat{\Theta}}_n \\
&+ \sum_{j=1}^{n} \frac{\|\Psi_j^T \rho_j(\bar{x}_j)\|^2}{2} - \sum_{j=1}^{n-1} \frac{\beta_{j1}}{2}\|\tilde{\theta}_j\|^2 + \sum_{j=1}^{n-1} \frac{\beta_{j1}}{2}\|\theta_j\|^2 \\
&- \sum_{j=1}^{n-1} \frac{\beta_{j2}}{2}\|\tilde{\Theta}_j\|^2 + \sum_{j=1}^{n-1} \frac{\beta_{j2}}{2}\|\Theta_j\|^2 \\
&+ \sum_{j=1}^{n} \frac{\|\Theta_j\|^2}{2} + \psi\dot{\psi}. \qquad (49)
\end{aligned}
$$

Substituting (45)–(47) into (49), we obtain

$$
\begin{aligned}
\dot{V}_n \leq& -\sum_{j=1}^{n} z_j^T K_j z_j - \sum_{j=1}^{n-1} \gamma_j z_j^T \alpha_j + \sum_{j=1}^{n-1} \xi_{j1}\|z_j\|\|x_{j+1}\| \\
&- \gamma_n z_n^T u + \xi_{n1}\|z_n\|\|u\| + z_n^T G_n(\bar{x}_n)\Delta u + z_n^T K_n e \\
&- z_n^T F_n(\bar{x}_n)\tilde{\theta}_n - z_n^T \mathrm{Tanh}(z_n)\rho_n(\bar{x}_n)\tilde{\Theta}_n + e^T\dot{e} \\
&+ \tilde{\theta}_n^T \Lambda_{n1}^{-1}\dot{\hat{\theta}}_n + \tilde{\Theta}_n^T \Lambda_{n2}^{-1}\dot{\hat{\Theta}}_n + \sum_{j=1}^{n} \frac{\|\Psi_j^T \rho_j(\bar{x}_j)\|^2}{2} \\
&- \sum_{j=1}^{n-1} \frac{\beta_{j1}}{2}\|\tilde{\theta}_j\|^2 + \sum_{j=1}^{n-1} \frac{\beta_{j1}}{2}\|\theta_j\|^2 - \sum_{j=1}^{n-1} \frac{\beta_{j2}}{2}\|\tilde{\Theta}_j\|^2 \\
&+ \sum_{j=1}^{n-1} \frac{\beta_{j2}}{2}\|\Theta_j\|^2 + \sum_{j=1}^{n} \frac{\|\Theta_j\|^2}{2} - \frac{\|z_n\|^2 h(Z)}{\psi^2 + \|z_n\|^2} + \psi\dot{\psi} \\
\leq& -\sum_{j=1}^{n} z_j^T K_j z_j - e^T\left(K_{n2} - I_{m\times m}\right)e - z_n^T F_n(\bar{x}_n)\tilde{\theta}_n \\
&- z_n^T \mathrm{Tanh}(z_n)\rho_n(\bar{x}_n)\tilde{\Theta}_n + \tilde{\theta}_n^T \Lambda_{n1}^{-1}\dot{\hat{\theta}}_n + \tilde{\Theta}_n^T \Lambda_{n2}^{-1}\dot{\hat{\Theta}}_n \\
&- \sum_{j=1}^{n-1} \frac{\beta_{j1}}{2}\|\tilde{\theta}_j\|^2 + \sum_{j=1}^{n-1} \frac{\beta_{j1}}{2}\|\theta_j\|^2 - \sum_{j=1}^{n-1} \frac{\beta_{j2}}{2}\|\tilde{\Theta}_j\|^2 \\
&+ \sum_{j=1}^{n-1} \frac{\beta_{j2}}{2}\|\Theta_j\|^2 + \sum_{j=1}^{n} \frac{\|\Theta_j\|^2}{2} \\
&+ \frac{\psi^2 h(Z)}{\psi^2 + \|z_n\|^2} + \psi\dot{\psi}. \qquad (50)
\end{aligned}
$$

Invoking the third equation of (47), we have

$$\frac{\psi^2 h(Z)}{\psi^2 + \|z_n\|^2} + \psi\dot{\psi} = -k_v\psi^2. \qquad (51)$$

Substituting (51) into (50) yields

$$
\begin{aligned}
\dot{V}_n \leq& -\sum_{j=1}^{n} z_j^T K_j z_j - e^T\left(K_{n2} - I_{m\times m}\right)e - z_n^T F_n(\bar{x}_n)\tilde{\theta}_n \\
&- z_n^T \mathrm{Tanh}(z_n)\rho_n(\bar{x}_n)\tilde{\Theta}_n + \tilde{\theta}_n^T \Lambda_{n1}^{-1}\dot{\hat{\theta}}_n + \tilde{\Theta}_n^T \Lambda_{n2}^{-1}\dot{\hat{\Theta}}_n \\
&- \sum_{j=1}^{n-1} \frac{\beta_{j1}}{2}\|\tilde{\theta}_j\|^2 + \sum_{j=1}^{n-1} \frac{\beta_{j1}}{2}\|\theta_j\|^2 - \sum_{j=1}^{n-1} \frac{\beta_{j2}}{2}\|\tilde{\Theta}_j\|^2 \\
&+ \sum_{j=1}^{n-1} \frac{\beta_{j2}}{2}\|\Theta_j\|^2 + \sum_{j=1}^{n} \frac{\|\Theta_j\|^2}{2} - k_v\psi^2. \qquad (52)
\end{aligned}
$$

Consider the adaptive laws for $\hat{\theta}_n$ and $\hat{\Theta}_n$ as

$$\dot{\hat{\theta}}_n = \Lambda_{n1}(F_n^T(\bar{x}_n)z_n - \beta_{n1}\hat{\theta}_n) \qquad (53)$$

$$\dot{\hat{\Theta}}_n = \Lambda_{n2}(\rho_n(\bar{x}_n)\mathrm{Tanh}(z_n)z_n - \beta_{n2}\hat{\Theta}_n) \qquad (54)$$

where $\beta_{n1} > 0$ and $\beta_{n2} > 0$.

Substituting (53) and (54) into (52), similar with (19) and (20) we have

$$
\begin{aligned}
\dot{V}_n \leq& -\sum_{j=1}^{n} z_j^T K_j z_j - e^T\left(K_{n2} - I_{m\times m}\right)e - k_v\psi^2 \\
&- \sum_{j=1}^{n} \frac{\beta_{j1}}{2}\|\tilde{\theta}_j\|^2 + \sum_{i=1}^{n} \frac{\beta_{j1}}{2}\|\theta_j\|^2 - \sum_{j=1}^{n} \frac{\beta_{j2}}{2}\|\tilde{\Theta}_j\|^2 \\
&+ \sum_{i=1}^{n} \frac{\beta_{j2}}{2}\|\Theta_j\|^2 + \sum_{j=1}^{n} \frac{\|\Theta_j\|^2}{2} \\
\leq& -\kappa V_n + C \qquad (55)
\end{aligned}
$$

where

$$\kappa := \min\left(\begin{matrix} 2\lambda_{\min}\left(\sum_{j=1}^{n} K_j\right), 2\lambda_{\min}(K_{n2} - I_{m\times m}), \\ \sum_{j=1}^{n} \frac{2\beta_{j1}}{\lambda_{\max}(\Lambda_{j1}^{-1})}, \sum_{j=1}^{n} \frac{2\beta_{j2}}{\lambda_{\max}(\Lambda_{j2}^{-1})}, k_v \end{matrix}\right) \tag{56}$$

$$C := \sum_{j=1}^{n} \frac{\beta_{j1}}{2}\|\theta_j\|^2 + \sum_{j=1}^{n} \frac{\beta_{j2}}{2}\|\Theta_j\|^2 + \sum_{j=1}^{n} \frac{\|\Theta_j\|^2}{2}.$$

To ensure that $\kappa > 0$, the design parameter K_{n2} must make $K_{n2} - I_{m\times m} > 0$.

From (55), if $\kappa > 0$, we can conclude that z_1 converges to a compact set asymptotically, and therefore the control objective is reached when the input saturation constraint occurs, i.e., the desired trajectory of MIMO nonlinear system is followed in the presence of parametric uncertainties and disturbances under the saturation constraint. On the other hand, we can conclude that auxiliary design variables e and ψ, error signals z_i, $\tilde{\theta}_i$ and $\tilde{\Theta}_i$ converge to a compact set asymptotically.

It is worth pointing out that the above proof of Theorem 1 only contains the result when the states of the auxiliary design system (45) satisfy the condition $\|e\| \geq \sigma$, i.e., there exists input saturation. If $\|e\| < \sigma$ means that there does not exist input saturation, we have $\Delta u = 0$, i.e., $u = v$ and the control input u is bounded. Thus, v is bounded. The stability proof of Theorem 1 can be easily proved by considering Eqs. (48)–(55) when $\|e\| < \sigma$. The detailed proof is omitted. This concludes the proof. □

Remark 3. In this section, the robust adaptive tracking control is proposed for a class of uncertain MIMO nonlinear systems with non-symmetric input saturation constraints. To handle the non-symmetric input saturation, the auxiliary design system (45) is introduced to analyze the effect of saturation constraint, and the auxiliary variable e is used to design the robust adaptive control law. It is apparent that the constrained control u produced by the designed control command v can guarantee the closed-loop system stability. If $e \leq \sigma$ and $\dot{e} = 0$, it means that there is no saturation, i.e., there is $u = v$ according to (45) (Polycarpou et al., 2003). It implies that $v_{li\max} \leq v_i \leq v_{ri\max}$ and $g_{ri}(v_i) = g_{li}(v_i) = v_i$.

4. Constrained adaptive control design and stability analysis

Although the robust adaptive control for the uncertain MIMO nonlinear system (1) with non-symmetric input saturation constraints has been successfully developed in Section 3, physics constraints of virtual control laws have not been considered, and the analytic computations of time derivatives of virtual control laws α_i $(i = 1, \ldots, n-1)$ need to be done in the backstepping procedure. In fact, the vast analytic calculation of the virtual control derivatives is a drawback of backstepping control. Specially, the analytic calculation of time derivatives of virtual control laws is tedious for the MIMO nonlinear systems. In this section, we will investigate the constrained robust adaptive control which consider the mechanical or operating limitations of virtual control laws and control command, and eliminate the analytic computations of the virtual control law derivatives. Therefore, command filters are introduced to avoid the analytic calculation of the time derivatives of the virtual control laws.

Step 1: Define error variables $z_1 = x_1 - x_{1d}$ and $z_2 = x_2 - \alpha_1$. Considering (1) and differentiating z_1 with respect to time, we obtain

$$\begin{aligned}\dot{z}_1 &= F_1(x_1)\theta_1 + (G_1(x_1) + \Delta G_1(x_1))(z_2 + \alpha_1) \\ &\quad + D_1(x_1, t) - \dot{x}_{1d}\end{aligned} \tag{57}$$

where α_1 is a virtual control law which is produced by the nominal virtual control law α_{10}. The nominal virtual control law α_{10} is

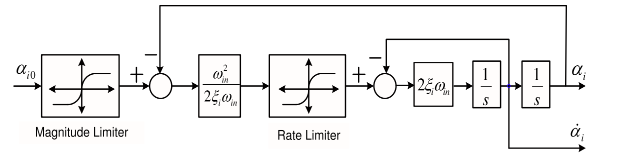

Fig. 2. Configuration of the command filter, where $i = 1, 2, \ldots, n-1$, $\alpha_n = v$, α_{i0} are the nominal virtual control law or the nominal control law, α_i are the virtual control law or the control law, ξ_i and ω_{ni} are the bandwidth parameters.

filtered to provide the magnitude, rate and bandwidth limited virtual control law α_1 and its derivatives $\dot{\alpha}_1$ which are within the operating envelope of the system. Such a command filter is shown in Fig. 2 to implement the emulate of any mechanical or operating constraints on virtual control law α_{10} (Polycarpou et al., 2004).

The nominal virtual control law α_{10} is given by

$$\begin{aligned}\alpha_{10} &= (G_1(x_1) + \gamma_1 I_{m\times m})^{-1}(-K_{10}(z_1 - \varphi_1) \\ &\quad - F_1(x_1)\hat{\theta}_1 - \mathrm{Tanh}(z_1)\rho_1(x_1)\hat{\Theta}_1 + \dot{x}_{1d})\end{aligned} \tag{58}$$

where $K_{10} = K_{10}^T > 0$, and $\varphi_1 \in R^m$ is the state vector of auxiliary design system which denotes the constraint effect due to the magnitude, rate and bandwidth limitation of the nominal virtual control law. Note the following facts $\|\alpha_1\| \leq \varepsilon_{10}$ with $\varepsilon_{10} > 0$, where ε_{10} denotes the magnitude limit of α_1 which is decided by the command filter. Let $\delta_1 = \zeta_1 + \sqrt{m}\gamma_1$. For convenience of constraint effect analysis (Chen et al., 2010), the auxiliary design system is given by

$$\dot{\varphi}_1 = \begin{cases} -K_{11}\varphi_1 - f_1(z_1, \varphi_1, \hat{\theta}_1, \hat{\Theta}_1)\varphi_1 \\ \quad + (G_1(x_1) + \gamma_1 I_{m\times m})(\alpha_1 - \alpha_{10}), & \|\varphi_1\| \geq \sigma_1 \\ 0, & \|\varphi_1\| < \sigma_1 \end{cases} \tag{59}$$

where $f_1(z_1, \varphi_1, \hat{\theta}_1, \hat{\Theta}_1) = \frac{\phi_1(z_1, \hat{\theta}_1, \hat{\Theta}_1)}{\|\varphi_1\|^2}$, $\phi_1(z_1, \hat{\theta}_1, \hat{\Theta}_1) = a_1\|K_{10}\| \|z_1\|^2 + \frac{1}{2}\|F_1(x_1)\hat{\theta}_1\|^2 + \frac{\delta_1\varepsilon_{10}^2}{2} + \zeta_1\varepsilon_{10}\|z_1\| + \|z_1\|\|F_1(x_1)\hat{\theta}_1\| + \frac{1}{2}\|\mathrm{Tanh}(z_1)\rho_1(x_1)\hat{\Theta}_1\|^2 + \|\dot{x}_{1d}\|^2 + \gamma_1 z_1^T\alpha_1 + \|z_1\|\|\mathrm{Tanh}(z_1)\rho_1(x_1)\hat{\Theta}_1\|$, $K_{11} = K_{11}^T > 0$, $a_1 > 0$ and σ_1 is a positive design parameter.

Consider the Lyapunov function candidate

$$V_1^* = \frac{1}{2}z_1^T z_1 + \frac{1}{2}\varphi_1^T\varphi_1. \tag{60}$$

Invoking (57)–(59), the time derivative of V_1 is

$$\begin{aligned}\dot{V}_1^* &= -c_1 z_1^T K_{10} z_1 + z_1^T F_1(x_1)\theta_1 + z_1^T G_1(x_1)(z_2 + \alpha_1) \\ &\quad + z_1^T \Delta G_1(x_1)x_2 + z_1^T D_1(x_1, t) - z_1^T\dot{x}_{1d} \\ &\quad + c_1 z_1^T K_{10} z_1 - \varphi_1^T K_{11}\varphi_1 - f_1(z_1, \varphi_1, \hat{\theta}_1, \hat{\Theta}_1)\|\varphi_1\|^2 \\ &\quad + \varphi_1^T(G_1(x_1) + \gamma_1 I_{m\times m})(\alpha_1 - \alpha_{10}) \\ &\leq -c_1 z_1^T K_{10} z_1 + z_1^T F_1(x_1)\theta_1 + z_1^T\mathrm{Tanh}(z_1)\rho_1(x_1)\Theta_1 \\ &\quad + \frac{\zeta_1}{2}\|z_1\|^2 + \frac{\zeta_1}{2}\|z_2\|^2 + \zeta_1\varepsilon_{10}\|z_1\| + \xi_{11}\|z_1\|\|x_2\| \\ &\quad + \frac{1}{2}\|z_1\|^2 + \frac{\|\dot{x}_{1d}\|^2}{2} + c_1\|K_{10}\|\|z_1\|^2 - \varphi_1^T K_{11}\varphi_1 \\ &\quad - f_1(z_1, \varphi_1, \theta_1, \Theta_1)\|\varphi_1\|^2 + \frac{\delta_1}{2}\|\varphi_1\|^2 + \frac{\delta_1\varepsilon_{10}^2}{2} \\ &\quad + \frac{\|K_{10}\|}{2}\|z_1\|^2 + \frac{3\|K_{10}\|}{2}\|\varphi_1\|^2 + \frac{3}{2}\|\varphi_1\|^2 \\ &\quad + \frac{1}{2}\|F_1(x_1)\hat{\theta}_1\|^2 + \frac{1}{2}\|\mathrm{Tanh}(z_1)\rho_1(x_1)\hat{\Theta}_1\|^2 + \frac{\|\dot{x}_{1d}\|^2}{2} \\ &\quad + z_1^T F_1(x_1)\hat{\theta}_1 + z_1^T\mathrm{Tanh}(z_1)\rho_1(x_1)\hat{\Theta}_1 - z_1^T F_1(x_1)\hat{\theta}_1 \\ &\quad - z_1^T\mathrm{Tanh}(z_1)\rho_1(x_1)\hat{\Theta}_1 + \frac{\|\Psi_1^T\rho_1(x_1)\|^2}{2} + \frac{\|\Theta_1\|^2}{2}\end{aligned}$$

$$\le -z_1^T\left(c_1K_{10}-\left(\frac{\zeta_1}{2}+\frac{1}{2}\right)I_{m\times m}\right)z_1$$
$$-\varphi_1^T\left(K_{11}-\left(\frac{3\|K_{10}\|}{2}+\frac{3}{2}+\frac{\delta_1}{2}\right)I_{m\times m}\right)\varphi_1$$
$$+\frac{\zeta_1}{2}\|z_2\|^2+\xi_{11}\|z_1\|\|x_2\|+\frac{\|\Psi_1^T\rho_1(x_1)\|^2}{2}$$
$$-\gamma_1z_1^T\alpha_1-z_1^TF_1(x_1)\tilde{\theta}_1-z_1^T\mathrm{Tanh}(z_1)\rho_1(x_1)\tilde{\Theta}_1$$
$$-(c_1+0.5)\|K_{10}\|\|z_1\|^2(a_{10}-1)+\frac{\|\Theta_1\|^2}{2} \quad (61)$$

where $c_1>0$, $a_{10}=\frac{a_1}{(c_1+0.5)}$.

Considering the error signals $\tilde{\theta}_1$ and $\tilde{\Theta}_1$, the augmented Lyapunov function candidate can be written as

$$V_1=V_1^*+\frac{1}{2}\tilde{\theta}_1^T\Lambda_{11}^{-1}\tilde{\theta}_1+\frac{1}{2}\tilde{\Theta}_1^T\Lambda_{12}^{-1}\tilde{\Theta}_1 \quad (62)$$

where $\Lambda_{11}=\Lambda_{11}^T>0$ and $\Lambda_{12}=\Lambda_{12}^T>0$.

Obviously, we can choose a_1 and c_1 to render $a_{10}-1>0$. Invoking (61), the time derivative of V_1 is

$$\dot{V}_1\le -z_1^T\left(c_1K_{10}-\left(\frac{\zeta_1}{2}+\frac{1}{2}\right)I_{m\times m}\right)z_1$$
$$-\varphi_1^T\left(K_{11}-\left(\frac{3\|K_{10}\|}{2}+\frac{3}{2}+\frac{\delta_1}{2}\right)I_{m\times m}\right)\varphi_1$$
$$+\frac{\zeta_1}{2}\|z_2\|^2+\xi_{11}\|z_1\|\|x_2\|+\frac{\|\Psi_1^T\rho_1(x_1)\|^2}{2}$$
$$-\gamma_1z_1^T\alpha_1-z_1^TF_1(x_1)\tilde{\theta}_1-z_1^T\mathrm{Tanh}(z_1)\rho_1(x_1)\tilde{\Theta}_1$$
$$+\tilde{\theta}_1^T\Lambda_{11}^{-1}\dot{\tilde{\theta}}_1+\tilde{\Theta}_1^T\Lambda_{12}^{-1}\dot{\tilde{\Theta}}_1. \quad (63)$$

Consider the adaptive laws for $\hat{\theta}_1$ and $\hat{\Theta}_1$ as

$$\dot{\hat{\theta}}_1=\Lambda_{11}(F_1^T(x_1)z_1-\beta_{11}\hat{\theta}_1) \quad (64)$$
$$\dot{\hat{\Theta}}_1=\Lambda_{12}(\rho_1(x_1)\mathrm{Tanh}(z_1)z_1-\beta_{12}\hat{\Theta}_1) \quad (65)$$

where $\beta_{11}>0$ and $\beta_{12}>0$.

Substituting (64) and (65) into (63), we obtain

$$\dot{V}_1\le -z_1^T\left(c_1K_{10}-\left(\frac{\zeta_1}{2}+\frac{1}{2}\right)I_{m\times m}\right)z_1$$
$$-\varphi_1^T\left(K_{11}-\left(\frac{3\|K_{10}\|}{2}+\frac{3}{2}+\frac{\delta_1}{2}\right)I_{m\times m}\right)\varphi_1$$
$$+\frac{\zeta_1}{2}\|z_2\|^2+\xi_{11}\|z_1\|\|x_2\|+\frac{\|\Psi_1^T\rho_1(x_1)\|^2}{2}$$
$$-\gamma_1z_1^T\alpha_1-\frac{\beta_{11}}{2}\|\tilde{\theta}_1\|^2+\frac{\beta_{11}}{2}\|\theta_1\|^2$$
$$-\frac{\beta_{12}}{2}\|\tilde{\Theta}_1\|^2+\frac{\beta_{12}}{2}\|\Theta_1\|^2+\frac{\|\Theta_1\|^2}{2}. \quad (66)$$

The first term and the second term on the right-hand side are negative if $c_1K_{10}-\left(\frac{\zeta_1}{2}+\frac{1}{2}\right)I_{m\times m}>0$ and $K_{11}-\left(\frac{3\|K_{10}\|}{2}+\frac{3}{2}+\frac{\delta_1}{2}\right)I_{m\times m}>0$. The other terms will be considered in the next step or the stability analysis of the closed-loop system.

Step i ($2\le i\le n-1$): Define the error variables $z_i=x_i-\alpha_{i-1}$ and $z_{i+1}=x_{i+1}-\alpha_i$. Considering (1) and differentiating z_i with respect to time, we obtain

$$\dot{z}_i=F_i(\bar{x}_i)\theta_i+G_i(\bar{x}_i)(z_{i+1}+\alpha_i)+\Delta G_i(\bar{x}_i)x_{i+1}$$
$$+D_i(\bar{x}_i,t)-\dot{\alpha}_{i-1} \quad (67)$$

where α_i is a virtual control law which is produced by the nominal virtual control law α_{i0}. The nominal virtual control law α_{i0} are filtered to provide the magnitude, rate and bandwidth limited virtual control law α_i and its derivatives $\dot{\alpha}_i$ which are within the operating envelope of the system. Such a command filter is similar to the first filter shown in Fig. 2 to implement any mechanical or operating constraints on virtual control law α_{i0}.

The nominal virtual control law α_{i0} is given by

$$\alpha_{i0}=(G_i(\bar{x}_i)+\gamma_iI_{m\times m})^{-1}\pi_i$$
$$\pi_i=-G_{i-1}^T(\bar{x}_{i-1})z_{i-1}-K_{i0}(z_i-\varphi_i)-F_i(\bar{x}_i)\hat{\theta}_i \quad (68)$$
$$-\mathrm{Tanh}(z_i)\rho_i(\bar{x}_i)\hat{\Theta}_i+\dot{\alpha}_{i-1}$$

where $K_{i0}=K_{i0}^T>0$, and $\varphi_i\in R^m$ is the state vector of the *ith* auxiliary design system which denotes the constraint effect due to the magnitude, rate and bandwidth limitation of nominal virtual control law α_{i0}. In nominal virtual control law (68), $\dot{\alpha}_{i-1}$ need not be computed here which can be directly obtained from the first command filter in Step $i-1$. Note the following facts $\|\alpha_i\|\le\varepsilon_{i0}$ and $\|\dot{\alpha}_{i-1}\|\le\varepsilon_{i1}$ with $\varepsilon_{i0}>0$ and $\varepsilon_{i1}>0$, where ε_{i0} and ε_{i1} denote the magnitude limit of α_i and the rate limit of $\dot{\alpha}_{i-1}$ which are decided by the command filter. Let $\delta_i=\zeta_i+\sqrt{m}\gamma_i$. For convenience of constraint effect analysis, the auxiliary design system is given by

$$\dot{\varphi}_i=\begin{cases}-K_{i1}\varphi_i-f_i(z_i,\varphi_i,\hat{\theta}_i,\hat{\Theta}_i)\varphi_i \\ \quad+(G_i(\bar{x}_i)+\gamma_iI_{m\times m})(\alpha_i-\alpha_{i0}), & \|\varphi_i\|\ge\sigma_i \\ 0, & \|\varphi_i\|<\sigma_i\end{cases} \quad (69)$$

where $f_i(z_i,\varphi_i,\hat{\theta}_i,\hat{\Theta}_i)=\frac{\phi_i(z_i,\hat{\theta}_i,\hat{\Theta}_i)}{\|\varphi_i\|^2}$, $\phi_i(z_i,\hat{\theta}_i,\hat{\Theta}_i)=a_i\|K_{i0}\|\|z_i\|^2+\frac{1}{2}\|\mathrm{Tanh}(z_i)\rho_i(\bar{x}_i)\hat{\Theta}_i\|^2+\frac{1}{2}\|F_i(\bar{x}_i)\hat{\theta}_i\|^2+\frac{\zeta_{i-1}}{2}\|z_{i-1}\|^2+\zeta_i\varepsilon_{i0}\|z_i\|+\|z_i\|\|F_i(\bar{x}_i)\hat{\theta}_i\|+(0.5+0.5\delta_i)\varepsilon_{i1}^2+\|z_i\|\|\mathrm{Tanh}(z_i)\rho_i(\bar{x}_i)\hat{\Theta}_i\|+\gamma_iz_i^T\alpha_i$, $K_{i1}=K_{i1}^T>0$, $a_i>0$ and σ_i is a positive design parameter.

Consider the Lyapunov function candidate

$$V_i^*=\frac{1}{2}z_i^Tz_i+\frac{1}{2}\varphi_i^T\varphi_i. \quad (70)$$

Invoking (67)–(69), the time derivative of V_i^* is

$$\dot{V}_i^*=-c_iz_i^TK_{i0}z_i+z_i^TF_i(\bar{x}_i)\theta_i+z_i^TG_i(\bar{x}_i)(z_{i+1}+\alpha_i)$$
$$+z_i^T\Delta G_i(\bar{x}_i)x_{i+1}+z_i^TD_i(\bar{x}_i,t)-z_i^T\dot{\alpha}_{i-1}$$
$$+c_iz_i^TK_{i0}z_i-\varphi_i^TK_{i1}\varphi_i-f_i(z_i,\varphi_i,\hat{\theta}_i,\hat{\Theta}_i)\|\varphi_i\|^2$$
$$+\varphi_i^T(G_i(\bar{x}_i)+\gamma_iI_{m\times m})(\alpha_i-\alpha_{i0})$$
$$\le -c_iz_i^TK_{i0}z_i+z_i^TF_i(\bar{x}_i)\theta_i+z_i^T\mathrm{Tanh}(z_i)\rho_i(\bar{x}_i)\Theta_i$$
$$+\frac{\zeta_i}{2}\|z_i\|^2+\frac{\zeta_i}{2}\|z_{i+1}\|^2+\zeta_i\varepsilon_{i0}\|z_i\|+\xi_{i1}\|z_i\|\|x_{i+1}\|$$
$$+\frac{1}{2}\|z_i\|^2+\frac{\varepsilon_{i1}^2}{2}+c_i\|K_{i0}\|\|z_i\|^2-\varphi_i^TK_{i1}\varphi_i$$
$$-f_i(z_i,\varphi_i,\hat{\theta}_i,\hat{\Theta}_i)\|\varphi_i\|^2+\frac{\delta_i}{2}\|\varphi_i\|^2+\frac{\delta_i\varepsilon_{i0}^2}{2}$$
$$+\frac{\zeta_{i-1}}{2}\|\varphi_i\|^2+\frac{\zeta_{i-1}}{2}\|z_{i-1}\|^2+\frac{\|K_{i0}\|}{2}\|z_i\|^2$$
$$+\frac{3\|K_{i0}\|}{2}\|\varphi_i\|^2+\frac{3}{2}\|\varphi_i\|^2+\frac{1}{2}\|F_i(\bar{x}_i)\hat{\theta}_i\|^2$$
$$+\frac{1}{2}\|\mathrm{Tanh}(z_i)\rho_i(\bar{x}_i)\hat{\Theta}_i\|^2+\frac{\varepsilon_{i1}^2}{2}+z_i^TF_i(\bar{x}_i)\hat{\theta}_i$$
$$+\frac{\|\Psi_i^T\rho_i(\bar{x}_i)\|^2}{2}+z_i^T\mathrm{Tanh}(z_i)\rho_i(\bar{x}_i)\hat{\Theta}_i-z_i^TF_i(\bar{x}_i)\hat{\theta}_i$$
$$-z_i^T\mathrm{Tanh}(z_i)\rho_i(\bar{x}_i)\hat{\Theta}_i+\frac{\|\Psi_i^T\rho_i(\bar{x}_i)\|^2}{2}+\frac{\|\Theta_i\|^2}{2}$$

$$\le -z_i^T\left(c_iK_{i0}-\left(\frac{\zeta_i}{2}+\frac{1}{2}\right)I_{m\times m}\right)z_i - \varphi_i^T\left(K_{i1}-\left(\frac{3\|K_{i0}\|}{2}+\frac{\zeta_{i-1}}{2}+\frac{3}{2}+\frac{\delta_i}{2}\right)I_{m\times m}\right)\varphi_i + \frac{\zeta_i}{2}\|z_{i+1}\|^2+\xi_{i1}\|z_i\|\|x_{i+1}\|-z_i^TF_i(\bar{x}_i)\tilde{\theta}_i - z_i^T\mathrm{Tanh}(z_i)\rho_i(\bar{x}_i)\tilde{\Theta}_i-\gamma_iz_i^T\alpha_i+\frac{\|\Psi_i^T\rho_i(\bar{x}_i)\|^2}{2} -(c_i+0.5)\|K_{i0}\|\|z_i\|^2(a_{i0}-1)+\frac{\|\Theta_i\|^2}{2} \tag{71}$$

where $c_i > 0$, $a_{i0}=\frac{a_i}{(c_i+0.5)}$.

Considering the error signals $\tilde{\theta}_i$ and $\tilde{\Theta}_i$, the augmented Lyapunov function candidate can be written as

$$V_i = V_{i-1}+V_i^*+\frac{1}{2}\tilde{\theta}_i^T\Lambda_{i1}^{-1}\tilde{\theta}_i+\frac{1}{2}\tilde{\Theta}_i^T\Lambda_{i2}^{-1}\tilde{\Theta}_i \tag{72}$$

where $\Lambda_{i1}=\Lambda_{i1}^T>0$ and $\Lambda_{i2}=\Lambda_{i2}^T>0$.

Similarly, we can choose a_i and c_i to render $a_{i0}-1>0$. Invoking (66) and (71), the time derivative of V_i is

$$\dot{V}_i \le -z_1^T\left(c_1K_{10}-\left(\frac{\zeta_1}{2}+\frac{1}{2}\right)I_{m\times m}\right)z_1 - \sum_{j=2}^{i}z_j^T\left(c_jK_{j0}-\left(\frac{\zeta_{j-1}}{2}+\frac{\zeta_j}{2}+\frac{1}{2}\right)I_{m\times m}\right)z_j - \varphi_1^T\left(K_{11}-\left(\frac{3\|K_{10}\|}{2}+\frac{3}{2}+\frac{\delta_1}{2}\right)I_{m\times m}\right)\varphi_1 - \sum_{j=2}^{i}\varphi_j^T\left(K_{j1}-\bar{K}_{j1}\right)\varphi_j+\frac{\zeta_i}{2}\|z_{i+1}\|^2-z_i^TF_i(\bar{x}_i)\tilde{\theta}_i - z_i^T\mathrm{Tanh}(z_i)\rho_i(\bar{x}_i)\tilde{\Theta}_i+\tilde{\theta}_i^T\Lambda_{i1}^{-1}\dot{\tilde{\theta}}_i+\tilde{\Theta}_i^T\Lambda_{i2}^{-1}\dot{\tilde{\Theta}}_i - \sum_{j=1}^{i-1}\frac{\beta_{j1}}{2}\|\tilde{\theta}_j\|^2+\sum_{j=1}^{i-1}\frac{\beta_{j1}}{2}\|\theta_j\|^2-\sum_{j=1}^{i-1}\frac{\beta_{j2}}{2}\|\tilde{\Theta}_j\|^2 + \sum_{j=1}^{i-1}\frac{\beta_{j2}}{2}\|\Theta_j\|^2+\sum_{j=1}^{i}\frac{\|\Theta_j\|^2}{2}-\sum_{j=1}^{i}\gamma_jz_j^T\alpha_j + \sum_{j=1}^{i}\xi_{j1}\|z_j\|\|x_{j+1}\|+\sum_{j=1}^{i}\frac{\|\Psi_j^T\rho_j(\bar{x}_j)\|^2}{2} \tag{73}$$

where $\bar{K}_{j1}=\left(\frac{3\|K_{j0}\|}{2}+\frac{\zeta_{j-1}}{2}+\frac{3}{2}+\frac{\delta_j}{2}\right)I_{m\times m}$.

Consider the adaptive laws for $\hat{\theta}_i$ and $\hat{\Theta}_i$ as

$$\dot{\hat{\theta}}_i=\Lambda_{i1}(F_i^T(\bar{x}_i)z_i-\beta_{i1}\hat{\theta}_i) \tag{74}$$

$$\dot{\hat{\Theta}}_i=\Lambda_{i2}(\rho_i(\bar{x}_i)\mathrm{Tanh}(z_i)z_i-\beta_{i2}\hat{\Theta}_i) \tag{75}$$

where $\beta_{i1}>0$ and $\beta_{i2}>0$.

Substituting (74) and (75) into (73), we obtain

$$\dot{V}_i \le -z_1^T\left(c_1K_{10}-\left(\frac{\zeta_1}{2}+\frac{1}{2}\right)I_{m\times m}\right)z_1 - \sum_{j=2}^{i}z_j^T\left(c_jK_{j0}-\left(\frac{\zeta_{j-1}}{2}+\frac{\zeta_j}{2}+\frac{1}{2}\right)I_{m\times m}\right)z_j - \varphi_1^T\left(K_{11}-\left(\frac{3\|K_{10}\|}{2}+\frac{3}{2}+\frac{\delta_1}{2}\right)I_{m\times m}\right)\varphi_1 - \sum_{j=2}^{i}\varphi_j^T\left(K_{j1}-\bar{K}_{j1}\right)\varphi_j+\frac{\zeta_i}{2}\|z_{i+1}\|^2-\sum_{j=1}^{i}\frac{\beta_{j1}}{2}\|\tilde{\theta}_j\|^2 + \sum_{j=1}^{i}\frac{\beta_{j1}}{2}\|\theta_j\|^2-\sum_{j=1}^{i}\frac{\beta_{j2}}{2}\|\tilde{\Theta}_j\|^2+\sum_{j=1}^{i}\frac{\beta_{j2}}{2}\|\Theta_j\|^2 + \sum_{j=1}^{i}\frac{\|\Theta_j\|^2}{2}-\sum_{j=1}^{i}\gamma_jz_j^T\alpha_j + \sum_{j=1}^{i}\xi_{j1}\|z_j\|\|x_{j+1}\|+\sum_{j=1}^{i}\frac{\|\Psi_j^T\rho_j(\bar{x}_j)\|^2}{2}. \tag{76}$$

The first four terms are negative if $c_1K_{10}-\left(\frac{\zeta_1}{2}+\frac{1}{2}\right)I_{m\times m}>0$, $c_jK_{j0}-\left(\frac{\zeta_{j-1}}{2}+\frac{\zeta_j}{2}+\frac{1}{2}\right)I_{m\times m}>0$ $(j=2,\ldots,i)$, $K_{11}-\left(\frac{3\|K_{10}\|}{2}+\frac{3}{2}+\frac{\delta_1}{2}\right)I_{m\times m}$ and $K_{j1}-\left(\frac{3\|K_{j0}\|}{2}+\frac{\zeta_{j-1}}{2}+\frac{3}{2}+\frac{\delta_j}{2}\right)I_{m\times m}$ $(j=2,\ldots,i)$. The other terms will be considered in the next step.

Step n: By differentiating $z_n=x_n-\alpha_{n-1}$ with respect to time yields

$$\dot{z}_n = F_n(\bar{x}_n)\theta_n+G_n(\bar{x}_n)u+\Delta G_n(\bar{x}_n)u + D_n(\bar{x}_n,t)-\dot{\alpha}_{n-1} \tag{77}$$

where u is a control law which is produced by the nominal control law v. The nominal control law v is filtered to provide the magnitude, rate and bandwidth limited virtual control law u and its derivatives $\dot{u}$ which are within the operating envelope of the system. Such a command filter is similar to the first filter shown in Fig. 2 to implement the mechanical or operating constraints on virtual control law v. Here, it is required that the command filter can implement the same position constraints on adaptive control v as shown in (2).

Define

$$h(Z) = \frac{1}{2}z_n^TK_{n0}^TK_nz_{n0}+\sum_{j=1}^{n-1}\xi_{j1}\|z_j\|\|x_{j+1}\| + \sum_{j=1}^{n-1}\gamma_j|z_j^T\alpha_j|+\sum_{j=1}^{n}\frac{\|\Psi_j^T\rho_j(\bar{x}_j)\|^2}{2} \tag{78}$$

where $K_{n0}=K_{n0}^T>0$ and $Z=[\alpha_j,z_j,\bar{x}_j]^T, j=1,2,\ldots,n$.

The nominal virtual control law v is given by

$$v=(G_n(\bar{x}_n)+\gamma_nI_{m\times m})^{-1}v_0$$

$$v_0 = -G_{n-1}^T(\bar{x}_{n-1})z_{n-1}-K_{n0}(z_n-\varphi_n)-F_n(\bar{x}_n)\hat{\theta}_n - \mathrm{Tanh}(z_n)\rho_n(\bar{x}_n)\hat{\Theta}_n+\dot{\alpha}_{n-1}-\frac{z_nh(Z)}{\psi^2+\|z_n\|^2} \tag{79}$$

$$\dot{\psi} = \begin{cases}-\dfrac{\psi h(Z)}{\psi^2+\|z_n\|^2}-k_v\psi, & \|z_n\|\ge\ell\\ 0, & \|z_n\|<\ell\end{cases} \tag{80}$$

where $\varphi_n\in R^m$ is the state vector of the auxiliary design system which denotes the constraint effect due to the magnitude, rate and bandwidth limitation of nominal virtual control law. In nominal control law (79), $\dot{\alpha}_{n-1}$ need not be computed here which can be directly obtained from the command filter in Step $n-1$. Note the following facts $\|u\|\le\varepsilon_{n0}$ and $\|\dot{\alpha}_{n-1}\|\le\varepsilon_{n1}$ with $\varepsilon_{n0}>0$ and $\varepsilon_{n1}>0$, where ε_{n0} and ε_{n1} denote the magnitude limit of u and the rate limit of $\dot{\alpha}_{n-1}$ which are decided by the command filter. Let $\delta_n=\zeta_n+\sqrt{m}\gamma_n$. For convenience of constraint effect analysis, the auxiliary design system is given by

$$\dot{\varphi}_n = \begin{cases}-K_{n1}\varphi_n-f_n(u,z_n,\varphi_n,\hat{\theta}_n,\hat{\Theta}_n)\varphi_n \\ \quad +(G_n(\bar{x}_n)+\gamma_nI_{m\times m})(u-v), & \|\varphi_n\|\ge\sigma_n\\ 0, & \|\varphi_n\|<\sigma_n\end{cases} \tag{81}$$

where $f_n(u,z_n,\varphi_n,\hat{\theta}_n,\hat{\Theta}_n)=\frac{\phi_n(u,z_n,\hat{\theta}_n,\hat{\Theta}_n)}{\|\varphi_n\|^2}$, $\phi_n(u,z_n,\hat{\theta}_n,\hat{\Theta}_n)=a_n\|K_{n0}\|\|z_n\|^2+\frac{\zeta_n}{2}\|u\|^2+\frac{1}{2}\|F_n(\bar{x}_n)\hat{\theta}_n\|^2+\frac{1}{2}\|\mathrm{Tanh}(z_n)\rho_n(\bar{x}_n)\hat{\Theta}_n\|^2+$

$\frac{\zeta_{n-1}}{2}\|z_{n-1}\|^2+\|z_n\|\|F_n(\bar{x}_n)\hat{\theta}_n\|+\|z_n\|\|\mathrm{Tanh}(z_n)\rho_n(\bar{x}_n)\hat{\Theta}_n\|+\frac{\delta_n\varepsilon_{n0}^2}{2}+\varepsilon_{n1}^2+|\gamma_n z_n^T u|+\xi_{n1}\|z_n\|\|u\|+\frac{1}{2}z_n^T K_{n0}^T K_n z_{n0}+\frac{\|z_n\|^2 h(Z)}{\psi^2+\|z_n\|^2}$, $K_{n1}=K_{n1}^T>0$, $a_n>0$ and σ_n is a positive design parameter.

Consider the Lyapunov function candidate

$$V_n^*=\frac{1}{2}z_n^T z_n+\frac{1}{2}\varphi_n^T\varphi_n. \tag{82}$$

Invoking (77), (79) and (81), the time derivative of V_n^* is

$$\begin{aligned}
\dot{V}_n^* &= -c_n z_n^T K_{n0}z_n+z_n^T F_n(\bar{x}_n)\theta_n+z_n^T G_n(\bar{x}_n)u\\
&\quad +z_n^T\Delta G_n(\bar{x}_n)u+z_n^T D_n(\bar{x}_n,t)-z_n^T\dot{\alpha}_{n-1}\\
&\quad +c_n z_n^T K_{n0}z_n-\varphi_n^T K_{n1}\varphi_n-f_n(z_n,\varphi_n,\hat{\theta}_n,\hat{\Theta}_n)\|\varphi_n\|^2\\
&\quad +\varphi_n^T(G_n(\bar{x}_n)+\gamma_n I_{m\times m})(u-v)\\
&\le -c_n z_n^T K_{n0}z_n+z_n^T F_n(\bar{x}_n)\theta_n+\frac{\zeta_n}{2}\|z_n\|^2\\
&\quad +z_n^T\mathrm{Tanh}(z_n)\rho_n(\bar{x}_n)\Theta_n+|\gamma_n z_n^T u|+\frac{\zeta_n}{2}\|u\|^2\\
&\quad +\xi_{n1}\|z_n\|\|u\|+\frac{1}{2}\|z_n\|^2+\frac{\varepsilon_{n1}^2}{2}+c_n\|K_{n0}\|\|z_n\|^2\\
&\quad -\varphi_n^T K_{n1}\varphi_n-f_n(z_n,\varphi_n,\hat{\theta}_n,\hat{\Theta}_n)\|\varphi_n\|^2\\
&\quad +\frac{\delta_n}{2}\|\varphi_n\|^2+\frac{\delta_n\varepsilon_{n0}^2}{2}+\frac{\zeta_{n-1}}{2}\|\varphi_n\|^2+\frac{\zeta_{n-1}}{2}\|z_{n-1}\|^2\\
&\quad +\frac{\|K_{n0}\|}{2}\|z_n\|^2+\frac{3\|K_{n0}\|}{2}\|\varphi_n\|^2+\frac{3}{2}\|\varphi_n\|^2\\
&\quad +\frac{1}{2}\|F_n(\bar{x}_n)\hat{\theta}_n\|^2+\frac{1}{2}\|\mathrm{Tanh}(z_n)\rho_n(\bar{x}_n)\hat{\Theta}_n\|^2\\
&\quad +\frac{\varepsilon_{n1}^2}{2}+z_n^T F_n(\bar{x}_n)\hat{\theta}_n+z_n^T\mathrm{Tanh}(z_n)\rho_n(\bar{x}_n)\hat{\Theta}_n\\
&\quad -z_n^T F_n(\bar{x}_n)\hat{\theta}_n-z_n^T\mathrm{Tanh}(z_n)\rho_n(\bar{x}_n)\hat{\Theta}_n\\
&\quad +\frac{\|\Psi_n^T\rho_n(\bar{x}_n)\|^2}{2}+\frac{\|\Theta_n\|^2}{2}-\frac{1}{2}z_n^T K_{n0}^T K_n z_{n0}\\
&\le -z_n^T\left(c_n K_{n0}-\left(\frac{\zeta_n}{2}+\frac{1}{2}\right)I_{m\times m}\right)z_n\\
&\quad -\varphi_n^T\left(K_{n1}-\bar{K}_{n1}\right)\varphi_n+\frac{1}{2}z_n^T K_{n0}^T K_n z_{n0}\\
&\quad -z_n^T F_n(\bar{x}_n)\tilde{\theta}_n-z_n^T\mathrm{Tanh}(z_n)\rho_n(\bar{x}_n)\tilde{\Theta}_n\\
&\quad -(c_n+0.5)\|K_{n0}\|\|z_n\|^2(a_{n0}-1)\\
&\quad +\frac{\|\Theta_n\|^2}{2}+\frac{\|\Psi_n^T\rho_n(\bar{x}_n)\|^2}{2}-\frac{\|z_n\|^2 h(Z)}{\psi^2+\|z_n\|^2}
\end{aligned}\tag{83}$$

where $\bar{K}_{n1}=\left(\frac{3\|K_{n0}\|+\zeta_{n-1}+\delta_n+3}{2}\right)I_{m\times m}$, $c_n>0$ and $a_{n0}=\frac{a_n}{(c_n+0.5)}$.

Considering the error signals $\tilde{\theta}_n$ and $\tilde{\Theta}_n$, the augmented Lyapunov function candidate can be written as

$$V_n=V_{n-1}+V_n^*+\frac{1}{2}\tilde{\theta}_n^T\Lambda_{n1}^{-1}\tilde{\theta}_n+\frac{1}{2}\tilde{\Theta}_n^T\Lambda_{n2}^{-1}\tilde{\Theta}_n+\frac{1}{2}\psi^2 \tag{84}$$

where $\Lambda_{n1}=\Lambda_{n1}^T>0$ and $\Lambda_{n2}=\Lambda_{n2}^T>0$.

Obviously, we can choose a_n and c_n to render $a_{n0}-1>0$. Invoking (76), (78), (79) and (83), the time derivative of V_n is

$$\begin{aligned}
\dot{V}_n &\le -z_1^T\left(c_1K_{10}-\left(\frac{\zeta_1}{2}+\frac{1}{2}\right)I_{m\times m}\right)z_1\\
&\quad -\sum_{j=2}^{n}z_j^T\left(c_jK_{j0}-\left(\frac{\zeta_{j-1}}{2}+\frac{\zeta_j}{2}+\frac{1}{2}\right)I_{m\times m}\right)z_j\\
&\quad -\varphi_1^T\left(K_{11}-\left(\frac{3\|K_{10}\|}{2}+\frac{3}{2}+\frac{\delta_1}{2}\right)I_{m\times m}\right)\varphi_1\\
&\quad -\sum_{j=2}^{n}\varphi_j^T\left(K_{j1}-\bar{K}_{j1}\right)\varphi_j-z_n^T F_n(\bar{x}_n)\tilde{\theta}_n\\
&\quad -z_n^T\mathrm{Tanh}(z_n)\rho_n(\bar{x}_n)\tilde{\Theta}_n+\tilde{\theta}_n^T\Lambda_{n1}^{-1}\dot{\hat{\theta}}_n+\tilde{\Theta}_n^T\Lambda_{n2}^{-1}\dot{\hat{\Theta}}_n\\
&\quad -\sum_{j=1}^{n-1}\frac{\beta_{j1}}{2}\|\tilde{\theta}_j\|^2+\sum_{j=1}^{n-1}\frac{\beta_{j1}}{2}\|\theta_j\|^2-\sum_{j=1}^{n-1}\frac{\beta_{j2}}{2}\|\tilde{\Theta}_j\|^2\\
&\quad +\sum_{j=1}^{n-1}\frac{\beta_{j2}}{2}\|\Theta_j\|^2+\sum_{j=1}^{n}\frac{\|\Theta_j\|^2}{2}\\
&\quad +\frac{\psi^2 h(Z)}{\psi^2+\|z_n\|^2}+\psi\dot{\psi}.
\end{aligned}\tag{85}$$

Consider the adaptive laws for $\hat{\theta}_n$ and $\hat{\Theta}_n$ as

$$\dot{\hat{\theta}}_n=\Lambda_{n1}(F_n^T(\bar{x}_n)z_n-\beta_{n1}\hat{\theta}_n) \tag{86}$$

$$\dot{\hat{\Theta}}_n=\Lambda_{n2}(\rho_n(\bar{x}_n)\mathrm{Tanh}(z_n)z_n-\beta_{n2}\hat{\Theta}_n) \tag{87}$$

where $\beta_{n1}>0$ and $\beta_{n2}>0$.

Substituting (78), (86) and (87) into (85), we obtain

$$\begin{aligned}
\dot{V}_n &\le -z_1^T\left(c_1K_{10}-\left(\frac{\zeta_1}{2}+\frac{1}{2}\right)I_{m\times m}\right)z_1\\
&\quad -\sum_{j=2}^{n}z_j^T\left(c_jK_{j0}-\left(\frac{\zeta_{j-1}}{2}+\frac{\zeta_j}{2}+\frac{1}{2}\right)I_{m\times m}\right)z_j\\
&\quad -\varphi_1^T\left(K_{11}-\left(\frac{3\|K_{10}\|}{2}+\frac{3}{2}+\frac{\delta_1}{2}\right)I_{m\times m}\right)\varphi_1\\
&\quad -\sum_{j=2}^{n}\varphi_j^T\left(K_{j1}-\bar{K}_{j1}\right)\varphi_j-\sum_{j=1}^{n}\frac{\beta_{j1}}{2}\|\tilde{\theta}_j\|^2\\
&\quad +\sum_{i=1}^{n}\frac{\beta_{j1}}{2}\|\theta_j\|^2-\sum_{j=1}^{n}\frac{\beta_{j2}}{2}\|\tilde{\Theta}_j\|^2+\sum_{i=1}^{n}\frac{\beta_{j2}}{2}\|\Theta_j\|^2\\
&\quad +\sum_{j=1}^{n}\frac{\|\Theta_j\|^2}{2}-k_v\psi^2\\
&\le -\kappa V_n+C
\end{aligned}\tag{88}$$

where

$$\kappa:=\min\left(\begin{array}{c}2\lambda_{\min}(Q_0),2\lambda_{\min}(Q_1),\\ 2\lambda_{\min}(Q_2),2\lambda_{\min}(Q_3),\\ \sum_{j=1}^{n}\frac{2\beta_{j1}}{\lambda_{\max}(\Lambda_{j1}^{-1})},\sum_{j=1}^{n}\frac{2\beta_{j2}}{\lambda_{\max}(\Lambda_{j2}^{-1})},k_v\end{array}\right),$$

$$C:=\sum_{j=1}^{n}\frac{\beta_{j1}}{2}\|\theta_j\|^2+\sum_{j=1}^{n}\frac{\beta_{j2}}{2}\|\Theta_j\|^2+\sum_{j=1}^{n}\frac{\|\Theta_j\|^2}{2},$$

$$Q_0=c_1K_{10}-\left(\frac{\zeta_1}{2}+\frac{1}{2}\right)I_{m\times m},$$

$$Q_1=\sum_{j=2}^{n}\left(c_jK_{j0}-\left(\frac{\zeta_{j-1}}{2}+\frac{\zeta_j}{2}+\frac{1}{2}\right)I_{m\times m}\right),$$

$$Q_2=K_{11}-\left(\frac{3\|K_{10}\|}{2}+\frac{3}{2}+\frac{\delta_1}{2}\right)I_{m\times m},$$

$$Q_3=\sum_{j=2}^{n}(K_{j1}-\bar{K}_{j1}),\quad j=2,\ldots,n. \tag{89}$$

To ensure the closed-loop stability, we can choose corresponding design parameters to make $Q_0>0$, $Q_1>0$, $Q_2>0$ and $Q_3>0$. The above design procedure can be summarized in the following theorem, which contains the results for the constrained adaptive control of an uncertain nonlinear system.

Theorem 2. *Considering the uncertain MIMO nonlinear system (1) satisfies Assumptions 1–5, and given that full state information is available. The control law is produced by nominal control law (79) using the command filter. Under the parameter adaptation laws (64), (65), (74), (75), (86), (87) and for any bounded initial condition, the closed-loop signals z_i, φ_i, ψ, $\tilde{\theta}_i$ and $\tilde{\Theta}_i$ $(i \le i \le n)$ are semi-globally stable in the sense that all of the closed-loop signals are bounded, where $i = 1, 2, \ldots, n$. The tracking error z_1 asymptotically converges to a compact set Ω_{z1} defined by*

$$\Omega_{z_1} := \left\{ z_1 \in R^m \mid \|z_1\| \le \sqrt{D} \right\}$$

where $D = 2(V_n(0) + \frac{C}{\kappa})$ with C and κ as defined in (89).

It is apparent that the Theorem 2 can be easily proved according to (84) and (88).

Remark 4. In the proposed constrained adaptive control, we can see that the satisfactory closed-loop stability with suitable transient performance can be achieved by properly adjusting design parameters $K_{i0}, K_{i1}, \beta_{i1}, \beta_{i2}, \Lambda_i$, and k, $i = 1, 2, \ldots, n$. For example, the tracking error could be decreased by increasing the value of K_{i0}, but that increase would also increase the control signal, and could excite unmodeled dynamics. Therefore, caution must be exercised in the choice of these parameters, due to the fact that there is some trade-off between the control performance and other issues.

Remark 5. In the developed constrained adaptive control, if $\varphi_i = 0$ and $\dot{\varphi}_i = 0$, there are $\alpha_{i0} = \alpha_i$ and $u = v$ according to (59), (69) and (81), i.e., there are no constraints. At the same time, the nominal virtual control law (58), (67) and nominal control law (79) are the same as the virtual control law (13), (35) and control law (47) of the proposed adaptive control in Section 3. It should be pointed out that we do not directly consider the input saturation constraint (2). However, the command filter can not only implement the same position and also rate constraints can be considered on the adaptive control v by choosing the appropriate design parameter.

Remark 6. In practice, it is apparent that the magnitude of the actual/virtual control input, as well as their derivations should be bounded due to the physical limitation. Thus, the command filter could be presented according to mechanical and operating constraints of actuator. Magnitude limit function and rate limit function can be chosen as conservative common saturation function or other limit functions. If limit functions are chosen as conservative common saturation functions, the relationship between the input and the output of the command filter can be found in Farrell et al. (2003).

5. Simulation results

Consider the uncertain MIMO nonlinear system with input saturation in the form of Chen et al. (2010)

$$\begin{aligned}
\dot{x}_1 &= F_1(x_1)\theta_1 + (G_1(x_1) + \Delta G_1(x_1))x_2 + D_1(x_1, t) \\
\dot{x}_2 &= F_2(\bar{x}_2)\theta_2 + (G_2(\bar{x}_2) + \Delta G_2(\bar{x}_2))u + D_2(\bar{x}_2, t) \\
y &= x_1
\end{aligned} \tag{90}$$

where

$$x_1 = [x_{11}, x_{12}]^T, \qquad x_2 = [x_{21}, x_{22}]^T,$$

$$F_1(x_1) = \begin{bmatrix} 0.2\sin(x_{11})\cos(x_{12}) \\ 0.2x_{11}x_{12} \end{bmatrix},$$

$$G_1(x_1) = \begin{bmatrix} g_{11}(x) & -2 \\ 5 & g_{22}(x) \end{bmatrix},$$

$$g_{11}(x) = 1.2 + \cos(x_{11})\sin(x_{12}),$$

$$g_{22}(x) = 1.3 - \cos(x_{12})\sin(x_{11}),$$

$$\Delta G_1(x_1) = \begin{bmatrix} 0.2\sin(x_{11}) & 0 \\ 0 & 0.1\cos(x_{12}) \end{bmatrix},$$

$$F_2(\bar{x}_2) = \begin{bmatrix} -x_{12}x_{21} & 0 \\ 0 & 2x_{11}x_{22} \end{bmatrix}$$

$$G_2(\bar{x}_2) = \begin{bmatrix} \cos(x_{21})\sin(x_{22}) & -\sin(x_{22}) \\ \sin(x_{22}) & \cos(x_{21}) \end{bmatrix},$$

$$\Delta G_2(\bar{x}_2) = \begin{bmatrix} 0.12\sin(x_{11}x_{21}) & 0.11\cos(x_{11}x_{21}) \\ 0.15\cos(x_{11}x_{21}) & 0.13\sin(x_{21}x_{22}) \end{bmatrix},$$

$$D_1(x_1, t) = \begin{bmatrix} 0.21(\cos(x_{12}))^2 + 0.04\sin(0.3x_{12}t) \\ 0.12(\sin(x_{11}))^2 + 0.03\sin(0.2x_{11}t) \end{bmatrix},$$

$$D_2(\bar{x}_2, t) = \begin{bmatrix} 0.13(\sin(x_{22}))^2 + 0.05\sin(0.2x_{22}t) \\ 0.11(\cos(x_{21}))^2 + 0.21\sin(0.3x_{21}t) \end{bmatrix}.$$

For simulation purposes, parameter values are set to $\theta_1 = -1$, $\theta_2 = 0.5$, $u_{1\max} = -u_{1\min} = 3.0$, $u_{2\max} = -u_{2\min} = 2.0$, $g_{ri}(v_i) = g_{li}(v_i) = v_i$, $\gamma_1 = 3.0$ and $\gamma_2 = 2.0$.

Now, the control objective is to design adaptive control and constraint adaptive control for system (90) such that the system output $y = x_1$ follows the desired trajectory x_{1d}, where the desired trajectories are taken as $x_{11d} = 0.5[\sin(1.5t) + \sin(0.5t)]$ and $x_{12d} = 0.8\sin(t) + 0.5\sin(0.5t)$.

The adaptive control is designed as follows:

$$\begin{aligned}
\alpha_1 &= (G_1(x_1) + \gamma_1 I_{m\times m})^{-1} \\
&\quad \left(-K_1 z_1 - F_1(x_1)\hat{\theta}_1 - \mathrm{Tanh}(z_1)\rho_1(x_1)\hat{\Theta}_1 + \dot{x}_{1d}\right)
\end{aligned}$$

$$\dot{e} = \begin{cases} -K_{22}e - \dfrac{1}{\|e\|^2} f(u, \Delta u, z_2, \bar{x}_2)e \\ \quad + (G_2(\bar{x}_2) + \gamma_2 I_{2\times 2})(v - u), & \|e\| \ge \sigma \\ 0, & \|e\| < \sigma \end{cases}$$

$$v = (G_2(\bar{x}_2) + \gamma_2 I_{m\times m})^{-1} v_0$$

$$\begin{aligned}
v_0 &= -G_1^T(x_1)z_1 - K_2(z_2 - e) - F_2(\bar{x}_2)\hat{\theta}_2 \\
&\quad - \mathrm{Tanh}(z_2)\rho_2(\bar{x}_2)\hat{\Theta}_2 + \dot{\alpha}_1 - \frac{z_2 h(Z)}{\psi^2 + \|z_2\|^2}
\end{aligned}$$

$$\dot{\psi} = \begin{cases} -\dfrac{\psi h(Z)}{\psi^2 + \|z_2\|^2} - k\psi, & \|z_2\| \ge \ell \\ 0, & \|z_2\| < \ell \end{cases}$$

where $k > 0$, $K_1 = K_1^T > 0$, $K_2 = K_2^T > 0$, $K_{22} = K_{22}^T > 0$, $f(u, \Delta u, z_2, \bar{x}_2) = |z_2^T G_2(\bar{x}_2)\Delta u| + 0.5(\gamma_2 + \zeta_2)^2 \Delta u^T \Delta u$ and $\sigma = 0.1$. The adaptive laws for $\hat{\theta}_1$ and $\hat{\Theta}_1$ are chosen as (17) and (18). The adaptive laws of $\hat{\theta}_2$ and $\hat{\Theta}_2$ are chosen as (53) and (54). The design parameters of the control are chosen as $K_1 = \mathrm{diag}\{18.0, 18.0\}$, $K_2 = \mathrm{diag}\{120.0, 180.0\}$, $K_{22} = \mathrm{diag}\{10.0, 10.0\}$ and $\Lambda_{11} = \Lambda_{12} = \Lambda_{21} = \Lambda_{21} = \mathrm{diag}\{0.01, 0.01\}$.

The simulation results of the tracking output are shown in Figs. 3 and 4 with initial states $x_{11} = 1.0$ and $x_{12} = 0.0$. It can be observed that the system output x_{11} and x_{12} follow the desired trajectory x_{11d} and x_{12d} well despite the unknown parameters, perturbation of the control coefficient matrices and input saturation. From Figs. 5 and 6, we can see that the control inputs are saturated in the initialization transient phase. These simulation results show that good tracking performance can be obtained under the proposed adaptive control.

To illustrate the effectiveness of the proposed constrained adaptive control, the nominal virtual control law and the control command are designed based on (58) and (79). Then, the nominal virtual control law α_{10} and the nominal control command v are used to produce the virtual control law α_1 and the system control

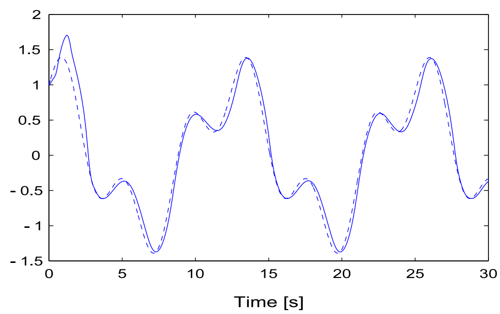

Fig. 3. Output x_{11} (solid line) follows desired trajectory x_{11d} (dashed line).

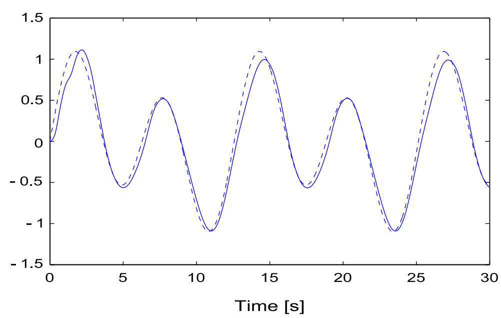

Fig. 4. Output x_{12} (solid line) follows desired trajectory x_{12d} (dashed line).

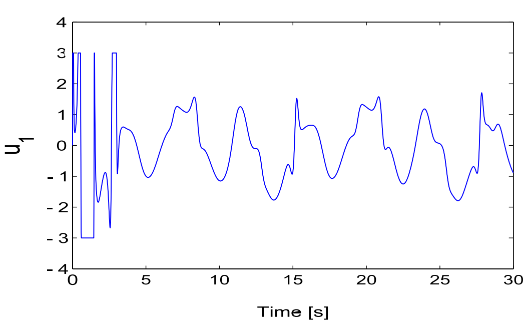

Fig. 5. Control signal u_1.

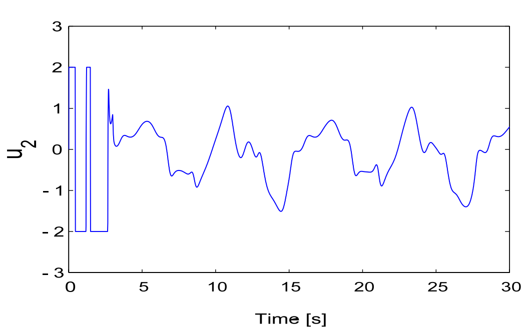

Fig. 6. Control signal u_2.

law u using the command filter as shown in Fig. 2. The design parameters of the filters are chosen as $\omega_{1n} = 10$, $\xi_1 = \xi_2 = 0.707$ and $\omega_{2n} = 100$. To observer the variety of closed-loop system control performance for the different design parameters under the constrained adaptive tracking control, the following two cases are considered:

Case 1: K_{10} and K_{20} are chosen as $K_{10} = \text{diag}\{18.0, 18.0\}$ and $K_{20} = \text{diag}\{120.0, 180.0\}$. Other design parameters are chosen as the same design parameters as the corresponding design parameters in the adaptive tracking control.

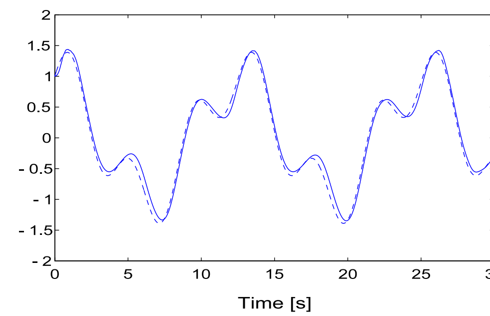

Fig. 7. Output x_{11} (solid line) follows desired trajectory x_{11d} (dashed line) for Case 1.

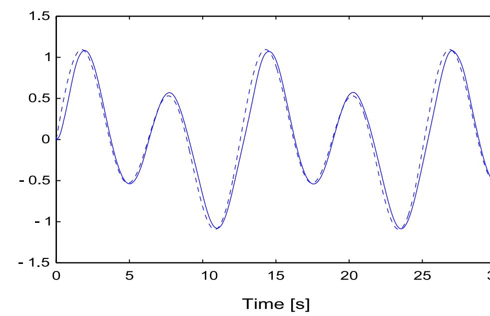

Fig. 8. Output x_{12} (solid line) follows desired trajectory x_{12d} (dashed line) for Case 1.

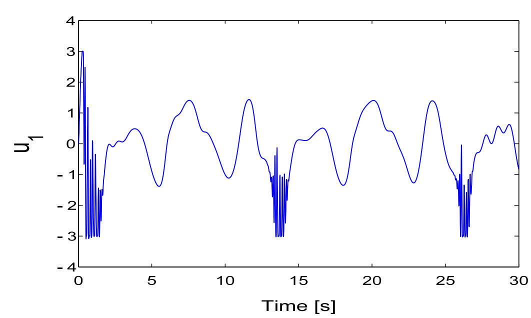

Fig. 9. Control signal u_1 for Case 1.

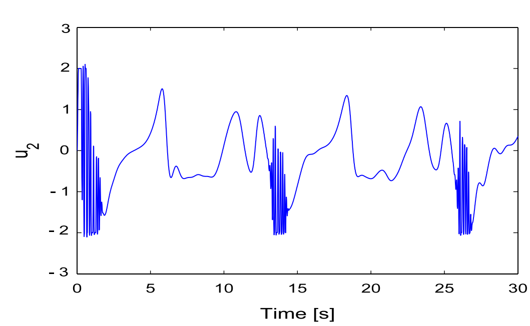

Fig. 10. Control signal u_2 for Case 1.

Case 2: The design parameters K_{10} and K_{20} are chosen as $K_{10} = \text{diag}\{10.0, 10.0\}$ and $K_{20} = \text{diag}\{120.0, 120.0\}$. Other design parameters are chosen as the same design parameters as the corresponding design parameters in the adaptive tracking control.

Under initial states are $x_{11} = 1.0$ and $x_{12} = 0.0$, the tracking results of the Case 1 are shown in Figs. 7 and 8. It can be observed that the outputs x_{11} and x_{12} of Case 1 still follow the desired trajectory x_{11d} and x_{12d} when the actuator constraints are considered. In accordance with Figs. 9 and 10, it is observed that the control inputs

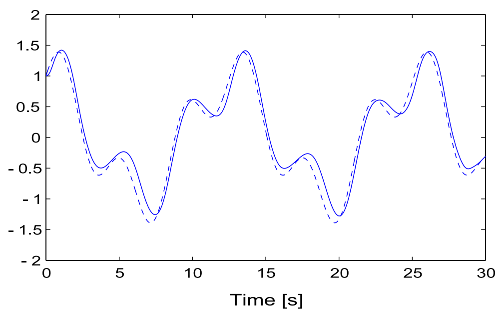

Fig. 11. Output x_{11} (solid line) follows desired trajectory x_{11d} (dashed line) for Case 2.

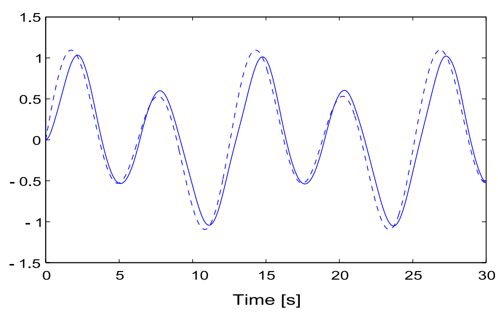

Fig. 12. Output x_{12} (solid line) follows desired trajectory x_{12d} (dashed line) for Case 2.

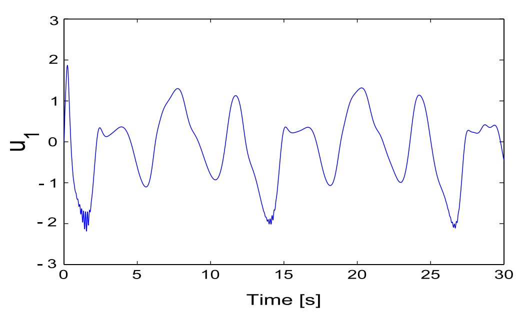

Fig. 13. Control signal u_1 for Case 2.

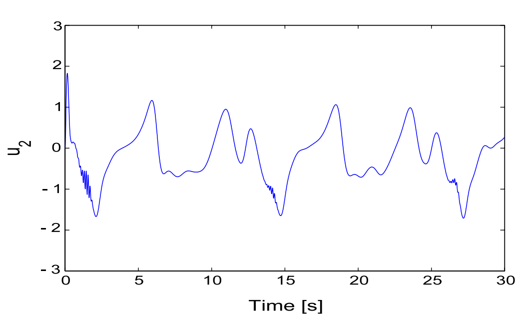

Fig. 14. Control signal u_2 for Case 2.

are saturated in the transient phase of Case 1. As a comparison, the corresponding simulation results of Case 2 are shown in Figs. 11–14. From Figs. 11 and 12, we can observed that the different tracking performance can be obtained by adjusting the design parameters of the constrained adaptive tracking control. According to Figs. 7, 8, 11 and 12, we obtain that the tracking error could be decreased by increasing the value of K_{i0}, but that increases would also increase the control signal and could excite unmodeled dynamics.

6. Conclusion

Model-based adaptive control has been investigated for the uncertain MIMO nonlinear systems with input constraints in this paper. Considering actuator physical constraints, the adaptive control and the constrained adaptive control in combination with the backstepping technique and Lyapunov synthesis have been proposed. In the development of adaptive control, the auxiliary design system has been introduced to analyze the effect of actuator physical constraint, and states of auxiliary design system are used to develop adaptive control. The cascade property of the studied systems has been fully utilized in developing the control structure and parameter adaptive laws. It has proved that both the proposed adaptive control and the constrained adaptive control are able to guarantee the asymptotical stability of all signals in the closed-loop system. Finally, simulation studies have been presented to illustrate the effectiveness of the proposed adaptive and the constrained adaptive control.

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