Adaptive Dynamic Surface Control for a Class of Strict-Feedback Nonlinear Systems with Unknown Backlash-Like Hysteresis

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Adaptive Dynamic Surface Control for a Class of Strict-Feedback Nonlinear Systems with Unknown Backlash-Like Hysteresis

Beibei Ren, Phyo Phyo San, Shuzhi Sam Ge and Tong Heng Lee

***Abstract*— In this paper, we investigate the control design for a class of strict-feedback nonlinear systems preceded by unknown backlash-like hysteresis. Using the characteristics of backlash-like hysteresis, adaptive dynamic surface control (DSC) is developed without constructing a hysteresis inverse. The explosion of complexity in traditional backstepping design is avoided by utilizing DSC. Function uncertainties are compensated for using neural networks due to their universal approximation capabilities. Through Lyapunov synthesis, the closed-loop control system is proved to be semi-globally uniformly ultimately bounded (SGUUB), and the tracking error converges to a small neighborhood of zero. Simulation results are provided to illustrate the performance of the proposed approach.**

***Index Terms*— Dynamic surface control (DSC), hysteresis, neural networks(NNs).**

I. INTRODUCTION

Hysteresis nonlinearities are common in many industrial processes, especially in position control of smart material-based actuators, including piezoceramics and shape memory alloys. The existence of hysteresis nonlinearities severely limit system performance such as giving rise to undesirable inaccuracy or oscillations and even may lead to instability [1]. Since hysteresis is a very complex phenomenon, modeling a general type of hysteresis is still an active research topic and there exist many hysteresis models in the literature, such as the Preisach model, the Ishlinskii hysteresis operator, the Prandtl-Ishlinskii hysteresis model, the Duhem hysteresis operator, the Bouc Wen model, an so on. Interested readers can refer to [2] for a review of the hysteresis models. Among of them, the backlash hysteresis model is the most familiar and simple model, which can be described by two parallel lines connected via horizontal line segments and will be considered in this paper.

Due to the nonsmooth characteristics of hysteresis nonlinearities, traditional control methods are inadequate in dealing with the effects of unknown hysteresis. Therefore, advanced control techniques to mitigate the effects of hysteresis have been called upon and have been studied for decades. One of the most common approaches is to construct an inverse operator to cancel the effects of the hysteresis as in [1] and [3]. However, it is a challenging task to construct the inverse operator for the hysteresis, due to its complexity and uncertainty. To circumvent these difficulties, alternative control approaches that do not need an inverse model have also been developed. In [4] and [5], robust adaptive control and adaptive backstepping control were, respectively, investigated for a class of nonlinear systems in a Brunovsky form with unknown backlash-like hysteresis and system parameters.

Motivated by the above works [4] and [5], in this paper, we extend the system to a class of nonlinear systems in strict-feedback form with unknown functions and disturbances. The function uncertainties are compensated for by neural networks due to their universal approximation capabilities [6]-[8]. For the control of strict-feedback nonlinear systems, though backstepping is one of the popular design methods, an obvious drawback in the traditional backstepping design is the problem of "explosion of complexity", which is caused by the repeated differentiations of certain nonlinear functions such as virtual controls. To overcome the "explosion of complexity", dynamic surface control (DSC) was proposed for a class of strict-feedback nonlinear systems with known $f_i(x_1, ..., x_i)$ and $g_i = 1$ by introducing first-order filtering of the synthetic virtual control input at each step of traditional backstepping approach [9]. The result was extended to a class of strict-feedback nonlinear systems with unknown functions f_i and virtual coefficients $g_i = 1$ by combining DSC control and neural networks [10]. In this paper, the virtual coefficients g_i of the strict-feedback nonlinear systems are considered as unknown constants further. The bounds of the "disturbance-like" terms, including disturbances and neural network approximation errors, are estimated by adaptive control.

The organization of this paper is as follows. The problem formulation and preliminaries are given in Section II. In Section III, adaptive dynamic surface control is developed for a class of unknown nonlinear systems in strict-feedback form with the unknown backlash-like hysteresis. The closed-loop system stability is analyzed as well. Results of extensive simulation studies are shown to demonstrate the effectiveness of the approach in Section IV, followed by the conclusion in Section V.

II. PROBLEM FORMULATION AND PRELIMINARIES

Throughout this paper, $\tilde{(\cdot)} = \hat{(\cdot)} - (\cdot)$, $\|\cdot\|$ denotes the 2-norm, $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the smallest and largest eigenvalues of a square matrix $(\cdot)$, respectively.

Consider a class of nonlinear systems in strict-feedback form described as follows:

$$\begin{aligned}\dot{x}_1 &= f_1(x_1) + g_1 x_2 + d_1(t)\\ &\vdots\end{aligned}$$

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$$
\begin{aligned}
\dot{x}_i &= f_i(\bar{x}_i) + g_i x_{i+1} + d_i(t), \quad i = 2, ..., n-1 \\
&\vdots \\
\dot{x}_n &= f_n(\bar{x}_n) + g_n u(v) + d_n(t) \\
y &= x_1
\end{aligned} \tag{1}
$$

where $\bar{x}_i = [x_1, ..., x_i]^T \in R^i$, $i = 1, ..., n$ are the states, y is the system output, g_i are the unknown constant virtual coefficients, $f_i(\cdot)$ are the unknown smooth functions, $d_i(\cdot)$ are the unknown bounded time varying disturbances, and $u \in R$ is the system input and the output of the backlash-like hysteresis, which is described as follows:

$$
\frac{du}{dt} = \alpha \left| \frac{dv}{dt} \right| (cv - u) + B_1 \frac{dv}{dt} \tag{2}
$$

where α, c, and B_1 are constants, $c > 0$ is the slope of lines satisfying $c > B_1$. Fig. 1 shows that the model (2) indeed generates a class of backlash-like hysteresis curve, where $\alpha = 1.0$, $c = 3.1635$, $B_1 = 0.345$ and the input signal $v = 6.5\sin(2.3t)$.

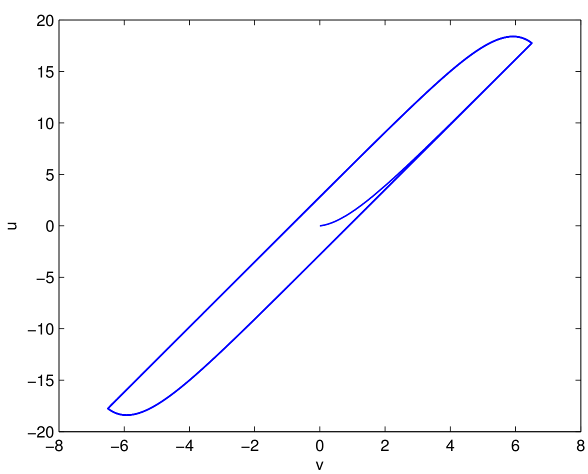

Fig. 1. Backlash-like Hysteris curve

Based on the analysis in [4], (2) can be solved explicitly as follows:

$$
u(t) = cv(t) + h(v) \tag{3}
$$

where

$$
\begin{aligned}
h(v) &= [u_0 - cv_0]e^{-\alpha(v-v_0)\mathrm{sgn}\dot{v}} \\
&\quad + e^{-\alpha v \mathrm{sgn}\dot{v}} \int_{v_0}^{v} [B_1 - c] e^{\alpha\zeta(\mathrm{sgn}\,\dot{v})} d\zeta
\end{aligned} \tag{4}
$$

Substituting (3) into (1), we have:

$$
\begin{aligned}
\dot{x}_1 &= f_1(x_1) + g_1 x_2 + d_1(t) \\
&\vdots \\
\dot{x}_i &= f_i(\bar{x}_i) + g_i x_{i+1} + d_i(t), \quad i = 2, ..., n-1 \\
&\vdots \\
\dot{x}_n &= f_n(\bar{x}_n) + g_n cv(t) + g_n h(v) + d_n(t) \\
y &= x_1
\end{aligned} \tag{5}
$$

The control objective is to design adaptive control law $v(t)$ for system (5) such that the output y follows the specified desired trajectory y_d.

To facilitate the control design later in Section III, the following assumptions are needed.

Assumption 1: The signs of g_i are known, and there exist constants $g_{i\max} \geq g_{i\min} > 0$ such that $g_{i\min} \leq |g_i| \leq g_{i\max}$.

Assumption 2: The desired trajectory vectors are continuous and available, and $[y_d, \dot{y}_d, \ddot{y}_d]^T \in \Omega_d$ with known compact set $\Omega_d = \{[y_d, \dot{y}_d, \ddot{y}_d]^T : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \leq B_0\} \subset R^3$, whose size B_0 is a known positive constant.

Assumption 3: [4] There exist constants $c_{\min}$ and $c_{\max}$ such that the slope c in (2) satisfies $c \in [c_{\min}, c_{\max}]$.

Assumption 4: [4] There exist a constant $h_{\max}$ such that $h(v) \leq h_{\max}$.

Assumption 5: There exist constants $d_{i\max}$ such that $d_i(t) \leq d_{i\max}$.

Remark 1: Assumption 1 implies that unknown constants g_i are strictly either positive or negative. Without losing generality, we will only consider the case when $g_i > 0$. Assumptions 3 and 4 assume the slop range of a backlash hysteresis and the upper bound of the hysteresis loop, which are reasonable according to the analysis in [4]. In Assumption 5, the disturbances are also required to be bounded, which is practical in reality. It should be noted that all these bounds $g_{\max}$, $g_{\min}$, $c_{\min}$, $c_{\max}$, $h_{\max}$ and $d_{i\max}$ are not required in implementation proposed control design. They are used only for analytical purposes.

III. Control Design and Stability Analysis

In this section, we will combine the dynamic surface control with backstepping and adaptive control for the nth-order system described by (5). Similar to traditional backstepping, the design of adaptive dynamic surface control is based on the following change of coordinates: $z_1 = x_1 - y_d$, $z_i = x_i - \omega_i$, $i = 2, ..., n$, where ω_i is the output of a first order filter with α_{i-1} as the input, and α_{i-1} is an intermediate control which shall be developed for the corresponding $(i-1)$th subsystem. Finally, an overall control law v is constructed at step n. The major difference of dynamic surface control with traditional backstepping is to replace, at each step of recursion, the quantity $\dot{\alpha}_{i-1}$ by $\dot{\omega}_i$ in determining the virtual control $\dot{\alpha}_i$. As a result, the operation of differentiation can be replaced by simpler algebraic operation. Before proceeding with the adaptive control, some notations are presented below: $\bar{z}_i = [z_1, ..., z_i]^T$, $\bar{y}_j = [y_2, ..., y_j]^T$, $\bar{\hat{W}}_i = [\hat{W}_1^T, ..., \hat{W}_i^T]^T$, where $i = 1, ..., n$, $y_j = \omega_j - \alpha_{j-1}$, $j = 2, ..., n$.

Step 1: Since $z_1 = x_1 - y_d$, and its derivative is

$$
\dot{z}_1 = \dot{x}_1 - \dot{y}_d = f_1(x_1) + g_1 x_2 + d_1(t) - \dot{y}_d \tag{6}
$$

Consider the following Lyapunov function candidate:

$$
V_{z_1} = \frac{1}{2g_1} z_1^2 \tag{7}
$$

Its derivative along (6) is

$$\dot{V}_{z_1} = \frac{1}{g_1} z_1 \dot{z}_1 = z_1[Q_1(Z_1) + x_2 + \frac{1}{g_1} d_1(t)] \quad (8)$$

where $Q_1(Z_1) = g_1^{-1} f_1(x_1) - g_1^{-1} \dot{y}_d$ with $Z_1 = [x_1, \dot{y}_d] \in \Omega_{Z_1} \subset R^2$. To compensate for the unknown function $Q_1(Z_1)$, we can use the radial basis function neural network (RBFNN) in [11], $\hat{W}_1^T S(Z_1)$, with $\hat{W}_1 \in R^{l\times 1}$, $S(Z_1) \in R^{l\times 1}$, and the NN node number $l > 1$, to approximate the function $Q_1(Z_1)$ on the compact set Ω_{Z_1} as follows

$$Q_1(Z_1) = \hat{W}_1^T S(Z_1) - \tilde{W}_1^T S(Z_1) + \varepsilon_1(Z_1) \quad (9)$$

where the approximation error $\varepsilon_1(Z_1)$ satisfies $|\varepsilon_1(Z_1)| \leq \varepsilon_1^*$ with a positive constant ε_1^*. Substituting (9) into (8) and according to Assumptions 1 and 5, we obtain

$$\dot{V}_{z_1} \leq z_1[\hat{W}_1^T S(Z_1) - \tilde{W}_1^T S(Z_1) + x_2] + |z_1| D_1 \quad (10)$$

where $D_1 = \frac{d_{1max}}{g_{1\min}} + \varepsilon_1^*$. Since $x_2 = z_2 + y_2 + \alpha_1$, (10) becomes

$$\dot{V}_{z_1} \leq z_1[\hat{W}_1^T S(Z_1) - \tilde{W}_1^T S(Z_1) + z_2 + y_2 + \alpha_1] + |z_1| D_1 \quad (11)$$

Choose the following virtual control law and adaptation laws:

$$\alpha_1 = -k_1 z_1 - \hat{W}_1^T S(Z_1) - \tanh(\frac{z_1}{\epsilon})\hat{D}_1 \quad (12)$$

$$\dot{\hat{W}}_1 = \Gamma_1[z_1 S(Z_1) - \sigma_1 \hat{W}_1] \quad (13)$$

$$\dot{\hat{D}}_1 = \gamma_{d_1}[z_1 \tanh(\frac{z_1}{\epsilon}) - \sigma_{d_1}\hat{D}_1] \quad (14)$$

where $k_1 > 0$, $\epsilon > 0$, $\hat{D}_1$ is the estimate of D_1, $\Gamma_1 = \Gamma_1^T \in R^{l\times l} > 0$, $\sigma_1 > 0$, $\gamma_{d_1} > 0$ and $\sigma_{d_1} > 0$.

Substituting (12) into (11), and using the following property of the hyperbolic tangent function $\tanh(\cdot)$:

$$0 \leq |z_1| - z_1 \tanh(\frac{z_1}{\epsilon}) \leq 0.2785\epsilon \quad (15)$$

we obtain that

$$\begin{aligned}\dot{V}_{z_1} &\leq -k_1 z_1^2 + z_1 z_2 + z_1 y_2 - z_1 \tilde{W}_1^T S(Z_1) \\ &\quad -z_1 \tanh(\frac{z_1}{\epsilon})\hat{D}_1 + |z_1| D_1 \\ &\leq -k_1 z_1^2 + z_1 z_2 + z_1 y_2 - z_1 \tilde{W}_1^T S(Z_1) \\ &\quad -z_1 \tanh(\frac{z_1}{\epsilon})\tilde{D}_1 + |z_1| D_1 - z_1 \tanh(\frac{z_1}{\epsilon}) D_1 \\ &\leq -k_1 z_1^2 + z_1 z_2 + z_1 y_2 - z_1 \tilde{W}_1^T S(Z_1) \\ &\quad -z_1 \tanh(\frac{z_1}{\epsilon})\tilde{D}_1 + 0.2785\epsilon D_1 \end{aligned} \quad (16)$$

where $\tilde{D} = \hat{D} - D$. Using the Young's inequality, the following inequalities hold:

$$z_1 z_2 \leq z_1^2 + \frac{1}{4} z_2^2 \quad (17)$$

$$z_1 y_2 \leq z_1^2 + \frac{1}{4} y_2^2 \quad (18)$$

Substituting (17) and (18) into (16) leads to

$$\begin{aligned}\dot{V}_{z_1} &\leq -(k_1 - 2) z_1^2 + \frac{1}{4} z_2^2 + \frac{1}{4} y_2^2 - z_1 \tilde{W}_1^T S(Z_1) \\ &\quad -z_1 \tanh(\frac{z_1}{\epsilon})\tilde{D}_1 + 0.2785\epsilon D_1 \end{aligned} \quad (19)$$

Define the filtered virtual control ω_2 in the following manner:

$$\tau_2 \dot{\omega}_2 + \omega_2 = \alpha_1, \quad \omega_2(0) = \alpha_1(0), \quad (20)$$

where τ_2 is a design constant that we will choose later.

Due to $y_2 = \omega_2 - \alpha_1$, from (20), we have $\dot{\omega}_2 = -\frac{y_2}{\tau_2}$. Therefore, we have

$$\begin{aligned}\dot{y}_2 &= \dot{\omega}_2 - \dot{\alpha}_1 = -\frac{y_2}{\tau_2} + [k_1 \dot{z}_1 + \dot{\hat{W}}_1^T S(Z_1) + \hat{W}_1^T \dot{S}(Z_1) \\ &\quad + \tanh(\frac{z_1}{\epsilon})\dot{\hat{D}}_1 + (1 - \tanh^2(\frac{z_1}{\epsilon}))\dot{z}_1 \hat{D}_1] \end{aligned} \quad (21)$$

As such,

$$\left| \dot{y}_2 + \frac{y_2}{\tau_2} \right| \leq \zeta_2(\bar{z}_2, y_2, \hat{W}_1, \hat{D}_1, y_d, \dot{y}_d, \ddot{y}_d) \quad (22)$$

where $\zeta_2(\bar{z}_2, y_2, \hat{W}_1, \hat{D}_1, y_d, \dot{y}_d, \ddot{y}_d)$ is a continuous function.

From (21) and (22), using the Young's inequality, we obtain that

$$y_2 \dot{y}_2 \leq -\frac{y_2^2}{\tau_2} + |y_2|\zeta_2 \leq -\frac{y_2^2}{\tau_2} + y_2^2 + \frac{1}{4}\zeta_2^2 \quad (23)$$

Consider the following Lyapunov function candidate:

$$V_1 = V_{z_1} + \frac{1}{2}\tilde{W}_1^T \Gamma_1^{-1} \tilde{W}_1 + \frac{1}{2\gamma_{d_1}}\tilde{D}_1^2 + \frac{1}{2} y_2^2 \quad (24)$$

Its time derivative along (19) and (23) is

$$\begin{aligned}\dot{V}_1 &= \dot{V}_{z_1} + \tilde{W}_1^T \Gamma_1^{-1} \dot{\hat{W}}_1 + \frac{1}{\gamma_{d_1}} \tilde{D}\dot{\hat{D}} + y_2 \dot{y}_2 \\ &\leq -(k_1 - 2) z_1^2 + \frac{1}{4} z_2^2 - z_1 \tilde{W}_1^T S(Z_1) \\ &\quad -z_1 \tanh(\frac{z_1}{\epsilon})\tilde{D}_1 + 0.2785\epsilon D_1 + \tilde{W}_1^T \Gamma_1^{-1} \dot{\hat{W}}_1 \\ &\quad + \frac{1}{\gamma_{d_1}} \tilde{D}\dot{\hat{D}} - \frac{y_2^2}{\tau_2} + 1\frac{1}{4} y_2^2 + \frac{1}{4}\zeta_2^2 \end{aligned} \quad (25)$$

Substituting (13) and (14) into (25) results in

$$\begin{aligned}\dot{V}_1 &\leq -(k_1 - 2) z_1^2 + \frac{1}{4} z_2^2 - \sigma_1 \tilde{W}_1^T \hat{W}_1 - \sigma_{d_1} \tilde{D}_1 \hat{D}_1 \\ &\quad -\frac{y_2^2}{\tau_2} + 1\frac{1}{4} y_2^2 + \frac{1}{4}\zeta_2^2 + 0.2785\epsilon D_1 \end{aligned} \quad (26)$$

Step i ($2 \leq i < n$)**:** The time derivative of z_i is

$$\dot{z}_i = f_i(\bar{x}_i) + g_i x_{i+1} + d_i(t) - \dot{\omega}_i \quad (27)$$

Consider the following Lyapunov function candidate:

$$V_{z_i} = \frac{1}{2 g_i} z_i^2 \quad (28)$$

Its derivative along (27) is

$$\dot{V}_{z_i} = \frac{1}{g_i} z_i \dot{z}_i = z_i[Q_i(Z_i) + x_{i+1} + \frac{1}{g_i} d_i(t)] \quad (29)$$

where $Q_i(Z_i) = g_i^{-1} f_i(\bar{x}_i) - g_i^{-1}\dot{\omega}_i$ with $Z_i = [\bar{x}_i, \dot{\omega}_i] \in \Omega_{Z_i} \subset R^{i+1}$. To compensate for the unknown function $Q_i(Z_i)$, we can use the radial basis function neural network (RBFNN), $\hat{W}_i^T S(Z_i)$, with $\hat{W}_i \in R^{l\times 1}$, $S(Z_i) \in R^{l\times 1}$, and the NN node number $l > 1$, to approximate the function $Q_i(Z_i)$ on the compact set Ω_{Z_i} as follows

$$Q_i(Z_i) = \hat{W}_i^T S(Z_i) - \tilde{W}_i^T S(Z_i) + \varepsilon_i(Z_i) \tag{30}$$

where the approximation error $\varepsilon_i(Z_i)$ satisfies $|\varepsilon_i(Z_i)| \le \varepsilon_i^*$ with a positive constant ε_i^*. Substituting (30) into (29), we obtain

$$\dot{V}_{z_i} \le z_i[\hat{W}_i^T S(Z_i) - \tilde{W}_i^T S(Z_i) + x_{i+1}] + |z_i|D_i \tag{31}$$

where $D_i = \frac{d_{1\max}}{g_{1\min}} + \varepsilon_i^*$. Since $x_{i+1} = z_{i+1} + y_{i+1} + \alpha_i$, (31) becomes

$$\begin{aligned}\dot{V}_{z_i} \le\ & z_i[\hat{W}_i^T S(Z_i) - \tilde{W}_i^T S(Z_i) + z_{i+1} + y_{i+1} + \alpha_i] \\ & + |z_i|D_i \end{aligned}\tag{32}$$

Choose the following virtual control law and adaptation laws:

$$\alpha_i = -k_i z_i - \hat{W}_i^T S(Z_i) - \tanh(\frac{z_i}{\epsilon})\hat{D}_i \tag{33}$$

$$\dot{\hat{W}}_i = \Gamma_i[z_i S(Z_i) - \sigma_i \hat{W}_i] \tag{34}$$

$$\dot{\hat{D}}_i = \gamma_{d_i}[z_i \tanh(\frac{z_i}{\epsilon}) - \sigma_{d_i}\hat{D}_i] \tag{35}$$

where $k_i > 0$, $\epsilon > 0$, $\hat{D}_i$ is the estimate of D_i, $\Gamma_i = \Gamma_i^T \in R^{l\times l} > 0$, $\sigma_i > 0$, $\gamma_{d_i} > 0$ and $\sigma_{d_i} > 0$.

Substituting (33) into (32) and using the property of the hyperbolic tangent function as (15), we obtain

$$\begin{aligned}\dot{V}_{z_i} \le\ & -k_i z_i^2 + z_i z_{i+1} + z_i y_{i+1} - z_i \tilde{W}_i^T S(Z_i) \\ & - z_i \tanh(\frac{z_i}{\epsilon})\tilde{D}_i + 0.2785\epsilon D_i \end{aligned}\tag{36}$$

Using the Young's inequality, the following inequalities hold:

$$z_i z_{i+1} \le z_i^2 + \frac{1}{4} z_{i+1}^2 \tag{37}$$

$$z_i y_{i+1} \le z_i^2 + \frac{1}{4} y_{i+1}^2 \tag{38}$$

Substituting (37) and (38) into (36) leads to

$$\begin{aligned}\dot{V}_{z_i} \le\ & -(k_i - 2)z_i^2 + \frac{1}{4}z_{i+1}^2 + \frac{1}{4}y_{i+1}^2 - z_i\tilde{W}_i^T S(Z_i) \\ & - z_i \tanh(\frac{z_i}{\epsilon})\tilde{D}_i + 0.2785\epsilon D_i \end{aligned}\tag{39}$$

Define the filtered virtual control ω_{i+1} in the following manner:

$$\tau_{i+1}\dot{\omega}_{i+1} + \omega_{i+1} = \alpha_i, \quad \omega_{i+1}(0) = \alpha_i(0), \tag{40}$$

where τ_{i+1} is a design constant that we will choose later.

Due to $y_{i+1} = \omega_{i+1} - \alpha_i$, from (40), we have $\dot{\omega}_{i+1} = -\frac{y_{i+1}}{\tau_{i+1}}$. Therefore, we have

$$\begin{aligned}\dot{y}_{i+1} &= \dot{\omega}_{i+1} - \dot{\alpha}_i \\ &= -\frac{y_{i+1}}{\tau_{i+1}} + [k_i \dot{z}_i + \dot{\hat{W}}_i^T S(Z_i) + \hat{W}_i^T \dot{S}(Z_i) \\ &\quad + \tanh(\frac{z_i}{\epsilon})\dot{\hat{D}}_i + (1 - \tanh^2(\frac{z_i}{\epsilon}))\dot{z}_i \hat{D}_i] \end{aligned}\tag{41}$$

As such,

$$\left|\dot{y}_{i+1} + \frac{y_{i+1}}{\tau_{i+1}}\right| \le \zeta_{i+1}(\bar{z}_{i+1}, \bar{y}_{i+1}, \bar{\hat{W}}_i, \bar{\hat{D}}_i, y_d, \dot{y}_d, \ddot{y}_d) \tag{42}$$

where $\zeta_{i+1}(\bar{z}_{i+1}, \bar{y}_{i+1}, \bar{\hat{W}}_i, \bar{\hat{D}}_i, y_d, \dot{y}_d, \ddot{y}_d)$ is a continuous function.

From (41) and (42), using the Young's inequality, we obtain that

$$\begin{aligned} y_{i+1}\dot{y}_{i+1} \le\ & -\frac{y_{i+1}^2}{\tau_{i+1}} + |y_{i+1}|\zeta_{i+1} \le -\frac{y_{i+1}^2}{\tau_{i+1}} + y_{i+1}^2 \\ & + \frac{1}{4}\zeta_{i+1}^2 \end{aligned}\tag{43}$$

Consider the following Lyapunov function candidate:

$$V_i = V_{z_i} + \frac{1}{2}\tilde{W}_i^T \Gamma_i^{-1}\tilde{W}_i + \frac{1}{2\gamma_{d_i}}\tilde{D}_i^2 + \frac{1}{2}y_{i+1}^2 \tag{44}$$

Its time derivative along (39) and (43) is

$$\begin{aligned}\dot{V}_i =\ & \dot{V}_{z_i} + \tilde{W}_i^T \Gamma_i^{-1}\dot{\hat{W}}_i + \frac{1}{\gamma_{d_i}}\tilde{D}\dot{\hat{D}}_i + y_{i+1}\dot{y}_{i+1} \\ \le\ & -(k_i - 2)z_i^2 + \frac{1}{4}z_{i+1}^2 - z_i\tilde{W}_i^T S(Z_i) + \tilde{W}_i^T\Gamma_i^{-1}\dot{\hat{W}}_i \\ & + \frac{1}{\gamma_{d_i}}\tilde{D}\dot{\hat{D}}_i - \frac{y_{i+1}^2}{\tau_{i+1}} + 1\frac{1}{4}y_{i+1}^2 + \frac{1}{4}\zeta_{i+1}^2 \end{aligned}\tag{45}$$

Substituting (34) and (35) into (45) results in

$$\begin{aligned}\dot{V}_i \le\ & -(k_i - 2)z_i^2 + \frac{1}{4}z_{i+1}^2 - \sigma_i\tilde{W}_i^T\hat{W}_i - \sigma_{d_i}\tilde{D}_i\hat{D}_i \\ & - \frac{y_{i+1}^2}{\tau_{i+1}} + 1\frac{1}{4}y_{i+1}^2 + \frac{1}{4}\zeta_{i+1}^2 + 0.2785\epsilon D_i \end{aligned}\tag{46}$$

Step n: In this final step, we will design the control input $v(t)$. Since $z_n = x_n - \omega_n$, the time derivative of z_n is

$$\dot{z}_n = f_n(\bar{x}_n) + g_n c v(t) + g_n h(v) + d_n(t) - \dot{\omega}_n \tag{47}$$

Consider the following Lyapunov function candidate:

$$V_{z_n} = \frac{1}{2g_n c}z_n^2 \tag{48}$$

Its derivative along (47) is

$$\begin{aligned}\dot{V}_{z_n} =\ & \frac{1}{g_n c}z_n\dot{z}_n = z_n[Q_i(Z_n) + v(t) + \frac{h(v)}{c} \\ & + \frac{1}{g_n c}d_n(t)] \end{aligned}\tag{49}$$

where $Q_n(Z_n) = (g_n c)^{-1} f_n(\bar{x}_n) - (g_n c)^{-1}\dot{\omega}_n$ with $Z_n = [\bar{x}_n, \dot{\omega}_n] \in \Omega_{Z_n i} \subset R^{n+1}$. To compensate for the unknown function $Q_n(Z_n)$, we can use the radial basis function neural network (RBFNN), $\hat{W}_n^T S(Z_n)$, with $\hat{W}_n \in R^{l\times 1}$, $S(Z_n) \in R^{l\times 1}$, and the NN node number $l > 1$, to approximate the function $Q_n(Z_n)$ on the compact set Ω_{Z_n} as follows

$$Q_n(Z_n) = \hat{W}_n^T S(Z_n) - \tilde{W}_n^T S(Z_n) + \varepsilon_n(Z_n) \tag{50}$$

where the approximation error $\varepsilon_n(Z_n)$ satisfies $|\varepsilon_n(Z_n)| \le \varepsilon_n^*$ with a positive constant ε_n^*. Substituting (50) into (53)and according to Assumptions 1, 3-5, we obtain that

$$\dot{V}_{z_n} \le z_n[\hat{W}_n^T S(Z_n) - \tilde{W}_n^T S(Z_n) + v(t)] + |z_n|D_n \tag{51}$$

where $D_n = \frac{h_{\max}}{c_{\min}} + \frac{d_{n\max}}{g_{n\min}c_{\min}} + \varepsilon_n^*$. Choose the following control law:

$$v(t) = -k_n z_n - \hat{W}_n^T S(Z_n) - \tanh(\frac{z_n}{\epsilon})\hat{D}_n \tag{52}$$

where $k_n > 0$, $\epsilon > 0$, $\hat{D}_n$ is the estimate of D_n. Substituting (52) into (51), and using the property of the hyperbolic tangent function as (15), we obtain that

$$\begin{aligned}\dot{V}_{z_n} &\le -k_n z_n^2 - z_n\tilde{W}_n^T S(Z_n) - z_n \tanh(\frac{z_n}{\epsilon})\tilde{D}_n \\ &\quad +0.2785\epsilon D_n\end{aligned} \tag{53}$$

where $\tilde{D}_n = \hat{D}_n - D_n$.

Consider the following Lyapunov function candidate:

$$V_n = V_{z_n} + \frac{1}{2}\tilde{W}_n^T\Gamma_n^{-1}\tilde{W}_n + \frac{1}{2\gamma_{d_n}}\tilde{D}_n^2 \tag{54}$$

where $\Gamma_n = \Gamma_n^T \in R^{l\times l} > 0$, $\gamma_{d_n} > 0$. Its time derivative along (53) is

$$\begin{aligned}\dot{V}_n &= \dot{V}_{z_n} + \tilde{W}_n^T\Gamma_n^{-1}\dot{\hat{W}}_n + \frac{1}{\gamma_{d_n}}\tilde{D}_n\dot{\hat{D}}_n \\ &\le -k_n z_n^2 - z_n\tilde{W}_n^T S(Z_n) - z_n\tanh(\frac{z_n}{\epsilon})\tilde{D}_n \\ &\quad +0.2785\epsilon D_n + \tilde{W}_n^T\Gamma_n^{-1}\dot{\hat{W}}_n + \frac{1}{\gamma_{d_n}}\tilde{D}_n\dot{\hat{D}}_n\end{aligned} \tag{55}$$

Choose the following adaptation laws:

$$\dot{\hat{W}}_n = \Gamma_n[z_n S(Z_n) - \sigma_n\hat{W}_n] \tag{56}$$

$$\dot{\hat{D}}_n = \gamma_{d_n}[z_n\tanh(\frac{z_n}{\epsilon}) - \sigma_{d_n}\hat{D}_n] \tag{57}$$

where $\sigma_n > 0$ and $\sigma_{d_n} > 0$. Substituting (56) and (57) into (55) results in

$$\dot{V}_n \le -k_n z_n^2 - \sigma_n\tilde{W}_n^T\hat{W}_n - \sigma_{d_n}\tilde{D}_n\hat{D}_n + 0.2785\epsilon D_n \tag{58}$$

The following theorem shows the stability and control performance of the closed-loop adaptive system.

Theorem 1: Consider the closed-loop system consisting of the plant (5) under Assumptions 1-5, the controller (52), and adaption laws (34)(35). For bounded initial conditions, there exist constants $p > 0$, $k_i > 0$, $\tau_i > 0$, $\lambda_{\max}(\Gamma_i^{-1})$, $\sigma_i > 0$, $\gamma_{d_i}$and $\sigma_{d_i} > 0$, satisfying $V = \sum_{i=1}^n V_i \le p$, such that the overall closed-loop control system is semi-globally stable in the sense that all of the signals in the closed-loop system are bounded, and the tracking error is smaller than a prescribed error bound.

Proof: Consider the Lyapunov function candidate $V = \sum_{i=1}^n V_i$. Its derivative with respect to time is:

$$\dot{V} = \sum_{i=1}^n \dot{V}_i \tag{59}$$

Substitute (26)(46) and (58) into (59), it follows that

$$\begin{aligned}\dot{V} &\le -(k_1-2)z_1^2 - \sum_{i=2}^{n-1}(k_i - 2\frac{1}{4})z_i^2 - (k_n - \frac{1}{4})z_n^2 \\ &\quad -\sum_{i=1}^n \sigma_i\tilde{W}_i^T\hat{W}_i - \sum_{i=1}^n \sigma_{d_i}\tilde{D}_i\hat{D}_i + \sum_{i=1}^{n-1}\Big[-\frac{y_{i+1}^2}{\tau_{i+1}} \\ &\quad +1\frac{1}{4}y_{i+1}^2 + \frac{1}{4}\zeta_{i+1}^2\Big] + \sum_{i=1}^n 0.2785\epsilon D_i\end{aligned} \tag{60}$$

Since for any $B_0 > 0$ and $p > 0$, the sets $\Omega_d = \{(y_d, \dot{y}_d, \ddot{y}_d) : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \le B_0\}$ and $\Omega_i = \{[\bar{z}_i^T, \bar{y}_i^T, \bar{\hat{W}}_i^T]^T : \sum_{j=1}^i V_j \le p\}$, $i = 1, ..., n$ are compact in R^3 and $R^{2i-1+\sum_{j=1}^i l_j}$, respectively. Therefore, ζ_{i+1} has a maximum M_{i+1} on $\Omega_d \times \Omega_i$.

By completion of squares, the following inequalities hold:

$$-\sigma_i\tilde{W}_i^T\hat{W}_i \le -\frac{\sigma_i\|\tilde{W}_i\|^2}{2} + \frac{\sigma_i\|W_i^*\|^2}{2} \tag{61}$$

$$-\sigma_{d_i}\tilde{D}_i\hat{D}_i \le -\frac{\sigma_{d_i}\tilde{D}_i^2}{2} + \frac{\sigma_{d_i}D_i^2}{2} \tag{62}$$

Substituting (61) and (62) into (60) leads to

$$\begin{aligned}\dot{V} &\le -(k_1-2)z_1^2 - \sum_{i=2}^{n-1}(k_i - 2\frac{1}{4})z_i^2 - (k_n - \frac{1}{4})z_n^2 \\ &\quad -\sum_{i=1}^n \frac{\sigma_i\|\tilde{W}_i\|^2}{2} - \sum_{i=1}^n \frac{\sigma_{d_i}\tilde{D}_i^2}{2} + \sum_{i=1}^{n-1}\Big[-\frac{y_{i+1}^2}{\tau_{i+1}} \\ &\quad +1\frac{1}{4}y_{i+1}^2\Big] + \mu\end{aligned} \tag{63}$$

where

$$\begin{aligned}\mu &= \sum_{i=1}^n \frac{\sigma_i\|W_i^*\|^2}{2} + \sum_{i=1}^n \frac{\sigma_{d_i}D_i^2}{2} + \frac{1}{4}\sum_{i=1}^{n-1}M_{i+1}^2 \\ &\quad +\sum_{i=1}^n 0.2785\epsilon D_i\end{aligned} \tag{64}$$

Choosing

$$\begin{aligned}\alpha_0 &\le \min\{\sigma_{d_i}\gamma_{d_i}, \frac{\sigma_i}{\lambda_{\max}(\Gamma_i^{-1})}\}, \quad i = 1, ..., n \\ k_1 &\ge 2 + \frac{\alpha_0}{2g_{1\min}} \\ k_i &\ge 2\frac{1}{4} + \frac{\alpha_0}{2g_{i\min}}, \quad i = 2, ..., n-1 \\ k_n &\ge \frac{1}{4} + \frac{\alpha_0}{2g_{i\min}c_{\min}} \\ \frac{1}{\tau_i} &\ge 1\frac{1}{4} + \frac{\alpha_0}{2}, \quad i = 2, ..., n\end{aligned} \tag{65}$$

and substituting them into (60), we obtain that

$$\dot{V} \le -\alpha_0 V + \mu \tag{66}$$

If $V = p$ and $\alpha_0 > \frac{\mu}{p}$, then $\dot{V} \le 0$. If implies that $V(t) \le p$, $\forall t \ge 0$ for $V(0) \le p$. Multiplying (66) by $e^{\alpha_0 t}$ and integrating over $[0, t]$ yields

$$0 \le V(t) \le \frac{\mu}{\alpha_0} + \Big[V(0) - \frac{\mu}{\alpha_0}\Big]e^{-\alpha_0 t} \tag{67}$$

Therefore, all signals of the closed-loop system, i.e., z_i, y_i and $\hat{W}_i$ are uniformly ultimately bounded. Furthermore, x_i, α_i and Ω_i are also uniformly ultimately bounded. From (64) and (65), we know that for any given constants B_0, p, σ_{d_i} and σ_i, we can decrease $\lambda_{\max}(\Gamma_i^{-1})$ to make $\frac{\mu}{\alpha_0}$ arbitrarily small. Thus, the tracking error z_1 becomes arbitrarily small. This completes the proof. ■

IV. Simulation Studies

To demonstrate the effectiveness of the proposed approach, we consider the plant used in [4] and [5]:

$$\begin{aligned} \dot{x} &= a\frac{1-e^{-x(t)}}{1+e^{-x(t)}} + bu(v) \\ y &= x \end{aligned} \tag{68}$$

where $a = 1$, $b = 1$, and $u(v)$ represents an output of the following backlash-like hysteresis:

$$\frac{du}{dt} = \alpha\left|\frac{dv}{dt}\right|(cv - u) + B_1\frac{dv}{dt} \tag{69}$$

with $\alpha = 1$, $c = 3.1635$, and $B_1 = 0.345$. As discussed in [4], without control, i.e., $u(v) = 0$, (68) is unstable, since $\dot{x} = a\frac{1-e^{-x(t)}}{1+e^{-x(t)}} > 0$ for $x > 0$, and $\dot{x} = a\frac{1-e^{-x(t)}}{1+e^{-x(t)}} < 0$ for $x < 0$. The objective is to control the system output y to follow a desired trajectory $y_d = 12.5\sin(2.3t)$.

We adopt the control law and adaption laws in (52) (56) (57). The following initial conditions and control design parameters are chosen as: $x(0) = u(0) = v(0) = 0.0$, $\hat{W}(0) = \hat{D}(0) = 0.0$, $k_1 = 0.3$, $\Gamma = 0.1I_{25}$, $\sigma = 0.1$, $\gamma_d = 0.1$,$\sigma_d = 0.1$, $\epsilon = 0.05$.

The simulation results are shown in Figs. 2 and 3. From Fig. 2, we observe that good tracking performance is achieved and the tracking error converges to a small neighborhood of zero. At the same time, the control signal v and hysteresis output u are kept bounded, as seen in Figs. 3. It is noted that there is a large difference between the signals v and u in Fig. 3, which indicates the significant hysteresis effect.

V. Conclusion

Adaptive dynamic surface control (DSC) using neural networks has been proposed for a class of nonlinear systems in strict-feedback form with back-lash hysteresis input, where the hysteresis is modeled as a differential equation. The developed adaptive control can guarantee that all signals involved are semi-globally uniformly ultimately bounded (SGUUB) without constructing a hysteresis inverse. Simulation results have been provided to show the effectiveness of the proposed approach.

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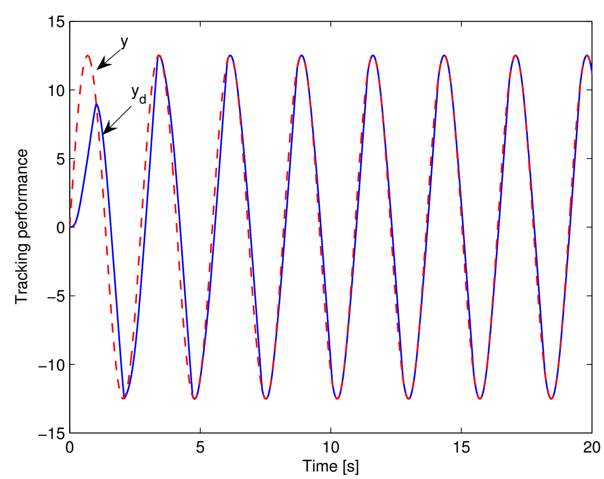

Fig. 2. Tracking performance

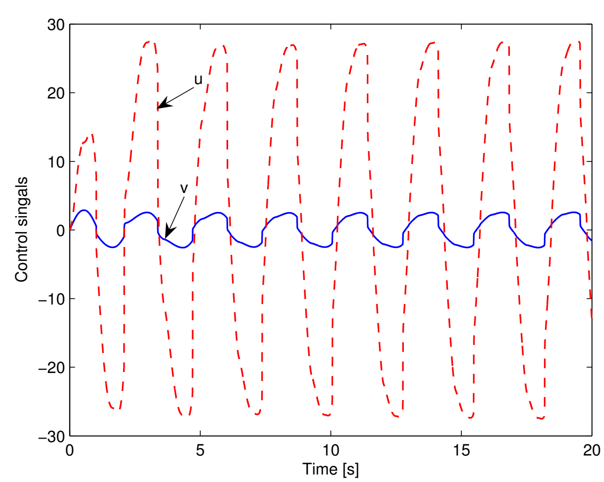

Fig. 3. Control inputs