MATH 3342-004: EXAM 1 INFO/LOGISTICS/ADVICE

- **INFO:**

WHEN:	Friday (02/12) at 10:00am	DURATION:	50 mins
PROBLEM COUNT:	Appropriate for a 50-min exam	BONUS COUNT:	At least one

  - TOPICS CANDIDATE FOR THE EXAM:
    - DEVORE 1.1: The Need for Probability & Statistics; Data, Variables, Samples, Populations
    - DEVORE 1.2: Frequency Tables, Histograms; Modality & Skewness of Data, Outliers
    - DEVORE 1.3: Measures of Center & Rank: Mean, Median, Trimmed Mean, Percentiles, Quartiles, Hinges
    - DEVORE 1.4: Measures of Spread: Range, Variance, Std Dev, IQR, IHR; Boxplots
    - DEVORE 2.1: Sets, Sequences; Outcomes, Sample Spaces, Events; Unions, Intersections, Complements
    - DEVORE 2.2: Probability Axioms & Properties; Measures of Countable Sets; Equally Likely Outcomes
    - DEVORE 2.4: Conditional Probability, Law of Total Probability, Bayes' Theorem
    - DEVORE 2.5: Independence of Two Events
  - TOPICS CANDIDATE FOR BONUS QUESTIONS: (**Maximum Bonus Points Possible = 12**)
    - DEVORE 2.3: Counting Tuples in Multi-Stage Experiments; Permutations, Combinations (entire section)
    - ?????
  - TOPICS NOT COVERED AT ALL:
    - **Proofs of any kind**
    - DEVORE 1.1: Enumerative vs. Analytic Studies (pg 9-10), Collecting Data (pg 10)
    - DEVORE 1.2: Dotplots (pg 13-15), Stem-and-Leaf Displays (pg 15-16)
    - DEVORE 1.3: Mean of a **finite population**: $\mu = \frac{1}{N}\sum_{k=1}^{N} x_k$
    - DEVORE 1.4: Deviations from Mean, Classification of Outliers, Std Error, Variance of a **finite population**
    - DEVORE 2.2: Measures of 1-D, 2-D, and 3-D sets [Slide #17 in 2.2 Slides]
    - DEVORE 2.5: Mutual Independence of More than Two Events (pg 88)
    - **Any applications mentioned in other sections**

- **LOGISTICS:**
  - **All you need to bring are pencil(s), eraser(s), calculator(s) & your Raidercard**.
  - Clear your desk of everything except pencil(s), eraser(s) and calculator(s).
  - **Formula Sheet (next two pages) will be provided**.
  - **Books, notes, notecards NOT PERMITTED.**
  - Mobile devices (phones, tablets, laptops, music, headphones, ...) are to be shut off and put away.
  - Tissues will be furnished – for allergies, not for sobbing. No talking or cheating!
  - **When you turn in your exam, be prepared to show me your Raidercard if I don't recognize you**.
  - **If you ask to use the restroom during the exam, either hold it or turn in your exam for grading**.

- **ADVICE:**
  - **Use the restroom before the exam, if needed**.
  - **Do not be late to the exam**.
  - Review the slides, past homework, and perhaps even work some similar problems in the textbook.
  - **Know how the use all formulas on the provided Formula Sheet (next two pages)**
  - Use flashcards to aid in memorization of hard formulas.
  - Study for the exam together in groups.
  - **If you need more review, show up to the last-minute help session Thursday evening (02/11)**.
  - **SHOW APPROPRIATE WORK! Attempt bonus question(s)**.

MATH 3342: EXAM 1 FORMULA SHEET

DEVORE 1.3

- SORTED SAMPLES:

  Given a sample with n data points $\quad x: \; x_1, x_2, \ldots, x_{n-1}, x_n$

  Then the corresponding **sorted sample** is $\quad x: \; x_{(1)}, x_{(2)}, \ldots, x_{(n-1)}, x_{(n)}$

  where the data points are sorted in ascending order: $\quad x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n-1)} \leq x_{(n)}$

- ROUNDING (COMPACT NOTATION): It is convenient to have mathematical notation for **rounding numbers**.

$$\text{Always Round Up:} \quad \lceil 3 \rceil = 3 \quad \lceil 3.1 \rceil = 4 \quad \lceil 3.5 \rceil = 4 \quad \lceil 3.9 \rceil = 4$$

- MEAN OF A SAMPLE: The **mean**, denoted $\overline{x}$, is the average of the sample.

$$\overline{x} := \frac{1}{n} \sum_{k=1}^{n} x_k = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

- MEDIAN OF A SAMPLE: The **median**, denoted $\widetilde{x}$, is the middle value of the sorted sample.

$$\widetilde{x} := \begin{cases} x_{([n+1]/2)} & \text{, if } n \text{ is odd} \\ \dfrac{x_{(n/2)} + x_{(1+[n/2])}}{2} & \text{, if } n \text{ is even} \end{cases}$$

- TRIMMED MEAN OF A SAMPLE: The $p\%$ **trimmed mean**, $\overline{x}_{tr(p\%)}$, is the mean of the dataset resulting from eliminating the smallest $p\%$ and largest $p\%$ of the sorted sample.

  e.g. $\overline{x}_{tr(10\%)} :=$ Mean of sorted sample x with largest 10% & smallest 10% removed

- PERCENTILES OF A SAMPLE: The **p-th percentile**, denoted $x_{p/100}$, is the smallest data point such that $p\%$ of the sample is less than or equal to that data point:

$$x_{p/100} := x_{(\lceil np/100 \rceil)} = \left(\left\lceil \frac{np}{100} \right\rceil \right)\text{-th data point in sorted sample}$$

$$\text{e.g.} \quad x_{0.37} \equiv (37^{th} \text{ percentile of sample } x)$$

$$\text{e.g.} \quad y_{0.98} \equiv (98^{th} \text{ percentile of sample } y)$$

- QUARTILES OF A SAMPLE:

$$x_{Q1} := x_{0.25} \equiv 1^{st} \textbf{ quartile } (25^{th} \text{ percentile) of sample } x$$

$$x_{Q3} := x_{0.75} \equiv 3^{rd} \textbf{ quartile } (75^{th} \text{ percentile) of sample } x$$

- HINGES OF A SAMPLE:

  The **lower hinge**, x_{LH}, is the median of the lower half of sorted sample.

  The **upper hinge**, x_{UH}, is the median of the upper half of sorted sample.

DEVORE 1.4

- RANGE OF A SAMPLE: Sample **range**, denoted x_R, is the **difference** between largest & smallest data points:

$$x_R := x_{(n)} - x_{(1)}$$

- VARIANCE OF A SAMPLE: Sample **variance**, denoted s^2 or s_x^2, is the following:

$$s^2 = \frac{S_{xx}}{n-1} \qquad \text{where} \qquad S_{xx} = \sum_{k=1}^{n} x_k^2 - \frac{1}{n} \left(\sum_{k=1}^{n} x_k \right)^2$$

- STANDARD DEVIATION OF A SAMPLE: **Standard deviation**, denoted s or s_x, is the square root of variance:

$$s := \sqrt{s^2}$$

- INTERQUARTILE RANGE (IQR): $\quad x_{IQR} := x_{Q3} - x_{Q1}$
- INTERHINGE RANGE (IHR): $\quad x_{IHR} := x_{UH} - x_{LH}$

DEVORE 2.1

- <u>BASIC TERMINOLOGY:</u> The **outcomes** $\omega_1, \omega_2, \ldots$ of an experiment are the different possible results.

  The **sample space** Ω of an experiment is the set of all possible outcomes.

  An **event** E is a subset of the sample space: $E \subseteq \Omega$

- <u>THE EMPTY SET:</u> The **empty set**, $\emptyset$, is the event with no outcomes in it.
- <u>UNIONS, INTERSECTIONS, COMPLEMENTS OF EVENTS (PROPERTIES):</u> Let $A, B, C \subseteq \Omega$. Then:

$$(A^c)^c = A, \quad A \cup B = B \cup A, \quad A \cap B = B \cap A$$
$$(A \cup B) \cup C = A \cup (B \cup C), \quad (A \cap B) \cap C = A \cap (B \cap C)$$
$$(A \cup B)^c = A^c \cap B^c, \quad (A \cap B)^c = A^c \cup B^c$$
$$A \cup \emptyset = A, \quad A \cap \emptyset = \emptyset, \quad \Omega^c = \emptyset, \quad \emptyset^c = \Omega$$
$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C), \quad (A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$

- <u>MUTUAL EXCLUSIVITY OF TWO EVENTS:</u> Events E, F are **mutually exclusive** (or **disjoint**) if $E \cap F = \emptyset$
- <u>MUTUAL EXCLUSIVITY OF THREE EVENTS:</u> Events E, F, G are **mutually exclusive** (or **<u>pairwise</u> disjoint**) if:

$$E \cap F = \emptyset \text{ and } E \cap G = \emptyset \text{ and } F \cap G = \emptyset$$

- <u>MUTUAL EXCLUSIVITY OF MANY EVENTS:</u> Events $E_1, E_2, \ldots, E_n$ are **mutually exclusive** (or **<u>pairwise</u> disjoint**) if they have no outcomes in common: $E_i \cap E_j = \emptyset$ for $i \neq j$

DEVORE 2.2

- <u>CHAIN OF UNIONS:</u> $\bigcup_{k=1}^{n} A_k = A_1 \cup A_2 \cup \cdots \cup A_{n-1} \cup A_n \qquad \bigcup_{k=1}^{\infty} A_k = A_1 \cup A_2 \cup A_3 \cup \cdots$
- <u>AXIOMS & PROPERTIES OF PROBABILITY:</u> Let $E, F \subseteq \Omega$ be two events from sample space Ω of an experiment.

  Let $E_1, E_2, E_3, \ldots$ be an infinite collection of <u>pairwise disjoint</u> events. Then:

$$\mathbb{P}(E) \geq 0, \quad \mathbb{P}(\Omega) = 1, \quad \mathbb{P}(\emptyset) = 0, \quad \mathbb{P}\left(\bigcup_{k=1}^{\infty} E_k\right) = \sum_{k=1}^{\infty} \mathbb{P}(E_k), \quad \mathbb{P}\left(\bigcup_{k=1}^{n} E_k\right) = \sum_{k=1}^{n} \mathbb{P}(E_k)$$
$$\mathbb{P}(E^c) = 1 - \mathbb{P}(E), \quad \mathbb{P}(E) \leq 1, \quad \mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F) - \mathbb{P}(E \cap F)$$

- <u>INCLUSION-EXCLUSION:</u> $\mathbb{P}(E \cup F \cup G) = \mathbb{P}(E) + \mathbb{P}(F) + \mathbb{P}(G) - \mathbb{P}(E \cap F) - \mathbb{P}(E \cap G) - \mathbb{P}(F \cap G) + \mathbb{P}(E \cap F \cap G)$
- <u>MEASURE OF A SET:</u> The **measure** of a **countable set** is $|E| := (\#\text{ of elements in } E)$, $\quad |\emptyset| := 0$
- <u>PROBABILITY WITH EQUALLY LIKELY OUTCOMES:</u>

  Let Ω be the sample space of an experiment with **equally likely outcomes**. Let E be an event of the experiment.

  Then the **probability** of event E occurring is defined as: $\mathbb{P}(E) = \dfrac{|E|}{|\Omega|}$

DEVORE 2.4

- <u>CONDITIONAL PROBABILITY:</u> Let $E, F \subseteq \Omega$ be two events such that $\mathbb{P}(F) > 0$.

  Then the **conditional probability** of E **given that** F **has occurred** is: $\mathbb{P}(E|F) := \dfrac{\mathbb{P}(E \cap F)}{\mathbb{P}(F)}$

- <u>PARTITIONS:</u> Events $E_1, \ldots, E_k$ **partition** sample space Ω if they are <u>pairwise disjoint</u> AND $\bigcup_{i=1}^{k} E_i = \Omega$
- <u>LAW OF TOTAL PROBABILITY:</u> Let $E_1, \ldots, E_k$ <u>partition</u> Ω. Then $\mathbb{P}(F) = \sum_{i=1}^{k} \mathbb{P}(F \cap E_i) = \sum_{i=1}^{k} \mathbb{P}(F|E_i) \cdot \mathbb{P}(E_i)$
- <u>BAYES' THEOREM:</u> Let $E_1, \ldots, E_k$ <u>partition</u> Ω. Then $\mathbb{P}(E_j|F) = \dfrac{\mathbb{P}(F \cap E_j)}{\sum_{i=1}^{k} \mathbb{P}(F \cap E_i)} = \dfrac{\mathbb{P}(F|E_j) \cdot \mathbb{P}(E_j)}{\sum_{i=1}^{k} \mathbb{P}(F|E_i) \cdot \mathbb{P}(E_i)}$ for $j = 1, \ldots, k$

DEVORE 2.5

- <u>INDEPENDENCE OF TWO EVENTS:</u> Two events E, F are **independent** if $\mathbb{P}(E|F) = \mathbb{P}(E)$
- <u>INDEPENDENCE OF TWO EVENTS:</u> Two events E, F are **independent** if and only if $\mathbb{P}(E \cap F) = \mathbb{P}(E) \cdot \mathbb{P}(F)$