MATH 3342-004: EXAM 2 INFO/LOGISTICS/ADVICE

- **INFO:**

WHEN:	Friday (03/11) at 10:00am	DURATION:	50 mins
PROBLEM COUNT:	Appropriate for a 50-min exam	BONUS COUNT:	At least one

  - TOPICS CANDIDATE FOR THE EXAM:
    - DEVORE 3.1: Random Variables (rv's): Definitions, Examples
    - DEVORE 3.2: Discrete rv's: pmf's, cdf's, Parameters, Probability
    - DEVORE 3.3: Discrete rv's: Expected Value, Variance, Std Dev
    - DEVORE 3.4: Discrete rv's: Bernoulli rv's, Binomial rv's
    - DEVORE 3.6: Discrete rv's: Poisson rv's
    - DEVORE 4.1: Continuous rv's: pdf's, Parameters, Probability
    - DEVORE 4.2: Continuous rv's: cdf's, Expected Value, Variance, Std Dev, Median, Percentiles
    - DEVORE 4.3: Continuous rv's: Uniform rv's, Normal rv's, Standard Normal Distribution
    - DEVORE 4.4: Continuous rv's: Exponential rv's, Gamma rv's
  - TOPICS CANDIDATE FOR BONUS QUESTIONS: (**Maximum Bonus Points Possible = 12**)
    - ?????
  - TOPICS NOT COVERED AT ALL:
    - **Proofs of any kind**, **Sketching Graphs/Histograms/Boxplots of any kind**
    - **The sections to ignore or skip mentioned at the end of the following slides: 3.4, 3.6, 4.3, 4.4**
    - DEVORE 3.5: Discrete rv's: Hypergeometric, Geometric & Negative Binomial rv's (entire section)
    - DEVORE 4.5: Continuous rv's: Lognormal, Weibull & Beta rv's (entire section)
    - DEVORE 4.6: Probability Plots (entire section)

- **LOGISTICS:**
  - **All you need to bring are pencil(s), eraser(s), calculator(s) & your Raidercard**.
  - Clear your desk of everything except pencil(s), eraser(s) and calculator(s).
  - **Formula Sheet (next two pages) will be provided**.
  - **Books, notes, notecards NOT PERMITTED.**
  - Mobile devices (phones, tablets, laptops, music, headphones, ...) are to be shut off and put away.
  - Tissues will be furnished – for allergies, not for sobbing. No talking or cheating!
  - **When you turn in your exam, be prepared to show me your Raidercard if I don't recognize you**.
  - **If you ask to use the restroom during the exam, either hold it or turn in your exam for grading**.

- **ADVICE:**
  - **Use the restroom before the exam, if needed**.
  - **Do not be late to the exam**.
  - Review the slides, past homework, and perhaps even work some similar problems in the textbook.
  - **Know how the use all formulas on the provided Formula Sheet (next four pages)**
  - Use flashcards to aid in memorization of hard formulas.
  - Study for the exam together in groups.
  - **If you need more review, show up to the last-minute help session Thursday evening (03/10)**.
  - **SHOW APPROPRIATE WORK! Attempt bonus question(s)**.

MATH 3342: EXAM 2 FORMULA SHEET

DEVORE 3.2 (The symbol $\forall$ translates to "for all" or "for every" or "for each")

- **PROBABILITY MASS FUNCTION (PMF):** Let X be a **discrete** random variable.
  Then, its **pmf**, denoted as $p_X(k)$, is defined as follows: $p_X(k) := \mathbb{P}(X = k) \quad \forall k \in \text{Supp}(X)$
- **CUMULATIVE DENSITY FUNCTION (CDF):** Let X be a **discrete** random variable s.t. $\text{Supp}(X) = \{k_1, k_2, k_3, \cdots\}$
  Then, its **cdf**, denoted as $F_X(x)$, is defined as follows: $F_X(x) := \mathbb{P}(X \leq x) = \sum_{k_i \leq x} p_X(k_i) \quad \forall x \in \mathbb{R}$

DEVORE 3.3

- **EXPECTED VALUE (MEAN) OF A DISCRETE R.V.:** $\mu_X = \mathbb{E}[X] := \sum_{k \in \text{Supp}(X)} k \cdot p_X(k)$
- **EXPECTED VALUE OF A FUNCTION OF DISCRETE R.V.:** $\mu_{h(X)} = \mathbb{E}[h(X)] := \sum_{k \in \text{Supp}(X)} h(k) \cdot p_X(k)$
- **VARIANCE OF A DISCRETE R.V.:** $\sigma_X^2 = \mathbb{V}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$
- **STANDARD DEVIATION OF A DISCRETE R.V.:** $\sigma_X := \sqrt{\mathbb{V}[X]}$

DEVORE 3.4

- **BERNOULLI RANDOM VARIABLES (SUMMARY):**

Notation	$X \sim \text{Bernoulli}(p), \quad 0 < p < 1, \quad q := 1 - p$	
Parameter(s)	$p \equiv \mathbb{P}(\text{Bernoulli Trial is a Success})$	
Support	$\text{Supp}(X) = \{0, 1\}$	
pmf	$p_X(k; p) := p^k q^{1-k} = p^k (1-p)^{1-k}$	
Mean, Variance	$\mathbb{E}[X] = p$	$\mathbb{V}[X] = pq = p(1-p)$
Model(s)	Result of One Bernoulli Trial: $1 \equiv \text{Success}, 0 \equiv \text{Failure}$	
Assumption(s)	Random process has its sample space partitioned into Successes and Failures	

- **BINOMIAL RANDOM VARIABLES (SUMMARY):**

Notation	$X \sim \text{Binomial}(n, p), \quad n \geq 1, \quad 0 < p < 1, \quad q := 1 - p$	
Parameter(s)	(Same parameters as Bernoulli(p))	
Support	$\text{Supp}(X) = \{0, 1, 2, \cdots, n-1, n\}$	
pmf	$p_X(k; n, p) := \binom{n}{k} p^k q^{n-k} = \binom{n}{k} p^k (1-p)^{n-k}$	
cdf	$\text{Bi}(x; n, p)$	
Mean, Variance	$\mathbb{E}[X] = np$	$\mathbb{V}[X] = npq = np(1-p)$
Model(s)	# Successes of n independent Bernoulli Trials	

- **COMBINATION:** $\binom{n}{k} := \frac{n!}{k!(n-k)!}$ where **factorial** is defined by $k! := k(k-1)(k-2)\cdots(3)(2)(1)$

DEVORE 3.6

- **POISSON RANDOM VARIABLES (SUMMARY):**

Notation	$X \sim \text{Poisson}(\lambda), \quad \lambda > 0$			
Parameter(s)	$\lambda = \alpha \Delta t$ s.t.	$\alpha \equiv$ Expected/Average # Arrivals per Unit Time	and	$\Delta t \equiv$ Time period
Support	$\text{Supp}(X) = \{0, 1, 2, 3, 4, \cdots\}$			
pmf	$p_X(k; \lambda) := \frac{e^{-\lambda} \lambda^k}{k!}$			
cdf	$\text{Pois}(x; \lambda)$			
Mean, Variance	$\mathbb{E}[X] = \lambda$	$\mathbb{V}[X] = \lambda$		
Model(s)	Number of arrivals over a fixed time period Δt			

DEVORE 4.1

- **PROBABILITY DENSITY FUNCTION (PDF):** The **pdf** of **continuous** rv X, denoted $f_X(x)$, satisfies:

$$\mathbb{P}(X \le a) = \int_{-\infty}^{a} f_X(x)\ dx, \qquad \mathbb{P}(a \le X \le b) = \int_{a}^{b} f_X(x)\ dx, \qquad \mathbb{P}(X \ge b) = \int_{b}^{\infty} f_X(x)\ dx, \qquad \int_{-\infty}^{\infty} f_X(x)\ dx = 1$$

DEVORE 4.2

- **CUMULATIVE DENSITY FCN (CDF):** Let X be a **continuous** random variable with pdf $f_X(x)$.

  Then, its **cdf**, denoted as $F_X(x)$, is defined as follows: $F_X(x) := \mathbb{P}(X \le x) = \int_{-\infty}^{x} f_X(t)\ dt$

DEVORE 4.3 ($\mu \in \mathbb{R}$ means μ can be any real number)

- **UNIFORM RANDOM VARIABLES (SUMMARY):**

Notation	$X \sim \text{Uniform}(a,b), \quad a < b$	
Parameter(s)	$a, b \in \mathbb{R}$	
Support	$\text{Supp}(X) = [a,b]$	
pdf	$f_X(x;a,b) = \dfrac{1}{b-a}$	
Mean, Variance	$\mathbb{E}[X] = \frac{1}{2}(b+a)$	$\mathbb{V}[X] = \frac{1}{12}(b-a)^2$

- **NORMAL RANDOM VARIABLES (SUMMARY):**

Notation	$X \sim \text{Normal}(\mu, \sigma^2), \quad \mu \in \mathbb{R},\ \sigma^2 > 0$	
Parameter(s)	$\mu \equiv$ Mean	$\sigma^2 \equiv$ Variance
Support	$\text{Supp}(X) = (-\infty, \infty)$	
pdf	$f_X(x;\mu,\sigma) = \dfrac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)}$	
cdf	$\Phi\left(\dfrac{x-\mu}{\sigma}\right)$	
Mean, Variance	$\mathbb{E}[X] = \mu$	$\mathbb{V}[X] = \sigma^2$

DEVORE 4.4

- **EXPONENTIAL RANDOM VARIABLES (SUMMARY):**

Notation	$X \sim \text{Exponential}(\lambda), \quad \lambda > 0$	
Parameter(s)	$\lambda \equiv$ Arrival Rate ($= \alpha$ of Poisson Process)	
Support	$\text{Supp}(X) = [0, \infty)$	
pdf	$f_X(x;\lambda) = \lambda e^{-\lambda x}$	
Mean, Variance	$\mathbb{E}[X] = 1/\lambda$	$\mathbb{V}[X] = 1/\lambda^2$

- **GAMMA RANDOM VARIABLES (SUMMARY):** **Gamma Function**

Notation	$X \sim \text{Gamma}(\alpha, \beta), \quad \alpha, \beta > 0$	
Parameter(s)	$\alpha \equiv$ Shape parameter	$\beta \equiv$ Scale parameter
Support	$\text{Supp}(X) = [0, \infty)$	
pdf	$f_X(x;\alpha,\beta) = \dfrac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}$	
cdf	$\gamma(x/\beta;\alpha)$	
Mean, Variance	$\mathbb{E}[X] = \alpha\beta$	$\mathbb{V}[X] = \alpha\beta^2$

- **GAMMA FUNCTION:** $\Gamma(n) = (n-1)!$ where n is a positive integer. $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$, where $\alpha > 0$

BINOMIAL CDF

$$\mathbf{Bi}(x; n, p) := \sum_{k \le x} \binom{n}{k} p^k (1-p)^{n-k}$$

n = 10	**Success Probability** (p)										
x	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**
0	0.34868	0.10737	0.05631	0.02825	0.00605	0.00098	0.00010	0.00001	0.00000	0.00000	0.00000
1	0.73610	0.37581	0.24403	0.14931	0.04636	0.01074	0.00168	0.00014	0.00003	0.00000	0.00000
2	0.92981	0.67780	0.52559	0.38278	0.16729	0.05469	0.01229	0.00159	0.00042	0.00008	0.00000
3	0.98720	0.87913	0.77588	0.64961	0.38228	0.17187	0.05476	0.01059	0.00351	0.00086	0.00001
4	0.99837	0.96721	0.92187	0.84973	0.63310	0.37695	0.16624	0.04735	0.01973	0.00637	0.00015
5	0.99985	0.99363	0.98027	0.95265	0.83376	0.62305	0.36690	0.15027	0.07813	0.03279	0.00163
6	0.99999	0.99914	0.99649	0.98941	0.94524	0.82812	0.61772	0.35039	0.22412	0.12087	0.01280
7	1.00000	0.99992	0.99958	0.99841	0.98771	0.94531	0.83271	0.61722	0.47441	0.32220	0.07019
8	1.00000	1.00000	0.99997	0.99986	0.99832	0.98926	0.95364	0.85069	0.75597	0.62419	0.26390
9	1.00000	1.00000	1.00000	0.99999	0.99990	0.99902	0.99395	0.97175	0.94369	0.89263	0.65132
10	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

POISSON CDF

$$\mathbf{Pois}(x; \lambda) := \sum_{k \le x} \frac{e^{-\lambda} \lambda^k}{k!}$$

	Parameter (λ)									
x	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0	0.36788	0.13534	0.04979	0.01832	0.00674	0.00248	0.00091	0.00034	0.00012	0.00005
1	0.73576	0.40601	0.19915	0.09158	0.04043	0.01735	0.00730	0.00302	0.00123	0.00050
2	0.91970	0.67668	0.42319	0.23810	0.12465	0.06197	0.02964	0.01375	0.00623	0.00277
3	0.98101	0.85712	0.64723	0.43347	0.26503	0.15120	0.08177	0.04238	0.02123	0.01034
4	0.99634	0.94735	0.81526	0.62884	0.44049	0.28506	0.17299	0.09963	0.05496	0.02925
5	0.99941	0.98344	0.91608	0.78513	0.61596	0.44568	0.30071	0.19124	0.11569	0.06709
6	0.99992	0.99547	0.96649	0.88933	0.76218	0.60630	0.44971	0.31337	0.20678	0.13014
7	0.99999	0.99890	0.98810	0.94887	0.86663	0.74398	0.59871	0.45296	0.32390	0.22022
8	1.00000	0.99976	0.99620	0.97864	0.93191	0.84724	0.72909	0.59255	0.45565	0.33282
9	1.00000	0.99995	0.99890	0.99187	0.96817	0.91608	0.83050	0.71662	0.58741	0.45793
10	1.00000	0.99999	0.99971	0.99716	0.98630	0.95738	0.90148	0.81589	0.70599	0.58304
11	1.00000	1.00000	0.99993	0.99908	0.99455	0.97991	0.94665	0.88808	0.80301	0.69678
12	1.00000	1.00000	0.99998	0.99973	0.99798	0.99117	0.97300	0.93620	0.87577	0.79156
13	1.00000	1.00000	1.00000	0.99992	0.99930	0.99637	0.98719	0.96582	0.92615	0.86446
14	1.00000	1.00000	1.00000	0.99998	0.99977	0.99860	0.99428	0.98274	0.95853	0.91654
15	1.00000	1.00000	1.00000	1.00000	0.99993	0.99949	0.99759	0.99177	0.97796	0.95126
16	1.00000	1.00000	1.00000	1.00000	0.99998	0.99983	0.99904	0.99628	0.98889	0.97296
17	1.00000	1.00000	1.00000	1.00000	0.99999	0.99994	0.99964	0.99841	0.99468	0.98572
18	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998	0.99987	0.99935	0.99757	0.99281
19	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99996	0.99975	0.99894	0.99655
20	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99991	0.99956	0.99841
21	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99997	0.99983	0.99930
22	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99993	0.99970
23	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998	0.99988
24	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99995

STANDARD NORMAL CDF $\Phi(z) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2}\, dt$

z	+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900

INCOMPLETE GAMMA FUNCTION $\gamma(x;\alpha) := \int_{0}^{x} t^{\alpha-1} e^{-t}\, dt$

	Shape Parameter (α)					
x	**0.5**	**1**	**2**	**3**	**4**	**5**
0.5	0.68269	0.39347	0.09020	0.01439	0.00175	0.00017
1	0.84270	0.63212	0.26424	0.08030	0.01899	0.00366
1.5	0.91674	0.77687	0.44217	0.19115	0.06564	0.01858
2	0.95450	0.86466	0.59399	0.32332	0.14288	0.05265
2.5	0.97465	0.91792	0.71270	0.45619	0.24242	0.10882
3	0.98569	0.95021	0.80085	0.57681	0.35277	0.18474
3.5	0.99185	0.96980	0.86411	0.67915	0.46337	0.27456