MATH 3342-004: EXAM 3 INFO/LOGISTICS/ADVICE

- **INFO:**

WHEN:	Friday (04/22) at 10:00am	DURATION:	50 mins
PROBLEM COUNT:	Appropriate for a 50-min exam	BONUS COUNT:	At least one

  - TOPICS CANDIDATE FOR THE EXAM:
    * DEVORE 5.3: Sampling Distribution of a Statistic, iid, Random Samples
    * DEVORE 5.4: Central Limit Theorem (CLT), Normal Approximation to Binomial & Poisson
    * DEVORE 6.1: Point Estimation: Unbiased Estimators, Standard Error
    * DEVORE 7.1: Overview of Confidence Intervals (CI's)
    * DEVORE 7.2: Large-Sample z-CI's for any Population Mean (σ unknown)
    * DEVORE 7.2: Wilson Score CI's for any Population Proportion
    * DEVORE 7.3: Small-Sample t-CI's for a Normal Population Mean (σ unknown)
    * DEVORE 7.4: Small-Sample χ^2-CI's for a Normal Population Variance or Std Dev
    * DEVORE 8.1: Overview of Hypothesis Tests, Type I/II Errors, P-values, α-levels
    * DEVORE 8.2: Large-Sample z-Tests about any Population Mean (σ unknown)
    * DEVORE 8.3: Small-Sample t-Tests about a Normal Population Mean (σ unknown)
    * DEVORE 8.4: Large-Sample z-Tests about any Population Proportion
  - TOPICS CANDIDATE FOR BONUS QUESTIONS: (**Maximum Bonus Points Possible = 12**)
    * ?????
  - TOPICS NOT COVERED AT ALL:
    * **Proofs of any kind**, **Sketching Graphs/Histograms/Boxplots of any kind**
    * **The sections to ignore or skip mentioned at the end of the Ch7 & Ch8 slides**
    * DEVORE 7.1: z-CI's for Population Mean **when σ is actually known**
    * DEVORE 8.2: z-Tests about Normal Population Mean **when σ is actually known**
    * DEVORE 8.5: Further Aspects of Hypothesis Testing (entire section)
    * **Computation of Type II Probability** β sprinkled throughout Chapter 8

- **LOGISTICS:**
  - **All you need to bring are pencil(s), eraser(s), calculator(s) & your Raidercard**.
  - Clear your desk of everything except pencil(s), eraser(s) and calculator(s).
  - **Formula Sheet (next four pages) will be provided**.
  - **Books, notes, notecards NOT PERMITTED.**
  - Mobile devices (phones, tablets, laptops, music, headphones, ...) are to be shut off and put away.
  - Tissues will be furnished – for allergies, not for sobbing. No talking or cheating!
  - **When you turn in your exam, be prepared to show me your Raidercard if I don't recognize you**.
  - **If you ask to use the restroom during the exam, either hold it or turn in your exam for grading**.

- **ADVICE:**
  - **Use the restroom before the exam, if needed**. **Do not be late to the exam**.
  - Review the slides, past homework, and perhaps even work some similar problems in the textbook.
  - **Know how the use all formulas & tables on the provided Formula Sheet (next six pages)**
  - Study for the exam together in groups.
  - **If you need more review, show up to the last-minute help session Thursday evening (04/21)**.
  - **SHOW APPROPRIATE WORK! Attempt bonus question(s)**.

MATH 3342: EXAM 3 FORMULA SHEET

DEVORE 5.3 — SAMPLING DISTRIBUTION OF A STATISTIC (PROCEDURE):

GIVEN: Random sample $\mathbf{X} := (X_1, \dots, X_n)$ of finite discrete population w/ pmf $p_X(k)$.

TASK: Find the sampling distribution $p_T(k)$ of statistic $T(\mathbf{X})$ of random sample.

(1) Enumerate all meaningful simultaneous values of the X_i's. Use the support of X_1, Supp(X_1), as guidance.

(2) For each set of meaningful simultaneous values of the X_i's, compute statistic $T(\mathbf{X})$ & joint probability:

$$\mathbb{P}(X_1 = j_1 \cap X_2 = j_2 \cap \cdots \cap X_n = j_n) \overset{iid}{=} p_X(j_1) \cdot p_X(j_2) \cdots p_X(j_n)$$

(3) The support of statistic $T(\mathbf{X})$, Supp(T), is the set of all values of $T(\mathbf{X})$ attained.

(4) The probability of $T(\mathbf{X})$ being a value is the sum of the joint probabilities corresponding to that value.

DEVORE 5.4

- **NORMAL APPROXIMATION TO THE BINOMIAL:** Let $X \sim \text{Binomial}(n, p)$. Then:

$$X \overset{approx}{\sim} \text{Normal}(\mu = np, \sigma^2 = npq) \quad \Longrightarrow \quad \mathbb{P}(X \le x) = \text{Bi}(x; n, p) \approx \Phi\left(\frac{x + 0.5 - np}{\sqrt{npq}}\right) \qquad (\text{where } q = 1 - p)$$

Requirement for this normal approximation to be valid: $\min\{np, nq\} \ge 10$ (i.e. $np \ge 10$ and $nq \ge 10$)

- **NORMAL APPROXIMATION TO THE POISSON:** Let $X \sim \text{Poisson}(\lambda)$. Then:

$$X \overset{approx}{\sim} \text{Normal}(\mu = \lambda, \sigma^2 = \lambda) \quad \Longrightarrow \quad \mathbb{P}(X \le x) = \text{Pois}(x; \lambda) \approx \Phi\left(\frac{x + 0.5 - \lambda}{\sqrt{\lambda}}\right)$$

Requirement for this normal approximation to be valid: $\lambda > 20$

DEVORE 7.2

- **LARGE-SAMPLE CI FOR POPULATION MEAN:**

Given any population with mean μ. Let $x_1, \dots, x_n$ be a large sample ($n > 40$) taken from the population.

Then the $100(1-\alpha)\%$ **large-sample CI for** μ is approximately $\left(\overline{x} - z^*_{\alpha/2} \cdot \frac{s}{\sqrt{n}},\ \overline{x} + z^*_{\alpha/2} \cdot \frac{s}{\sqrt{n}}\right)$

- **WILSON SCORE CI FOR POPULATION PROPORTION:**

Given any population with proportion p of some "success." Let $x_1, \dots, x_n$ be a sample taken from the population.

Then the $100(1-\alpha)\%$ **Wilson score CI for** p is approximately

$$\frac{n\widehat{p} + 0.5(z^*_{\alpha/2})^2}{n + (z^*_{\alpha/2})^2} \pm z^*_{\alpha/2} \cdot \frac{\sqrt{n\widehat{p}\widehat{q} + 0.25(z^*_{\alpha/2})^2}}{n + (z^*_{\alpha/2})^2} \quad \text{where} \quad \widehat{p} := \frac{\#\text{ Successes in Sample}}{\text{Sample Size}} \quad \text{and} \quad \widehat{q} := 1 - \widehat{p}$$

DEVORE 7.3 — SMALL-SAMPLE CI FOR NORMAL POPULATION MEAN μ :

Given a normal population with unknown mean μ and std dev σ.

Let $x_1, \dots, x_n$ be a small sample taken from the population.

Then the $100(1-\alpha)\%$ **small-sample CI for** μ is $\left(\overline{x} - t^*_{n-1,\alpha/2} \cdot \frac{s}{\sqrt{n}},\ \overline{x} + t^*_{n-1,\alpha/2} \cdot \frac{s}{\sqrt{n}}\right)$

DEVORE 7.4 — SMALL-SAMPLE CI FOR NORMAL POPULATION VARIANCE σ^2 :

Given a normal population with unknown mean μ and variance σ^2.

Let $x_1, \dots, x_n$ be a small sample taken from the population.

Then the $100(1-\alpha)\%$ **small-sample CI for** σ^2 is $\left(\frac{(n-1)s^2}{\chi^{2*}_{n-1,\alpha/2}},\ \frac{(n-1)s^2}{\chi^{2*}_{n-1,1-\alpha/2}}\right)$

The following info is the same for the hypothesis tests that follow on this page:

- Random Sample: $\mathbf{X} := (X_1, X_2, \ldots, X_n)$
- Realized Sample: $\mathbf{x} := (x_1, x_2, \ldots, x_n)$
- Decision Rule: If P-value $\leq \alpha$ then reject H_0 in favor of H_A
  If P-value $> \alpha$ then accept H_0 (i.e. fail to reject H_0)

DEVORE 8.2

- **LARGE-SAMPLE z-TEST ABOUT ANY POPULATION MEAN μ (SUMMARY):**

Population:	Any Population with std dev σ unknown
Approx. Test Statistic $W(\mathbf{X};\mu_0)$ Test Statistic Value $W(\mathbf{x};\mu_0)$	$Z = \dfrac{\overline{X} - \mu_0}{S/\sqrt{n}}$ $\quad z = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$
	Sample Size $n > 40$

HYPOTHESIS TEST:	P-VALUE DETERMINATION:
$H_0: \mu = \mu_0$ vs. $H_A: \mu > \mu_0$	P-value $\approx \mathbb{P}(Z \geq z) = 1 - \Phi(z)$
$H_0: \mu = \mu_0$ vs. $H_A: \mu < \mu_0$	P-value $\approx \mathbb{P}(Z \leq z) = \Phi(z)$
$H_0: \mu = \mu_0$ vs. $H_A: \mu \neq \mu_0$	P-value $\approx \mathbb{P}(\lvert Z\rvert \geq \lvert z\rvert) = 2 \cdot [1 - \Phi(\lvert z\rvert)]$

DEVORE 8.3

- **SMALL-SAMPLE t-TEST ABOUT NORMAL POPULATION MEAN μ (SUMMARY):**

Population:	Normal Population with std dev σ unknown
Test Statistic $W(\mathbf{X};\mu_0)$ Test Statistic Value $W(\mathbf{x};\mu_0)$	$T = \dfrac{\overline{X} - \mu_0}{S/\sqrt{n}}$ $\quad t = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$

HYPOTHESIS TEST:	P-VALUE DETERMINATION:
$H_0: \mu = \mu_0$ vs. $H_A: \mu > \mu_0$	P-value $= \mathbb{P}(T \geq t) = 1 - \Phi_t(t; \nu = n-1)$
$H_0: \mu = \mu_0$ vs. $H_A: \mu < \mu_0$	P-value $= \mathbb{P}(T \leq t) = \Phi_t(t; \nu = n-1)$
$H_0: \mu = \mu_0$ vs. $H_A: \mu \neq \mu_0$	P-value $= \mathbb{P}(\lvert T\rvert \geq \lvert t\rvert) = 2 \cdot [1 - \Phi_t(\lvert t\rvert; \nu = n-1)]$

DEVORE 8.4

- **LARGE-SAMPLE z-TEST ABOUT POPULATION PROPORTION p (SUMMARY):**

Population:	Unknown proportion p of "successes"
Test Statistic $W(\mathbf{X};p_0)$ Test Statistic Value $W(\mathbf{x};p_0)$	$Z = \dfrac{(X/n) - p_0}{\sqrt{p_0 q_0/n}}$ $\quad z = \dfrac{\widehat{p} - p_0}{\sqrt{p_0 q_0/n}}$
	$\widehat{p} := x/n,\ q_0 := 1 - p_0,\ \min\{np_0, nq_0\} \geq 10$ $x \equiv \#$ "Successes" in realized sample $\mathbf{x}$

HYPOTHESIS TEST:	P-VALUE DETERMINATION:
$H_0: p = p_0$ vs. $H_A: p > p_0$	P-value $= \mathbb{P}(Z \geq z) = 1 - \Phi(z)$
$H_0: p = p_0$ vs. $H_A: p < p_0$	P-value $= \mathbb{P}(Z \leq z) = \Phi(z)$
$H_0: p = p_0$ vs. $H_A: p \neq p_0$	P-value $= \mathbb{P}(\lvert Z\rvert \geq \lvert z\rvert) = 2 \cdot [1 - \Phi(\lvert z\rvert)]$

GOSSET'S t CRITICAL VALUES $t^*_{\nu,\alpha/2}$

ν	90% CI ($\alpha/2 = 0.05$)	95% CI ($\alpha/2 = 0.025$)	99% CI ($\alpha/2 = 0.005$)
1	6.314	12.706	63.657
2	2.920	4.303	9.925
3	2.353	3.182	5.841
4	2.132	2.776	4.604
5	2.015	2.571	4.032
6	1.943	2.447	3.707
7	1.895	2.365	3.499
8	1.860	2.306	3.355
9	1.833	2.262	3.250
10	1.812	2.228	3.169
11	1.796	2.201	3.106
12	1.782	2.179	3.055
13	1.771	2.160	3.012
14	1.761	2.145	2.977
15	1.753	2.131	2.947
16	1.746	2.120	2.921
17	1.740	2.110	2.898
18	1.734	2.101	2.878
19	1.729	2.093	2.861
20	1.725	2.086	2.845
21	1.721	2.080	2.831
22	1.717	2.074	2.819
23	1.714	2.069	2.807
24	1.711	2.064	2.797
25	1.708	2.060	2.787
26	1.706	2.056	2.779
27	1.703	2.052	2.771
28	1.701	2.048	2.763
29	1.699	2.045	2.756
30	1.697	2.042	2.750
31	1.696	2.040	2.744
32	1.694	2.037	2.738
33	1.692	2.035	2.733
34	1.691	2.032	2.728
35	1.690	2.030	2.724
36	1.688	2.028	2.719
37	1.687	2.026	2.715
38	1.686	2.024	2.712
39	1.685	2.023	2.708

HELMERT'S χ^2 CRITICAL VALUES

ν	90% CI ($\alpha/2 = 0.05$)		95% CI ($\alpha/2 = 0.025$)		99% CI ($\alpha/2 = 0.005$)	
	$\chi^{2*}_{\nu,1-\alpha/2}$	$\chi^{2*}_{\nu,\alpha/2}$	$\chi^{2*}_{\nu,1-\alpha/2}$	$\chi^{2*}_{\nu,\alpha/2}$	$\chi^{2*}_{\nu,1-\alpha/2}$	$\chi^{2*}_{\nu,\alpha/2}$
1	0.004	3.841	0.001	5.024	0.000	7.879
2	0.103	5.991	0.051	7.378	0.010	10.597
3	0.352	7.815	0.216	9.348	0.072	12.838
4	0.711	9.488	0.484	11.143	0.207	14.860
5	1.145	11.070	0.831	12.833	0.412	16.750
6	1.635	12.592	1.237	14.449	0.676	18.548
7	2.167	14.067	1.690	16.013	0.989	20.278
8	2.733	15.507	2.180	17.535	1.344	21.955
9	3.325	16.919	2.700	19.023	1.735	23.589
10	3.940	18.307	3.247	20.483	2.156	25.188
11	4.575	19.675	3.816	21.920	2.603	26.757
12	5.226	21.026	4.404	23.337	3.074	28.300
13	5.892	22.362	5.009	24.736	3.565	29.819
14	6.571	23.685	5.629	26.119	4.075	31.319
15	7.261	24.996	6.262	27.488	4.601	32.801
16	7.962	26.296	6.908	28.845	5.142	34.267
17	8.672	27.587	7.564	30.191	5.697	35.718
18	9.390	28.869	8.231	31.526	6.265	37.156
19	10.117	30.144	8.907	32.852	6.844	38.582
20	10.851	31.410	9.591	34.170	7.434	39.997
21	11.591	32.671	10.283	35.479	8.034	41.401
22	12.338	33.924	10.982	36.781	8.643	42.796
23	13.091	35.172	11.689	38.076	9.260	44.181
24	13.848	36.415	12.401	39.364	9.886	45.559
25	14.611	37.652	13.120	40.646	10.520	46.928
26	15.379	38.885	13.844	41.923	11.160	48.290
27	16.151	40.113	14.573	43.195	11.808	49.645
28	16.928	41.337	15.308	44.461	12.461	50.993
29	17.708	42.557	16.047	45.722	13.121	52.336
30	18.493	43.773	16.791	46.979	13.787	53.672
31	19.281	44.985	17.539	48.232	14.458	55.003
32	20.072	46.194	18.291	49.480	15.134	56.328
33	20.867	47.400	19.047	50.725	15.815	57.648
34	21.664	48.602	19.806	51.966	16.501	58.964
35	22.465	49.802	20.569	53.203	17.192	60.275
36	23.269	50.998	21.336	54.437	17.887	61.581
37	24.075	52.192	22.106	55.668	18.586	62.883
38	24.884	53.384	22.878	56.896	19.289	64.181

COMMON z CRITICAL VALUES:

	90% CI	95% CI	99% CI
$z^*_{\alpha/2}$	1.64	1.96	2.58

STANDARD NORMAL CDF $\Phi(z) := \mathbb{P}(Z \leq z)$

z	**+0.00**	**+0.01**	**+0.02**	**+0.03**	**+0.04**	**+0.05**	**+0.06**	**+0.07**	**+0.08**	**+0.09**
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997
4.0	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998	0.99998	0.99998	0.99998

GOSSET'S t CDF TABLE FOR $\Phi_t(t;\nu) := \mathbb{P}(T \leq t)$ [HERE, $\nu = 1, 2, \ldots, 15$]

	DEGREES OF FREEDOM (ν)														
t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
0.0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.1	0.532	0.535	0.537	0.537	0.538	0.538	0.538	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539
0.2	0.563	0.570	0.573	0.574	0.575	0.576	0.576	0.577	0.577	0.577	0.577	0.578	0.578	0.578	0.578
0.3	0.593	0.604	0.608	0.610	0.612	0.613	0.614	0.614	0.615	0.615	0.615	0.615	0.616	0.616	0.616
0.4	0.621	0.636	0.642	0.645	0.647	0.648	0.649	0.650	0.651	0.651	0.652	0.652	0.652	0.652	0.653
0.5	0.648	0.667	0.674	0.678	0.681	0.683	0.684	0.685	0.685	0.686	0.687	0.687	0.687	0.688	0.688
0.6	0.672	0.695	0.705	0.710	0.713	0.715	0.716	0.717	0.718	0.719	0.720	0.720	0.721	0.721	0.721
0.7	0.694	0.722	0.733	0.739	0.742	0.745	0.747	0.748	0.749	0.750	0.751	0.751	0.752	0.752	0.753
0.8	0.715	0.746	0.759	0.766	0.770	0.773	0.775	0.777	0.778	0.779	0.780	0.780	0.781	0.781	0.782
0.9	0.733	0.768	0.783	0.790	0.795	0.799	0.801	0.803	0.804	0.805	0.806	0.807	0.808	0.808	0.809
1.0	0.750	0.789	0.804	0.813	0.818	0.822	0.825	0.827	0.828	0.830	0.831	0.831	0.832	0.833	0.833
1.1	0.765	0.807	0.824	0.833	0.839	0.843	0.846	0.848	0.850	0.851	0.853	0.854	0.854	0.855	0.856
1.2	0.779	0.823	0.842	0.852	0.858	0.862	0.865	0.868	0.870	0.871	0.872	0.873	0.874	0.875	0.876
1.3	0.791	0.838	0.858	0.868	0.875	0.879	0.883	0.885	0.887	0.889	0.890	0.891	0.892	0.893	0.893
1.4	0.803	0.852	0.872	0.883	0.890	0.894	0.898	0.900	0.902	0.904	0.905	0.907	0.908	0.908	0.909
1.5	0.813	0.864	0.885	0.896	0.903	0.908	0.911	0.914	0.916	0.918	0.919	0.920	0.921	0.922	0.923
1.6	0.822	0.875	0.896	0.908	0.915	0.920	0.923	0.926	0.928	0.930	0.931	0.932	0.933	0.934	0.935
1.7	0.831	0.884	0.906	0.918	0.925	0.930	0.934	0.936	0.938	0.940	0.941	0.943	0.944	0.944	0.945
1.8	0.839	0.893	0.915	0.927	0.934	0.939	0.943	0.945	0.947	0.949	0.950	0.951	0.952	0.953	0.954
1.9	0.846	0.901	0.923	0.935	0.942	0.947	0.950	0.953	0.955	0.957	0.958	0.959	0.960	0.961	0.962
2.0	0.852	0.908	0.930	0.942	0.949	0.954	0.957	0.960	0.962	0.963	0.965	0.966	0.967	0.967	0.968
2.1	0.859	0.915	0.937	0.948	0.955	0.960	0.963	0.966	0.967	0.969	0.970	0.971	0.972	0.973	0.973
2.2	0.864	0.921	0.942	0.954	0.960	0.965	0.968	0.971	0.972	0.974	0.975	0.976	0.977	0.977	0.978
2.3	0.869	0.926	0.948	0.959	0.965	0.969	0.973	0.975	0.977	0.978	0.979	0.980	0.981	0.981	0.982
2.4	0.874	0.931	0.952	0.963	0.969	0.973	0.976	0.978	0.980	0.981	0.982	0.983	0.984	0.985	0.985
2.5	0.879	0.935	0.956	0.967	0.973	0.977	0.980	0.982	0.983	0.984	0.985	0.986	0.987	0.987	0.988
2.6	0.883	0.939	0.960	0.970	0.976	0.980	0.982	0.984	0.986	0.987	0.988	0.988	0.989	0.990	0.990
2.7	0.887	0.943	0.963	0.973	0.979	0.982	0.985	0.986	0.988	0.989	0.990	0.990	0.991	0.991	0.992
2.8	0.891	0.946	0.966	0.976	0.981	0.984	0.987	0.988	0.990	0.991	0.991	0.992	0.992	0.993	0.993
2.9	0.894	0.949	0.969	0.978	0.983	0.986	0.989	0.990	0.991	0.992	0.993	0.993	0.994	0.994	0.995
3.0	0.898	0.952	0.971	0.980	0.985	0.988	0.990	0.991	0.993	0.993	0.994	0.994	0.995	0.995	0.996
3.1	0.901	0.955	0.973	0.982	0.987	0.989	0.991	0.993	0.994	0.994	0.995	0.995	0.996	0.996	0.996
3.2	0.904	0.957	0.975	0.984	0.988	0.991	0.992	0.994	0.995	0.995	0.996	0.996	0.997	0.997	0.997
3.3	0.906	0.960	0.977	0.985	0.989	0.992	0.993	0.995	0.995	0.996	0.996	0.997	0.997	0.997	0.998
3.4	0.909	0.962	0.979	0.986	0.990	0.993	0.994	0.995	0.996	0.997	0.997	0.997	0.998	0.998	0.998
3.5	0.911	0.964	0.980	0.988	0.991	0.994	0.995	0.996	0.997	0.997	0.998	0.998	0.998	0.998	0.998
3.6	0.914	0.965	0.982	0.989	0.992	0.994	0.996	0.997	0.997	0.998	0.998	0.998	0.998	0.999	0.999
3.7	0.916	0.967	0.983	0.990	0.993	0.995	0.996	0.997	0.998	0.998	0.998	0.998	0.999	0.999	0.999
3.8	0.918	0.969	0.984	0.990	0.994	0.996	0.997	0.997	0.998	0.998	0.999	0.999	0.999	0.999	0.999
3.9	0.920	0.970	0.985	0.991	0.994	0.996	0.997	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999
4.0	0.922	0.971	0.986	0.992	0.995	0.996	0.997	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999

GOSSET'S t CDF TABLE FOR $\Phi_t(t;\nu) := \mathbb{P}(T \le t)$ [HERE, $\nu = \mathbf{16, 17, \ldots, 30}$]

	DEGREES OF FREEDOM (ν)														
t	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
0.0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.1	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539
0.2	0.578	0.578	0.578	0.578	0.578	0.578	0.578	0.578	0.578	0.578	0.578	0.579	0.579	0.579	0.579
0.3	0.616	0.616	0.616	0.616	0.616	0.616	0.617	0.617	0.617	0.617	0.617	0.617	0.617	0.617	0.617
0.4	0.653	0.653	0.653	0.653	0.653	0.653	0.653	0.654	0.654	0.654	0.654	0.654	0.654	0.654	0.654
0.5	0.688	0.688	0.688	0.689	0.689	0.689	0.689	0.689	0.689	0.689	0.689	0.689	0.690	0.690	0.690
0.6	0.722	0.722	0.722	0.722	0.722	0.723	0.723	0.723	0.723	0.723	0.723	0.723	0.723	0.723	0.723
0.7	0.753	0.753	0.754	0.754	0.754	0.754	0.754	0.755	0.755	0.755	0.755	0.755	0.755	0.755	0.755
0.8	0.782	0.783	0.783	0.783	0.783	0.784	0.784	0.784	0.784	0.784	0.785	0.785	0.785	0.785	0.785
0.9	0.809	0.810	0.810	0.810	0.811	0.811	0.811	0.811	0.811	0.812	0.812	0.812	0.812	0.812	0.812
1.0	0.834	0.834	0.835	0.835	0.835	0.836	0.836	0.836	0.836	0.837	0.837	0.837	0.837	0.837	0.837
1.1	0.856	0.857	0.857	0.857	0.858	0.858	0.858	0.859	0.859	0.859	0.859	0.859	0.860	0.860	0.860
1.2	0.876	0.877	0.877	0.878	0.878	0.878	0.879	0.879	0.879	0.879	0.880	0.880	0.880	0.880	0.880
1.3	0.894	0.895	0.895	0.895	0.896	0.896	0.896	0.897	0.897	0.897	0.897	0.898	0.898	0.898	0.898
1.4	0.910	0.910	0.911	0.911	0.912	0.912	0.912	0.913	0.913	0.913	0.913	0.914	0.914	0.914	0.914
1.5	0.923	0.924	0.925	0.925	0.925	0.926	0.926	0.926	0.927	0.927	0.927	0.927	0.928	0.928	0.928
1.6	0.935	0.936	0.936	0.937	0.937	0.938	0.938	0.938	0.939	0.939	0.939	0.939	0.940	0.940	0.940
1.7	0.946	0.946	0.947	0.947	0.948	0.948	0.948	0.949	0.949	0.949	0.949	0.950	0.950	0.950	0.950
1.8	0.955	0.955	0.956	0.956	0.957	0.957	0.957	0.958	0.958	0.958	0.958	0.958	0.959	0.959	0.959
1.9	0.962	0.963	0.963	0.964	0.964	0.964	0.965	0.965	0.965	0.965	0.966	0.966	0.966	0.966	0.966
2.0	0.969	0.969	0.970	0.970	0.970	0.971	0.971	0.971	0.972	0.972	0.972	0.972	0.972	0.973	0.973
2.1	0.974	0.975	0.975	0.975	0.976	0.976	0.976	0.977	0.977	0.977	0.977	0.977	0.978	0.978	0.978
2.2	0.979	0.979	0.979	0.980	0.980	0.980	0.981	0.981	0.981	0.981	0.982	0.982	0.982	0.982	0.982
2.3	0.982	0.983	0.983	0.984	0.984	0.984	0.984	0.985	0.985	0.985	0.985	0.985	0.985	0.986	0.986
2.4	0.986	0.986	0.986	0.987	0.987	0.987	0.987	0.988	0.988	0.988	0.988	0.988	0.988	0.988	0.989
2.5	0.988	0.989	0.989	0.989	0.989	0.990	0.990	0.990	0.990	0.990	0.990	0.991	0.991	0.991	0.991
2.6	0.990	0.991	0.991	0.991	0.991	0.992	0.992	0.992	0.992	0.992	0.992	0.993	0.993	0.993	0.993
2.7	0.992	0.992	0.993	0.993	0.993	0.993	0.993	0.994	0.994	0.994	0.994	0.994	0.994	0.994	0.994
2.8	0.994	0.994	0.994	0.994	0.994	0.995	0.995	0.995	0.995	0.995	0.995	0.995	0.995	0.996	0.996
2.9	0.995	0.995	0.995	0.995	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.997
3.0	0.996	0.996	0.996	0.996	0.996	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
3.1	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.998	0.998	0.998	0.998	0.998	0.998	0.998
3.2	0.997	0.997	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998
3.3	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999
3.4	0.998	0.998	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.6	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.7	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000
3.8	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.9	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4.0	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000