MATH 3342-004: FINAL EXAM INFO/LOGISTICS/ADVICE

- **INFO:**

LOCATION:	**Monday (05/16)** at **1:30pm** in MATH 014 (our usual room)	DURATION:	2.5 hrs
PROBLEM COUNT:	Appropriate for a 2.5-hr exam	BONUS COUNT:	At least one

  - The FINAL EXAM is **comprehensive**. Expect roughly equal coverage of all chapters covered in the course.
  - The FINAL EXAM is **not departmental**, meaning the instructor writes the FINAL EXAM.
  - TOPICS CANDIDATE FOR THE EXAM:
    - (**All 'TOPICS CANDIDATE FOR THE EXAM's from Info/Logistics/Advice for EXAMS 1,2,3**)
    - DEVORE 9.1: Large-Sample z-Tests & z-CI's for $\mu_1 - \mu_2$
    - DEVORE 9.2: Small-Sample t-Tests & t-CI's for $\mu_1 - \mu_2$
    - DEVORE 9.4: Large-Sample z-Tests & z-CI's for $p_1 - p_2$
  - TOPICS CANDIDATE FOR BONUS QUESTIONS: (**Maximum Bonus Points Possible = 9**)
    - (**All 'TOPICS CANDIDATE FOR BONUS ?'s from Info/Logistics/Advice for EXAMS 1,2,3**)
    - ???????
  - TOPICS NOT COVERED AT ALL:
    - (**All 'TOPICS NOT COVERED AT ALL' sections from Info/Logistics/Advice for EXAMS 1,2,3**)
    - DEVORE 9.3: Analysis of Paired Data, Paired t-Tests, Paired t-CI's (entire section)
    - DEVORE 9.5: Inferences concerning Two Population Variances, Snedecor's F Distribution (entire section)
    - DEVORE 3.5: Discrete rv's: Hypergeometric, Geometric & Negative Binomial rv's (entire section)
    - DEVORE 4.5: Continuous rv's: Lognormal, Weibull & Beta rv's (entire section)

- **LOGISTICS:**
  - **All you need to bring are pencil(s), eraser(s), calculator(s) & your Raidercard**.
  - Clear your desk of everything except pencil(s), eraser(s) and calculator(s).
  - **Formula Sheet (next twelve pages) will be provided**.
  - **Books, notes, notecards NOT PERMITTED.**
  - Mobile devices (phones, tablets, laptops, music, headphones, ...) are to be shut off and put away.
  - Tissues will be furnished – for allergies, not for sobbing. No talking or cheating!
  - **When you turn in your exam, be prepared to show me your Raidercard if I don't recognize you**.
  - **If you ask to use the restroom during the exam, either hold it or turn in your exam for grading**.

- **ADVICE:**
  - **Use the restroom before the exam, if needed**.
  - **Do not be late to the exam**.
  - Review the slides, past homework, and perhaps even work some similar problems in the textbook.
  - **Know how the use all formulas on the provided Formula Sheet (next twelve pages)**
  - Study for the exam together in groups.
  - **Show up to extended office hours Friday (05/13) from 8:30am to 3:00pm for last-minute help**.
  - **SHOW APPROPRIATE WORK! Attempt bonus question(s)**.

MATH 3342: EXAM 1 FORMULA SHEET

DEVORE 1.3

- **SORTED SAMPLES:**

  Given a sample with n data points $\quad x: \; x_1, x_2, \ldots, x_{n-1}, x_n$

  Then the corresponding **sorted sample** is $\quad x: \; x_{(1)}, x_{(2)}, \ldots, x_{(n-1)}, x_{(n)}$

  where the data points are sorted in ascending order: $\quad x_{(1)} \le x_{(2)} \le \cdots \le x_{(n-1)} \le x_{(n)}$

- **ROUNDING (COMPACT NOTATION):** It is convenient to have mathematical notation for **rounding numbers**.

$$\text{Always Round Up:} \quad \lceil 3 \rceil = 3 \quad \lceil 3.1 \rceil = 4 \quad \lceil 3.5 \rceil = 4 \quad \lceil 3.9 \rceil = 4$$

$$\text{Always Round Down:} \quad \lfloor 3 \rfloor = 3 \quad \lfloor 3.1 \rfloor = 3 \quad \lfloor 3.5 \rfloor = 3 \quad \lfloor 3.9 \rfloor = 3$$

- **MEAN OF A SAMPLE:** The **mean**, denoted $\overline{x}$, is the average of the sample.

$$\overline{x} := \frac{1}{n}\sum_{k=1}^{n} x_k = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

- **MEDIAN OF A SAMPLE:** The **median**, denoted $\widetilde{x}$, is the middle value of the sorted sample.

$$\widetilde{x} := \begin{cases} x_{([n+1]/2)} & \text{, if } n \text{ is odd} \\[2ex] \dfrac{x_{(n/2)} + x_{(1+[n/2])}}{2} & \text{, if } n \text{ is even} \end{cases}$$

- **TRIMMED MEAN OF A SAMPLE:** The p% **trimmed mean**, $\overline{x}_{tr(p\%)}$, is the mean of the dataset resulting from eliminating the smallest p% and largest p% of the sorted sample.

  e.g. $\quad \overline{x}_{tr(10\%)} :=$ Mean of sorted sample x with largest 10% & smallest 10% removed

- **PERCENTILES OF A SAMPLE:** The **p-th percentile**, denoted $x_{p/100}$, is the smallest data point such that p% of the sample is less than or equal to that data point:

$$x_{p/100} := x_{(\lceil np/100 \rceil)} = \left(\left\lceil \frac{np}{100} \right\rceil \right) \text{-th data point in sorted sample}$$

  e.g. $\quad x_{0.37} \equiv (37^{th}$ percentile of sample $x)$

  e.g. $\quad y_{0.98} \equiv (98^{th}$ percentile of sample $y)$

- **QUARTILES OF A SAMPLE:**

$$x_{Q1} := x_{0.25} \equiv 1^{st} \textbf{ quartile } (25^{th} \text{ percentile}) \text{ of sample } x$$

$$x_{Q3} := x_{0.75} \equiv 3^{rd} \textbf{ quartile } (75^{th} \text{ percentile}) \text{ of sample } x$$

- **HINGES OF A SAMPLE:**

  The **lower hinge**, x_{LH}, is the median of the lower half of sorted sample.

  The **upper hinge**, x_{UH}, is the median of the upper half of sorted sample.

DEVORE 1.4

- **RANGE OF A SAMPLE:** Sample **range**, denoted x_R, is the **difference** between largest & smallest data points:

$$x_R := x_{(n)} - x_{(1)}$$

- **VARIANCE OF A SAMPLE:** Sample **variance**, denoted s^2 or s_x^2, is the following:

$$s^2 = \frac{S_{xx}}{n-1} \qquad \text{where} \qquad S_{xx} = \sum_{k=1}^{n} x_k^2 - \frac{1}{n}\left(\sum_{k=1}^{n} x_k\right)^2$$

- **STANDARD DEVIATION OF A SAMPLE:** **Standard deviation**, denoted s or s_x, is the square root of variance:

$$s := \sqrt{s^2}$$

- **INTERQUARTILE RANGE (IQR):** $\quad x_{IQR} := x_{Q3} - x_{Q1}$
- **INTERHINGE RANGE (IHR):** $\quad x_{IHR} := x_{UH} - x_{LH}$

DEVORE 2.1

- **BASIC TERMINOLOGY:** The **outcomes** $\omega_1, \omega_2, \ldots$ of an experiment are the different possible results.

  The **sample space** Ω of an experiment is the set of all possible outcomes.

  An **event** E is a subset of the sample space: $E \subseteq \Omega$
- **THE EMPTY SET:** The **empty set**, $\emptyset$, is the event with no outcomes in it.
- **UNIONS, INTERSECTIONS, COMPLEMENTS OF EVENTS (PROPERTIES):** Let $A, B, C \subseteq \Omega$. Then:

$$(A^c)^c = A, \quad A \cup B = B \cup A, \quad A \cap B = B \cap A$$
$$(A \cup B) \cup C = A \cup (B \cup C), \quad (A \cap B) \cap C = A \cap (B \cap C)$$
$$(A \cup B)^c = A^c \cap B^c, \quad (A \cap B)^c = A^c \cup B^c$$
$$A \cup \emptyset = A, \quad A \cap \emptyset = \emptyset, \quad \Omega^c = \emptyset, \quad \emptyset^c = \Omega$$
$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C), \quad (A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$

- **MUTUAL EXCLUSIVITY OF TWO EVENTS:** Events E, F are **mutually exclusive** (or **disjoint**) if $E \cap F = \emptyset$
- MUTUAL EXCLUSIVITY OF THREE EVENTS: Events E, F, G are **mutually exclusive** (or **pairwise disjoint**) if:

$$E \cap F = \emptyset \text{ and } E \cap G = \emptyset \text{ and } F \cap G = \emptyset$$

- **MUTUAL EXCLUSIVITY OF MANY EVENTS:** Events $E_1, E_2, \ldots, E_n$ are **mutually exclusive** (or **pairwise disjoint**) if they have no outcomes in common: $E_i \cap E_j = \emptyset$ for $i \neq j$

DEVORE 2.2

- **CHAIN OF UNIONS:** $\bigcup_{k=1}^{n} A_k = A_1 \cup A_2 \cup \cdots \cup A_{n-1} \cup A_n$ $\qquad \bigcup_{k=1}^{\infty} A_k = A_1 \cup A_2 \cup A_3 \cup \cdots$
- AXIOMS & PROPERTIES OF PROBABILITY: Let $E, F \subseteq \Omega$ be two events from sample space Ω of an experiment.

  Let $E_1, E_2, E_3, \ldots$ be an infinite collection of pairwise disjoint events. Then:

$$\mathbb{P}(E) \geq 0, \quad \mathbb{P}(\Omega) = 1, \quad \mathbb{P}(\emptyset) = 0, \quad \mathbb{P}\left(\bigcup_{k=1}^{\infty} E_k\right) = \sum_{k=1}^{\infty} \mathbb{P}(E_k), \quad \mathbb{P}\left(\bigcup_{k=1}^{n} E_k\right) = \sum_{k=1}^{n} \mathbb{P}(E_k)$$
$$\mathbb{P}(E^c) = 1 - \mathbb{P}(E), \quad \mathbb{P}(E) \leq 1, \quad \mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F) - \mathbb{P}(E \cap F)$$

- **INCLUSION-EXCLUSION:** $\mathbb{P}(E \cup F \cup G) = \mathbb{P}(E) + \mathbb{P}(F) + \mathbb{P}(G) - \mathbb{P}(E \cap F) - \mathbb{P}(E \cap G) - \mathbb{P}(F \cap G) + \mathbb{P}(E \cap F \cap G)$
- **MEASURE OF A SET:** The **measure** of a **countable set** is $|E| := (\#$ of elements in $E)$, $\qquad |\emptyset| := 0$
- **PROBABILITY WITH EQUALLY LIKELY OUTCOMES:**

  Let Ω be the sample space of an experiment with **equally likely outcomes**. Let E be an event of the experiment.

  Then the **probability** of event E occurring is defined as: $\mathbb{P}(E) = \dfrac{|E|}{|\Omega|}$

DEVORE 2.4

- **CONDITIONAL PROBABILITY:** Let $E, F \subseteq \Omega$ be two events such that $\mathbb{P}(F) > 0$.

  Then the **conditional probability** of E **given that** F **has occurred** is: $\mathbb{P}(E|F) := \dfrac{\mathbb{P}(E \cap F)}{\mathbb{P}(F)}$
- **PARTITIONS:** Events $E_1, \ldots, E_k$ **partition** sample space Ω if they are pairwise disjoint AND $\bigcup_{i=1}^{k} E_i = \Omega$
- **LAW OF TOTAL PROBABILITY:** Let $E_1, \ldots, E_k$ partition Ω. Then $\mathbb{P}(F) = \sum_{i=1}^{k} \mathbb{P}(F \cap E_i) = \sum_{i=1}^{k} \mathbb{P}(F|E_i) \cdot \mathbb{P}(E_i)$
- BAYES' THEOREM: Let $E_1, \ldots, E_k$ partition Ω. Then $\mathbb{P}(E_j|F) = \dfrac{\mathbb{P}(F \cap E_j)}{\sum_{i=1}^{k} \mathbb{P}(F \cap E_i)} = \dfrac{\mathbb{P}(F|E_j) \cdot \mathbb{P}(E_j)}{\sum_{i=1}^{k} \mathbb{P}(F|E_i) \cdot \mathbb{P}(E_i)}$ for $j = 1, \ldots, k$

DEVORE 2.5

- **INDEPENDENCE OF TWO EVENTS:** Two events E, F are **independent** if $\mathbb{P}(E|F) = \mathbb{P}(E)$
- **INDEPENDENCE OF TWO EVENTS:** Two events E, F are **independent** if and only if $\mathbb{P}(E \cap F) = \mathbb{P}(E) \cdot \mathbb{P}(F)$

MATH 3342: EXAM 2 FORMULA SHEET

DEVORE 3.2 (The symbol $\forall$ translates to "for all" or "for every" or "for each")

- **PROBABILITY MASS FUNCTION (PMF):** Let X be a **discrete** random variable.
  Then, its **pmf**, denoted as $p_X(k)$, is defined as follows: $p_X(k) := \mathbb{P}(X = k) \;\; \forall k \in \text{Supp}(X)$
- **CUMULATIVE DENSITY FUNCTION (CDF):** Let X be a **discrete** random variable s.t. $\text{Supp}(X) = \{k_1, k_2, k_3, \cdots\}$
  Then, its **cdf**, denoted as $F_X(x)$, is defined as follows: $F_X(x) := \mathbb{P}(X \leq x) = \sum_{k_i \leq x} p_X(k_i) \;\; \forall x \in \mathbb{R}$

DEVORE 3.3

- **EXPECTED VALUE (MEAN) OF A DISCRETE R.V.:** $\mu_X = \mathbb{E}[X] := \sum_{k \in \text{Supp}(X)} k \cdot p_X(k)$
- **EXPECTED VALUE OF A FUNCTION OF DISCRETE R.V.:** $\mu_{h(X)} = \mathbb{E}[h(X)] := \sum_{k \in \text{Supp}(X)} h(k) \cdot p_X(k)$
- **VARIANCE OF A DISCRETE R.V.:** $\sigma_X^2 = \mathbb{V}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$
- **STANDARD DEVIATION OF A DISCRETE R.V.:** $\sigma_X := \sqrt{\mathbb{V}[X]}$

DEVORE 3.4

- **BERNOULLI RANDOM VARIABLES (SUMMARY):**

Notation	$X \sim \text{Bernoulli}(p), \quad 0 < p < 1, \quad q := 1 - p$	
Parameter(s)	$p \equiv \mathbb{P}(\text{Bernoulli Trial is a Success})$	
Support	$\text{Supp}(X) = \{0, 1\}$	
pmf	$p_X(k; p) := p^k q^{1-k} = p^k (1-p)^{1-k}$	
Mean, Variance	$\mathbb{E}[X] = p$	$\mathbb{V}[X] = pq = p(1-p)$
Model(s)	Result of One Bernoulli Trial: $1 \equiv \text{Success}, 0 \equiv \text{Failure}$	
Assumption(s)	Random process has its sample space partitioned into Successes and Failures	

- **BINOMIAL RANDOM VARIABLES (SUMMARY):**

Notation	$X \sim \text{Binomial}(n, p), \quad n \geq 1, \quad 0 < p < 1, \quad q := 1 - p$	
Parameter(s)	(Same parameters as Bernoulli(p))	
Support	$\text{Supp}(X) = \{0, 1, 2, \cdots, n-1, n\}$	
pmf	$p_X(k; n, p) := \binom{n}{k} p^k q^{n-k} = \binom{n}{k} p^k (1-p)^{n-k}$	
cdf	$\text{Bi}(x; n, p)$	
Mean, Variance	$\mathbb{E}[X] = np$	$\mathbb{V}[X] = npq = np(1-p)$
Model(s)	# Successes of n independent Bernoulli Trials	

- **COMBINATION:** $\binom{n}{k} := \frac{n!}{k!(n-k)!}$ where **factorial** is defined by $k! := k(k-1)(k-2)\cdots(3)(2)(1)$

DEVORE 3.6

- **POISSON RANDOM VARIABLES (SUMMARY):**

Notation	$X \sim \text{Poisson}(\lambda), \quad \lambda > 0$	
Parameter(s)	$\lambda = \alpha \Delta t$ s.t. $\alpha \equiv$ Expected/Average # Arrivals per Unit Time and $\Delta t \equiv$ Time period	
Support	$\text{Supp}(X) = \{0, 1, 2, 3, 4, \cdots\}$	
pmf	$p_X(k; \lambda) := \frac{e^{-\lambda} \lambda^k}{k!}$	
cdf	$\text{Pois}(x; \lambda)$	
Mean, Variance	$\mathbb{E}[X] = \lambda$	$\mathbb{V}[X] = \lambda$
Model(s)	Number of arrivals over a fixed time period Δt	

DEVORE 4.1

- **PROBABILITY DENSITY FUNCTION (PDF):** The **pdf** of **continuous** rv X, denoted $f_X(x)$, satisfies:

$$\mathbb{P}(X \le a) = \int_{-\infty}^{a} f_X(x)\, dx, \qquad \mathbb{P}(a \le X \le b) = \int_{a}^{b} f_X(x)\, dx, \qquad \mathbb{P}(X \ge b) = \int_{b}^{\infty} f_X(x)\, dx, \qquad \int_{-\infty}^{\infty} f_X(x)\, dx = 1$$

DEVORE 4.2

- **CUMULATIVE DENSITY FCN (CDF):** Let X be a **continuous** random variable with pdf $f_X(x)$.

  Then, its **cdf**, denoted as $F_X(x)$, is defined as follows: $F_X(x) := \mathbb{P}(X \le x) = \int_{-\infty}^{x} f_X(t)\, dt$

DEVORE 4.3 ($\mu \in \mathbb{R}$ means μ can be any real number)

- **UNIFORM RANDOM VARIABLES (SUMMARY):**

Notation	$X \sim \text{Uniform}(a, b), \quad a < b$	
Parameter(s)	$a, b \in \mathbb{R}$	
Support	$\text{Supp}(X) = [a, b]$	
pdf	$f_X(x; a, b) = \dfrac{1}{b-a}$	
Mean, Variance	$\mathbb{E}[X] = \frac{1}{2}(b+a)$	$\mathbb{V}[X] = \frac{1}{12}(b-a)^2$

- **NORMAL RANDOM VARIABLES (SUMMARY):**

Notation	$X \sim \text{Normal}(\mu, \sigma^2), \quad \mu \in \mathbb{R}, \; \sigma^2 > 0$	
Parameter(s)	$\mu \equiv \text{Mean}$	$\sigma^2 \equiv \text{Variance}$
Support	$\text{Supp}(X) = (-\infty, \infty)$	
pdf	$f_X(x; \mu, \sigma) = \dfrac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)}$	
cdf	$\Phi\left(\dfrac{x-\mu}{\sigma}\right)$	
Mean, Variance	$\mathbb{E}[X] = \mu$	$\mathbb{V}[X] = \sigma^2$

DEVORE 4.4

- **EXPONENTIAL RANDOM VARIABLES (SUMMARY):**

Notation	$X \sim \text{Exponential}(\lambda), \quad \lambda > 0$	
Parameter(s)	$\lambda \equiv$ Arrival Rate ($= \alpha$ of Poisson Process)	
Support	$\text{Supp}(X) = [0, \infty)$	
pdf	$f_X(x; \lambda) = \lambda e^{-\lambda x}$	
Mean, Variance	$\mathbb{E}[X] = 1/\lambda$	$\mathbb{V}[X] = 1/\lambda^2$

- **GAMMA RANDOM VARIABLES (SUMMARY):** **Gamma Function**

Notation	$X \sim \text{Gamma}(\alpha, \beta), \quad \alpha, \beta > 0$	
Parameter(s)	$\alpha \equiv$ Shape parameter	$\beta \equiv$ Scale parameter
Support	$\text{Supp}(X) = [0, \infty)$	
pdf	$f_X(x; \alpha, \beta) = \dfrac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}$	
cdf	$\gamma(x/\beta; \alpha)$	
Mean, Variance	$\mathbb{E}[X] = \alpha\beta$	$\mathbb{V}[X] = \alpha\beta^2$

- **GAMMA FUNCTION:** $\Gamma(n) = (n-1)!$ where n is a positive integer. $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$, where $\alpha > 0$

MATH 3342: EXAM 3 FORMULA SHEET

DEVORE 5.3 — SAMPLING DISTRIBUTION OF A STATISTIC (PROCEDURE):

GIVEN: Random sample $\mathbf{X} := (X_1, \ldots, X_n)$ of finite discrete population w/ pmf $p_X(k)$.

TASK: Find the sampling distribution $p_T(k)$ of statistic $T(\mathbf{X})$ of random sample.

(1) Enumerate all meaningful simultaneous values of the X_i's. Use the support of X_1, $\text{Supp}(X_1)$, as guidance.

(2) For each set of meaningful simultaneous values of the X_i's, compute statistic $T(\mathbf{X})$ & joint probability:

$$\mathbb{P}(X_1 = j_1 \cap X_2 = j_2 \cap \cdots \cap X_n = j_n) \overset{iid}{=} p_X(j_1) \cdot p_X(j_2) \cdots p_X(j_n)$$

(3) The support of statistic $T(\mathbf{X})$, $\text{Supp}(T)$, is the set of all values of $T(\mathbf{X})$ attained.

(4) The probability of $T(\mathbf{X})$ being a value is the sum of the joint probabilities corresponding to that value.

DEVORE 5.4

- **NORMAL APPROXIMATION TO THE BINOMIAL:** Let $X \sim \text{Binomial}(n, p)$. Then:

$$X \overset{approx}{\sim} \text{Normal}(\mu = np, \sigma^2 = npq) \implies \mathbb{P}(X \le x) = \text{Bi}(x; n, p) \approx \Phi\left(\frac{x + 0.5 - np}{\sqrt{npq}}\right) \quad \text{(where } q = 1 - p)$$

Requirement for this normal approximation to be valid: $\min\{np, nq\} \ge 10$ (i.e. $np \ge 10$ and $nq \ge 10$)

- **NORMAL APPROXIMATION TO THE POISSON:** Let $X \sim \text{Poisson}(\lambda)$. Then:

$$X \overset{approx}{\sim} \text{Normal}(\mu = \lambda, \sigma^2 = \lambda) \implies \mathbb{P}(X \le x) = \text{Pois}(x; \lambda) \approx \Phi\left(\frac{x + 0.5 - \lambda}{\sqrt{\lambda}}\right)$$

Requirement for this normal approximation to be valid: $\lambda > 20$

DEVORE 7.2

- **LARGE-SAMPLE TWO-SIDED CI FOR POPULATION MEAN:**

Given any population with mean μ. Let $x_1, \ldots, x_n$ be a large sample ($n > 40$) taken from the population.

Then the $100(1-\alpha)\%$ **large-sample CI for** μ is approximately $\left(\overline{x} - z^*_{\alpha/2} \cdot \frac{s}{\sqrt{n}},\ \overline{x} + z^*_{\alpha/2} \cdot \frac{s}{\sqrt{n}}\right)$

- **WILSON SCORE TWO-SIDED CI FOR POPULATION PROPORTION:**

Given any population with proportion p of some "success." Let $x_1, \ldots, x_n$ be a sample taken from the population.

Then the $100(1-\alpha)\%$ **Wilson score CI for** p is approximately

$$\frac{n\widehat{p} + 0.5(z^*_{\alpha/2})^2}{n + (z^*_{\alpha/2})^2} \pm z^*_{\alpha/2} \cdot \frac{\sqrt{n\widehat{p}\widehat{q} + 0.25(z^*_{\alpha/2})^2}}{n + (z^*_{\alpha/2})^2} \quad \text{where} \quad \widehat{p} := \frac{\text{\# Successes in Sample}}{\text{Sample Size}} \quad \text{and} \quad \widehat{q} := 1 - \widehat{p}$$

DEVORE 7.3 — SMALL-SAMPLE TWO-SIDED CI FOR NORMAL POPULATION MEAN μ :

Given a normal population with unknown mean μ and std dev σ.

Let $x_1, \ldots, x_n$ be a small sample taken from the population.

Then the $100(1-\alpha)\%$ **small-sample CI for** μ is $\left(\overline{x} - t^*_{n-1,\alpha/2} \cdot \frac{s}{\sqrt{n}},\ \overline{x} + t^*_{n-1,\alpha/2} \cdot \frac{s}{\sqrt{n}}\right)$

DEVORE 7.4 — SMALL-SAMPLE TWO-SIDED CI FOR NORMAL POPULATION VARIANCE σ^2 :

Given a normal population with unknown mean μ and variance σ^2.

Let $x_1, \ldots, x_n$ be a small sample taken from the population.

Then the $100(1-\alpha)\%$ **small-sample CI for** σ^2 is $\left(\frac{(n-1)s^2}{\chi^{2*}_{n-1,\alpha/2}},\ \frac{(n-1)s^2}{\chi^{2*}_{n-1,1-\alpha/2}}\right)$

The following info is the same for the hypothesis tests that follow on this page:

- Random Sample: $\mathbf{X} := (X_1, X_2, \ldots, X_n)$
- Realized Sample: $\mathbf{x} := (x_1, x_2, \ldots, x_n)$
- Decision Rule:
  If P-value $\leq \alpha$ then reject H_0 in favor of H_A
  If P-value $> \alpha$ then accept H_0 (i.e. fail to reject H_0)

DEVORE 8.2

- **LARGE-SAMPLE z-TEST ABOUT ANY POPULATION MEAN μ (SUMMARY):**

Population:	Any Population with std dev σ unknown
Approx. Test Statistic $W(\mathbf{X};\mu_0)$ Test Statistic Value $W(\mathbf{x};\mu_0)$	$Z = \dfrac{\overline{X} - \mu_0}{S/\sqrt{n}}$ $\quad z = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$
	Sample Size $n > 40$

HYPOTHESIS TEST:	P-VALUE DETERMINATION:
$H_0: \mu = \mu_0$ vs. $H_A: \mu > \mu_0$	P-value $\approx \mathbb{P}(Z \geq z) = 1 - \Phi(z)$
$H_0: \mu = \mu_0$ vs. $H_A: \mu < \mu_0$	P-value $\approx \mathbb{P}(Z \leq z) = \Phi(z)$
$H_0: \mu = \mu_0$ vs. $H_A: \mu \neq \mu_0$	P-value $\approx \mathbb{P}(\lvert Z\rvert \geq \lvert z\rvert) = 2 \cdot [1 - \Phi(\lvert z\rvert)]$

DEVORE 8.3

- **SMALL-SAMPLE t-TEST ABOUT NORMAL POPULATION MEAN μ (SUMMARY):**

Population:	Normal Population with std dev σ unknown
Test Statistic $W(\mathbf{X};\mu_0)$ Test Statistic Value $W(\mathbf{x};\mu_0)$	$T = \dfrac{\overline{X} - \mu_0}{S/\sqrt{n}}$ $\quad t = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$

HYPOTHESIS TEST:	P-VALUE DETERMINATION:
$H_0: \mu = \mu_0$ vs. $H_A: \mu > \mu_0$	P-value $= \mathbb{P}(T \geq t) = 1 - \Phi_t(t; \nu = n-1)$
$H_0: \mu = \mu_0$ vs. $H_A: \mu < \mu_0$	P-value $= \mathbb{P}(T \leq t) = \Phi_t(t; \nu = n-1)$
$H_0: \mu = \mu_0$ vs. $H_A: \mu \neq \mu_0$	P-value $= \mathbb{P}(\lvert T\rvert \geq \lvert t\rvert) = 2 \cdot [1 - \Phi_t(\lvert t\rvert; \nu = n-1)]$

DEVORE 8.4

- **LARGE-SAMPLE z-TEST ABOUT POPULATION PROPORTION p (SUMMARY):**

Population:	Unknown proportion p of "successes"
Test Statistic $W(\mathbf{X};p_0)$ Test Statistic Value $W(\mathbf{x};p_0)$	$Z = \dfrac{(X/n) - p_0}{\sqrt{p_0 q_0 / n}}$ $\quad z = \dfrac{\widehat{p} - p_0}{\sqrt{p_0 q_0 / n}}$
	$\widehat{p} := x/n,\ q_0 := 1 - p_0,\ \min\{np_0, nq_0\} \geq 10$ $x \equiv$ # "Successes" in realized sample $\mathbf{x}$

HYPOTHESIS TEST:	P-VALUE DETERMINATION:
$H_0: p = p_0$ vs. $H_A: p > p_0$	P-value $= \mathbb{P}(Z \geq z) = 1 - \Phi(z)$
$H_0: p = p_0$ vs. $H_A: p < p_0$	P-value $= \mathbb{P}(Z \leq z) = \Phi(z)$
$H_0: p = p_0$ vs. $H_A: p \neq p_0$	P-value $= \mathbb{P}(\lvert Z\rvert \geq \lvert z\rvert) = 2 \cdot [1 - \Phi(\lvert z\rvert)]$

The following info is the same for the hypothesis tests/CI's that follow on this page:

- Random Samples: $\mathbf{X} := (X_1, X_2, \ldots, X_{n_1})$, $\mathbf{Y} := (Y_1, Y_2, \ldots, Y_{n_2})$ where random samples **X** & **Y** are **independent** of one another
- Realized Samples: $\mathbf{x} := (x_1, x_2, \ldots, x_{n_1})$ from 1^{st} population with mean $\overline{x}$ & std dev s_1; $\mathbf{y} := (y_1, y_2, \ldots, y_{n_2})$ from 2^{nd} population with mean $\overline{y}$ & std dev s_2
- Decision Rule: If P-value $\le \alpha$ then reject H_0 in favor of H_A; If P-value $> \alpha$ then accept H_0 (i.e. fail to reject H_0)

DEVORE 9.1

- **LARGE-SAMPLE z-TEST FOR $\mu_1 - \mu_2$ (σ_1, σ_2 UNKNOWN):** (Here $n_1 > 40$, $n_2 > 40$)

Populations:	Any Two Populations with σ_1, σ_2 unknown
Test Statistic Value $W(\mathbf{x}, \mathbf{y}; \delta_0)$	$z = \dfrac{(\overline{x} - \overline{y}) - \delta_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

HYPOTHESIS TEST:	P-VALUE DETERMINATION:
$H_0 : \mu_1 - \mu_2 = \delta_0$ vs. $H_A : \mu_1 - \mu_2 > \delta_0$	P-value $\approx 1 - \Phi(z)$
$H_0 : \mu_1 - \mu_2 = \delta_0$ vs. $H_A : \mu_1 - \mu_2 < \delta_0$	P-value $\approx \Phi(z)$
$H_0 : \mu_1 - \mu_2 = \delta_0$ vs. $H_A : \mu_1 - \mu_2 \neq \delta_0$	P-value $\approx 2 \cdot [1 - \Phi(\lvert z \rvert)]$

- **LARGE-SAMPLE z-CI FOR $\mu_1 - \mu_2$ (σ_1, σ_2 UNKNOWN):** (Here $n_1 > 40$, $n_2 > 40$)

The $100(1-\alpha)\%$ **large-sample CI for** $\mu_1 - \mu_2$ is approximately
$$\left((\overline{x} - \overline{y}) - z^*_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}, \quad (\overline{x} - \overline{y}) + z^*_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \right)$$

DEVORE 9.2

- **SMALL-SAMPLE t-TEST FOR $\mu_1 - \mu_2$ (σ_1, σ_2 UNKNOWN):**

Populations:	Any Two Normal Populations with σ_1, σ_2 unknown
Test Statistic Value $W(\mathbf{x}, \mathbf{y}; \delta_0)$	$t = \dfrac{(\overline{x} - \overline{y}) - \delta_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ $\quad \nu^* = \left\lfloor \dfrac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2} \right)^2}{\dfrac{(s_1^2/n_1)^2}{n_1 - 1} + \dfrac{(s_2^2/n_2)^2}{n_2 - 1}} \right\rfloor$

HYPOTHESIS TEST:	P-VALUE DETERMINATION:
$H_0 : \mu_1 - \mu_2 = \delta_0$ vs. $H_A : \mu_1 - \mu_2 > \delta_0$	P-value $= 1 - \Phi_t(t; \nu^*)$
$H_0 : \mu_1 - \mu_2 = \delta_0$ vs. $H_A : \mu_1 - \mu_2 < \delta_0$	P-value $= \Phi_t(t; \nu^*)$
$H_0 : \mu_1 - \mu_2 = \delta_0$ vs. $H_A : \mu_1 - \mu_2 \neq \delta_0$	P-value $= 2 \cdot [1 - \Phi_t(\lvert t \rvert; \nu^*)]$

- **SMALL-SAMPLE t-CI FOR $\mu_1 - \mu_2$ (σ_1, σ_2 UNKNOWN):**

The $100(1-\alpha)\%$ **small-sample CI for** $\mu_1 - \mu_2$ is
$$\left((\overline{x} - \overline{y}) - t^*_{\nu^*, \alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}, \quad (\overline{x} - \overline{y}) + t^*_{\nu^*, \alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \right)$$

- **ROUNDING (COMPACT NOTATION):** It is convenient to have mathematical notation for **rounding numbers**.

Always Round Up: $\lceil 3 \rceil = 3$ $\lceil 3.1 \rceil = 4$ $\lceil 3.5 \rceil = 4$ $\lceil 3.9 \rceil = 4$

Always Round Down: $\lfloor 3 \rfloor = 3$ $\lfloor 3.1 \rfloor = 3$ $\lfloor 3.5 \rfloor = 3$ $\lfloor 3.9 \rfloor = 3$

- **LARGE-SAMPLE z-TEST FOR $p_1 - p_2$:**

Populations:	Two Populations with proportions p_1, p_2 of some "success"
Realized Samples:	$\mathbf{x} := (x_1, x_2, \ldots, x_{n_1})$ $\quad$ $\mathbf{y} := (y_1, y_2, \ldots, y_{n_2})$ Samples $\mathbf{x}$ & $\mathbf{y}$ are **independent** of one another
Test Statistic Value $W(\mathbf{x}, \mathbf{y})$	$z = \dfrac{\widehat{p}_1 - \widehat{p}_2}{\sqrt{\widehat{p}\widehat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ where $\widehat{p}_1 := X/n_1,\ \widehat{p}_2 := Y/n_2$; $\widehat{p} := (X+Y)/(n_1+n_2)$; $\widehat{q} := 1 - \widehat{p}$; $\widehat{q}_1 := 1 - \widehat{p}_1, \quad \widehat{q}_2 := 1 - \widehat{p}_2$
	$\min\{n_1\widehat{p}_1, n_1\widehat{q}_1, n_2\widehat{p}_2, n_2\widehat{q}_2\} \geq 10$

HYPOTHESIS TEST:	P-VALUE DETERMINATION:
$H_0:\ p_1 - p_2 = 0$ vs. $H_A:\ p_1 - p_2 > 0$	P-value $\approx 1 - \Phi(z)$
$H_0:\ p_1 - p_2 = 0$ vs. $H_A:\ p_1 - p_2 < 0$	P-value $\approx \Phi(z)$
$H_0:\ p_1 - p_2 = 0$ vs. $H_A:\ p_1 - p_2 \neq 0$	P-value $\approx 2 \cdot [1 - \Phi(\lvert z \rvert)]$

Decision Rule:
- If P-value $\leq \alpha$ then reject H_0 in favor of H_A
- If P-value $> \alpha$ then accept H_0 (i.e. fail to reject H_0)

where

$$\widehat{p}_1 := \frac{X}{n_1} \equiv \frac{\#\ \text{Successes in sample } \mathbf{x}}{\text{Sample size of sample } \mathbf{x}}$$

$$\widehat{p}_2 := \frac{Y}{n_2} \equiv \frac{\#\ \text{Successes in sample } \mathbf{y}}{\text{Sample size of sample } \mathbf{y}}$$

$$\widehat{p} := \frac{X+Y}{n_1+n_2} \equiv \frac{\text{Total}\ \#\ \text{Successes in both samples}}{\text{Total Sample Size of both samples}}$$

- **LARGE-SAMPLE z-CI FOR $p_1 - p_2$:**

Given any two populations with proportions p_1 and p_2 of some "success".

Let $\mathbf{x} := (x_1, x_2, \ldots, x_{n_1})$ be a sample taken from the 1^{st} population.

Let $\mathbf{y} := (y_1, y_2, \ldots, y_{n_2})$ be a sample taken from the 2^{nd} population.

Moreover, assume that samples $\mathbf{x}$ & $\mathbf{y}$ are **independent** of one another.

Then the $100(1-\alpha)\%$ **large-sample CI for** $p_1 - p_2$ is approximately

$$\left((\widehat{p}_1 - \widehat{p}_2) - z^*_{\alpha/2} \cdot \sqrt{\frac{\widehat{p}_1\widehat{q}_1}{n_1} + \frac{\widehat{p}_2\widehat{q}_2}{n_2}}, \quad (\widehat{p}_1 - \widehat{p}_2) + z^*_{\alpha/2} \cdot \sqrt{\frac{\widehat{p}_1\widehat{q}_1}{n_1} + \frac{\widehat{p}_2\widehat{q}_2}{n_2}} \right)$$

— OR WRITTEN MORE COMPACTLY —

$$(\widehat{p}_1 - \widehat{p}_2) \pm z^*_{\alpha/2} \cdot \sqrt{\frac{\widehat{p}_1\widehat{q}_1}{n_1} + \frac{\widehat{p}_2\widehat{q}_2}{n_2}}$$

where

$$\widehat{p}_1 := \frac{X}{n_1} \equiv \frac{\#\ \text{Successes in sample } \mathbf{x}}{\text{Sample size of sample } \mathbf{x}}, \quad \widehat{p}_2 := \frac{Y}{n_2} \equiv \frac{\#\ \text{Successes in sample } \mathbf{y}}{\text{Sample size of sample } \mathbf{y}}$$

$$\widehat{q}_1 := 1 - \widehat{p}_1, \quad \widehat{q}_2 := 1 - \widehat{p}_2, \quad \min\{n_1\widehat{p}_1, n_1\widehat{q}_1, n_2\widehat{p}_2, n_2\widehat{q}_2\} \geq 10$$

BINOMIAL CDF $\mathbf{Bi}(x;n,p) := \mathbb{P}(X \leq x)$

n = 10	**Success Probability** (p)										
x	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**
0	0.34868	0.10737	0.05631	0.02825	0.00605	0.00098	0.00010	0.00001	0.00000	0.00000	0.00000
1	0.73610	0.37581	0.24403	0.14931	0.04636	0.01074	0.00168	0.00014	0.00003	0.00000	0.00000
2	0.92981	0.67780	0.52559	0.38278	0.16729	0.05469	0.01229	0.00159	0.00042	0.00008	0.00000
3	0.98720	0.87913	0.77588	0.64961	0.38228	0.17187	0.05476	0.01059	0.00351	0.00086	0.00001
4	0.99837	0.96721	0.92187	0.84973	0.63310	0.37695	0.16624	0.04735	0.01973	0.00637	0.00015
5	0.99985	0.99363	0.98027	0.95265	0.83376	0.62305	0.36690	0.15027	0.07813	0.03279	0.00163
6	0.99999	0.99914	0.99649	0.98941	0.94524	0.82812	0.61772	0.35039	0.22412	0.12087	0.01280
7	1.00000	0.99992	0.99958	0.99841	0.98771	0.94531	0.83271	0.61722	0.47441	0.32220	0.07019
8	1.00000	1.00000	0.99997	0.99986	0.99832	0.98926	0.95364	0.85069	0.75597	0.62419	0.26390
9	1.00000	1.00000	1.00000	0.99999	0.99990	0.99902	0.99395	0.97175	0.94369	0.89263	0.65132
10	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

POISSON CDF $\mathbf{Pois}(x;\lambda) := \mathbb{P}(X \leq x)$

	Parameter (λ)									
x	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0	0.36788	0.13534	0.04979	0.01832	0.00674	0.00248	0.00091	0.00034	0.00012	0.00005
1	0.73576	0.40601	0.19915	0.09158	0.04043	0.01735	0.00730	0.00302	0.00123	0.00050
2	0.91970	0.67668	0.42319	0.23810	0.12465	0.06197	0.02964	0.01375	0.00623	0.00277
3	0.98101	0.85712	0.64723	0.43347	0.26503	0.15120	0.08177	0.04238	0.02123	0.01034
4	0.99634	0.94735	0.81526	0.62884	0.44049	0.28506	0.17299	0.09963	0.05496	0.02925
5	0.99941	0.98344	0.91608	0.78513	0.61596	0.44568	0.30071	0.19124	0.11569	0.06709
6	0.99992	0.99547	0.96649	0.88933	0.76218	0.60630	0.44971	0.31337	0.20678	0.13014
7	0.99999	0.99890	0.98810	0.94887	0.86663	0.74398	0.59871	0.45296	0.32390	0.22022
8	1.00000	0.99976	0.99620	0.97864	0.93191	0.84724	0.72909	0.59255	0.45565	0.33282
9	1.00000	0.99995	0.99890	0.99187	0.96817	0.91608	0.83050	0.71662	0.58741	0.45793
10	1.00000	0.99999	0.99971	0.99716	0.98630	0.95738	0.90148	0.81589	0.70599	0.58304
11	1.00000	1.00000	0.99993	0.99908	0.99455	0.97991	0.94665	0.88808	0.80301	0.69678
12	1.00000	1.00000	0.99998	0.99973	0.99798	0.99117	0.97300	0.93620	0.87577	0.79156
13	1.00000	1.00000	1.00000	0.99992	0.99930	0.99637	0.98719	0.96582	0.92615	0.86446
14	1.00000	1.00000	1.00000	0.99998	0.99977	0.99860	0.99428	0.98274	0.95853	0.91654
15	1.00000	1.00000	1.00000	1.00000	0.99993	0.99949	0.99759	0.99177	0.97796	0.95126

INCOMPLETE GAMMA FUNCTION $\gamma(x;\alpha) := \int_0^x t^{\alpha-1}e^{-t}\,dt$

	Shape Parameter (α)					
x	**0.5**	**1**	**2**	**3**	**4**	**5**
0.5	0.68269	0.39347	0.09020	0.01439	0.00175	0.00017
1	0.84270	0.63212	0.26424	0.08030	0.01899	0.00366
1.5	0.91674	0.77687	0.44217	0.19115	0.06564	0.01858
2	0.95450	0.86466	0.59399	0.32332	0.14288	0.05265
2.5	0.97465	0.91792	0.71270	0.45619	0.24242	0.10882
3	0.98569	0.95021	0.80085	0.57681	0.35277	0.18474

GOSSET'S t CRITICAL VALUES $t^*_{\nu,\alpha/2}$

ν	90% CI ($\alpha/2 = 0.05$)	95% CI ($\alpha/2 = 0.025$)	99% CI ($\alpha/2 = 0.005$)
1	6.314	12.706	63.657
2	2.920	4.303	9.925
3	2.353	3.182	5.841
4	2.132	2.776	4.604
5	2.015	2.571	4.032
6	1.943	2.447	3.707
7	1.895	2.365	3.499
8	1.860	2.306	3.355
9	1.833	2.262	3.250
10	1.812	2.228	3.169
11	1.796	2.201	3.106
12	1.782	2.179	3.055
13	1.771	2.160	3.012
14	1.761	2.145	2.977
15	1.753	2.131	2.947
16	1.746	2.120	2.921
17	1.740	2.110	2.898
18	1.734	2.101	2.878
19	1.729	2.093	2.861
20	1.725	2.086	2.845
21	1.721	2.080	2.831
22	1.717	2.074	2.819
23	1.714	2.069	2.807
24	1.711	2.064	2.797
25	1.708	2.060	2.787
26	1.706	2.056	2.779
27	1.703	2.052	2.771
28	1.701	2.048	2.763
29	1.699	2.045	2.756
30	1.697	2.042	2.750
31	1.696	2.040	2.744
32	1.694	2.037	2.738
33	1.692	2.035	2.733
34	1.691	2.032	2.728
35	1.690	2.030	2.724
36	1.688	2.028	2.719
37	1.687	2.026	2.715
38	1.686	2.024	2.712
39	1.685	2.023	2.708

HELMERT'S χ^2 CRITICAL VALUES

ν	90% CI ($\alpha/2 = 0.05$)		95% CI ($\alpha/2 = 0.025$)		99% CI ($\alpha/2 = 0.005$)	
	$\chi^{2*}_{\nu,1-\alpha/2}$	$\chi^{2*}_{\nu,\alpha/2}$	$\chi^{2*}_{\nu,1-\alpha/2}$	$\chi^{2*}_{\nu,\alpha/2}$	$\chi^{2*}_{\nu,1-\alpha/2}$	$\chi^{2*}_{\nu,\alpha/2}$
1	0.004	3.841	0.001	5.024	0.000	7.879
2	0.103	5.991	0.051	7.378	0.010	10.597
3	0.352	7.815	0.216	9.348	0.072	12.838
4	0.711	9.488	0.484	11.143	0.207	14.860
5	1.145	11.070	0.831	12.833	0.412	16.750
6	1.635	12.592	1.237	14.449	0.676	18.548
7	2.167	14.067	1.690	16.013	0.989	20.278
8	2.733	15.507	2.180	17.535	1.344	21.955
9	3.325	16.919	2.700	19.023	1.735	23.589
10	3.940	18.307	3.247	20.483	2.156	25.188
11	4.575	19.675	3.816	21.920	2.603	26.757
12	5.226	21.026	4.404	23.337	3.074	28.300
13	5.892	22.362	5.009	24.736	3.565	29.819
14	6.571	23.685	5.629	26.119	4.075	31.319
15	7.261	24.996	6.262	27.488	4.601	32.801
16	7.962	26.296	6.908	28.845	5.142	34.267
17	8.672	27.587	7.564	30.191	5.697	35.718
18	9.390	28.869	8.231	31.526	6.265	37.156
19	10.117	30.144	8.907	32.852	6.844	38.582
20	10.851	31.410	9.591	34.170	7.434	39.997
21	11.591	32.671	10.283	35.479	8.034	41.401
22	12.338	33.924	10.982	36.781	8.643	42.796
23	13.091	35.172	11.689	38.076	9.260	44.181
24	13.848	36.415	12.401	39.364	9.886	45.559
25	14.611	37.652	13.120	40.646	10.520	46.928
26	15.379	38.885	13.844	41.923	11.160	48.290
27	16.151	40.113	14.573	43.195	11.808	49.645
28	16.928	41.337	15.308	44.461	12.461	50.993
29	17.708	42.557	16.047	45.722	13.121	52.336
30	18.493	43.773	16.791	46.979	13.787	53.672
31	19.281	44.985	17.539	48.232	14.458	55.003
32	20.072	46.194	18.291	49.480	15.134	56.328
33	20.867	47.400	19.047	50.725	15.815	57.648
34	21.664	48.602	19.806	51.966	16.501	58.964
35	22.465	49.802	20.569	53.203	17.192	60.275
36	23.269	50.998	21.336	54.437	17.887	61.581
37	24.075	52.192	22.106	55.668	18.586	62.883
38	24.884	53.384	22.878	56.896	19.289	64.181

COMMON z CRITICAL VALUES:

	90% CI	95% CI	99% CI
$z^*_{\alpha/2}$	1.64	1.96	2.58

STANDARD NORMAL CDF $\Phi(z) := \mathbb{P}(Z \leq z)$

z	**+0.00**	**+0.01**	**+0.02**	**+0.03**	**+0.04**	**+0.05**	**+0.06**	**+0.07**	**+0.08**	**+0.09**
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997
4.0	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998	0.99998	0.99998	0.99998

GOSSET'S t CDF TABLE FOR $\Phi_t(t;\nu) := \mathbb{P}(T \le t)$ [HERE, $\nu = 1, 2, \ldots, 15$]

	DEGREES OF FREEDOM (ν)														
t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
0.0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.1	0.532	0.535	0.537	0.537	0.538	0.538	0.538	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539
0.2	0.563	0.570	0.573	0.574	0.575	0.576	0.576	0.577	0.577	0.577	0.577	0.578	0.578	0.578	0.578
0.3	0.593	0.604	0.608	0.610	0.612	0.613	0.614	0.614	0.615	0.615	0.615	0.615	0.616	0.616	0.616
0.4	0.621	0.636	0.642	0.645	0.647	0.648	0.649	0.650	0.651	0.651	0.652	0.652	0.652	0.652	0.653
0.5	0.648	0.667	0.674	0.678	0.681	0.683	0.684	0.685	0.685	0.686	0.687	0.687	0.687	0.688	0.688
0.6	0.672	0.695	0.705	0.710	0.713	0.715	0.716	0.717	0.718	0.719	0.720	0.720	0.721	0.721	0.721
0.7	0.694	0.722	0.733	0.739	0.742	0.745	0.747	0.748	0.749	0.750	0.751	0.751	0.752	0.752	0.753
0.8	0.715	0.746	0.759	0.766	0.770	0.773	0.775	0.777	0.778	0.779	0.780	0.780	0.781	0.781	0.782
0.9	0.733	0.768	0.783	0.790	0.795	0.799	0.801	0.803	0.804	0.805	0.806	0.807	0.808	0.808	0.809
1.0	0.750	0.789	0.804	0.813	0.818	0.822	0.825	0.827	0.828	0.830	0.831	0.831	0.832	0.833	0.833
1.1	0.765	0.807	0.824	0.833	0.839	0.843	0.846	0.848	0.850	0.851	0.853	0.854	0.854	0.855	0.856
1.2	0.779	0.823	0.842	0.852	0.858	0.862	0.865	0.868	0.870	0.871	0.872	0.873	0.874	0.875	0.876
1.3	0.791	0.838	0.858	0.868	0.875	0.879	0.883	0.885	0.887	0.889	0.890	0.891	0.892	0.893	0.893
1.4	0.803	0.852	0.872	0.883	0.890	0.894	0.898	0.900	0.902	0.904	0.905	0.907	0.908	0.908	0.909
1.5	0.813	0.864	0.885	0.896	0.903	0.908	0.911	0.914	0.916	0.918	0.919	0.920	0.921	0.922	0.923
1.6	0.822	0.875	0.896	0.908	0.915	0.920	0.923	0.926	0.928	0.930	0.931	0.932	0.933	0.934	0.935
1.7	0.831	0.884	0.906	0.918	0.925	0.930	0.934	0.936	0.938	0.940	0.941	0.943	0.944	0.944	0.945
1.8	0.839	0.893	0.915	0.927	0.934	0.939	0.943	0.945	0.947	0.949	0.950	0.951	0.952	0.953	0.954
1.9	0.846	0.901	0.923	0.935	0.942	0.947	0.950	0.953	0.955	0.957	0.958	0.959	0.960	0.961	0.962
2.0	0.852	0.908	0.930	0.942	0.949	0.954	0.957	0.960	0.962	0.963	0.965	0.966	0.967	0.967	0.968
2.1	0.859	0.915	0.937	0.948	0.955	0.960	0.963	0.966	0.967	0.969	0.970	0.971	0.972	0.973	0.973
2.2	0.864	0.921	0.942	0.954	0.960	0.965	0.968	0.971	0.972	0.974	0.975	0.976	0.977	0.977	0.978
2.3	0.869	0.926	0.948	0.959	0.965	0.969	0.973	0.975	0.977	0.978	0.979	0.980	0.981	0.981	0.982
2.4	0.874	0.931	0.952	0.963	0.969	0.973	0.976	0.978	0.980	0.981	0.982	0.983	0.984	0.985	0.985
2.5	0.879	0.935	0.956	0.967	0.973	0.977	0.980	0.982	0.983	0.984	0.985	0.986	0.987	0.987	0.988
2.6	0.883	0.939	0.960	0.970	0.976	0.980	0.982	0.984	0.986	0.987	0.988	0.988	0.989	0.990	0.990
2.7	0.887	0.943	0.963	0.973	0.979	0.982	0.985	0.986	0.988	0.989	0.990	0.990	0.991	0.991	0.992
2.8	0.891	0.946	0.966	0.976	0.981	0.984	0.987	0.988	0.990	0.991	0.991	0.992	0.992	0.993	0.993
2.9	0.894	0.949	0.969	0.978	0.983	0.986	0.989	0.990	0.991	0.992	0.993	0.993	0.994	0.994	0.995
3.0	0.898	0.952	0.971	0.980	0.985	0.988	0.990	0.991	0.993	0.993	0.994	0.994	0.995	0.995	0.996
3.1	0.901	0.955	0.973	0.982	0.987	0.989	0.991	0.993	0.994	0.994	0.995	0.995	0.996	0.996	0.996
3.2	0.904	0.957	0.975	0.984	0.988	0.991	0.992	0.994	0.995	0.995	0.996	0.996	0.997	0.997	0.997
3.3	0.906	0.960	0.977	0.985	0.989	0.992	0.993	0.995	0.995	0.996	0.996	0.997	0.997	0.997	0.998
3.4	0.909	0.962	0.979	0.986	0.990	0.993	0.994	0.995	0.996	0.997	0.997	0.997	0.998	0.998	0.998
3.5	0.911	0.964	0.980	0.988	0.991	0.994	0.995	0.996	0.997	0.997	0.998	0.998	0.998	0.998	0.998
3.6	0.914	0.965	0.982	0.989	0.992	0.994	0.996	0.997	0.997	0.998	0.998	0.998	0.998	0.999	0.999
3.7	0.916	0.967	0.983	0.990	0.993	0.995	0.996	0.997	0.998	0.998	0.998	0.998	0.999	0.999	0.999
3.8	0.918	0.969	0.984	0.990	0.994	0.996	0.997	0.997	0.998	0.998	0.999	0.999	0.999	0.999	0.999
3.9	0.920	0.970	0.985	0.991	0.994	0.996	0.997	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999
4.0	0.922	0.971	0.986	0.992	0.995	0.996	0.997	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999