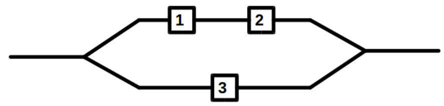
EX 2.1.2: Consider the following system of connected components shown below.

Components 1 & 2 are connected in series with component 3 connected to them in parallel.

For the entire system to function, either components 1 & 2 must function or component 3 must function.

The experiment consists of observing each component's condition and labeling it as either (S)uccess or (F)ailure.

So, for example, the outcome for components 1 & 2 functioning but 3 not functioning would be denoted as SSF.



(a) Determine the sample space Ω for the experiment.

$\Omega =$ (The set of all possible outcomes) $= \boxed{\{FFF,\ FFS,\ FSF,\ FSS,\ SFF,\ SSF,\ SFS,\ SSS\}}$

(b) Find the outcomes for event $A \equiv$ "Component 1 functions".

$A = \boxed{\{SFF,\ SSF,\ SFS,\ SSS\}}$ OR $A = \boxed{\{(S,F,F),\ (S,S,F),\ (S,F,S),\ (S,S,S)\}}$

(c) Find the outcomes for event $B \equiv$ "Component 2 functions".

$B = \boxed{\{FSF,\ FSS,\ SSF,\ SSS\}}$ OR $B = \boxed{\{(F,S,F),\ (F,S,S),\ (S,S,F),\ (S,S,S)\}}$

(d) Find the outcomes for event $C \equiv$ "Component 3 functions".

$C = \boxed{\{FFS, FSS, SFS, SSS\}}$ OR $C = \boxed{\{(F,F,S),\ (F,S,S),\ (S,F,S),\ (S,S,S)\}}$

(e) Find the outcomes for event $D \equiv$ "Exactly two components function".

$D = \boxed{\{FSS,\ SSF,\ SFS\}}$ OR $D = \boxed{\{(F,S,S),\ (S,S,F),\ (S,F,S)\}}$

(f) List the outcomes for event $E \equiv$ "At most two components function".

$E = \boxed{\{FFF,\ FFS,\ FSF,\ FSS,\ SFF,\ SSF,\ SFS\}}$

(g) List the outcomes for event $F \equiv$ "The entire system functions".

$F = \boxed{\{SSF,\ FFS,\ SSS,\ SFS,\ FSS\}}$

(h) Compute F^c. What does event F^c represent in the context of the experiment?

$F^c =$ (The set of all outcomes not in F) $= \boxed{\{FFF,\ FSF,\ SFF\}}$

$F^c \equiv$ The entire system does not function

(i) Compute $A \cup B$. What does event $A \cup B$ represent in the context of the experiment?

$A \cup B =$ (The set of all outcomes in A or B) $= \boxed{\{FSF,\ FSS,\ SFF,\ SFS,\ SSF,\ SSS\}}$

$A \cup B \equiv$ Component 1 functions or Component 2 functions

(j) Compute $A \cap B$. What does event $A \cap B$ represent in the context of the experiment?

$A \cap B =$ (The set of all outcomes in A and B) $= \boxed{\{SSF,\ SSS\}}$

$A \cap B \equiv$ Component 1 functions and Component 2 functions

(k) Compute $(A \cup B)^c$. What does event $(A \cup B)^c$ represent in the context of the experiment?

$(A \cup B)^c =$ (The set of all outcomes neither in A nor B) $= \boxed{\{FFF,\ FFS\}}$

$(A \cup B)^c \equiv$ Neither Component 1 nor Component 2 functions

(l) Compute $A \cap B^c \cap C$. What does event $A \cap B^c \cap C$ represent in the context of the experiment?

$A \cap B^c \cap C = (A \cap B^c) \cap C = (\ \{SFF,\ SSF,\ SFS,\ SSS\} \cap \{FFF,\ FFS,\ SFF,\ SFS\}\ ) \cap \{FFS,\ FSS,\ SFS,\ SSS\}$

$= \{SFF,\ SFS\} \cap \{FFS,\ FSS,\ SFS,\ SSS\} = \boxed{\{SFS\}}$

$A \cap B^c \cap C \equiv$ Component 1 functions and Component 2 does not function and Component 3 functions