PROBABILITY: SETS, SAMPLE SPACES, EVENTS [DEVORE 2.1]

- **THE NEED FOR PROBABILITY:** Life is full of processes whose outcome cannot be predicted ahead of time.
- **SETS & SEQUENCES (DEFINITION):** (See 2.1 Slides for examples of sets & sequences)

  A **set** is a possibly infinite, unique, unordered list of elements.

  A **sequence** is a possibly infinite, varying, arbitrary, ordered list of elements.

  Sets are written as comma-separated lists enclosed by braces.

  Sequences are written as comma-separated lists enclosed by parentheses.

  Finite sequences can also be written as lists w/o commas and parentheses.

  A set can contain sets or sequences: $\{(1,2), \{a,b\}, (a,c), bc, \{1,2\}\}$

  Be careful with number sequences: $(1,2,3,4) = 1234$ BUT $(12,34) \neq 1234$
- **BASIC TERMINOLOGY:**

  A **random process** is a process whose outcome cannot be predicted a priori.

  An **experiment** is any observation of a random process.

  The **outcomes** $\omega_1, \omega_2, \ldots$ of an experiment are the different possible results.

  The **sample space** Ω of an experiment is the set of all possible outcomes.

  An **event** E is a subset of the sample space: $E \subseteq \Omega$
- **THE EMPTY SET:** The **empty set**, $\emptyset$, is the event with no outcomes in it.

  The empty set is always a subset of the sample space: $\emptyset \subseteq \Omega$
- **GAMES OF CHANCE:** Some experiments involve **games of chance** (flipping coins, rolling dice, picking cards, ...)

  A **fair coin** has an equal chance of either side showing up upon flipping.

  A **fair die** has an equal chance of any of its sides showing up upon rolling.

  Games of chance requiring a deck of cards (blackjack, poker, ...) will not be considered in this course.
- **UNIONS, INTERSECTIONS, COMPLEMENTS OF EVENTS (DEFINTIONS):**

  The **union** of events E & F, $E \cup F$, is the set of all outcomes in E or F.

  The **intersection** of events E & F, $E \cap F$, is the set of all outcomes in E and F.

  The **complement** of event E, E^c, is the set of all outcomes not in E.

  The **complement** of event $E \cup F$, $(E \cup F)^c$, is the set of all outcomes neither in E nor in F.
- **UNIONS, INTERSECTIONS, COMPLEMENTS OF EVENTS (PROERTIES):**

  Let $A, B, C \subseteq \Omega$ be events of some experiment. Then:

(S1)	$(A^c)^c = A$	Complementing a Set twice is itself
(S2)	$A \cup B = B \cup A$	Commutativity of Unions
(S3)	$A \cap B = B \cap A$	Commutativity of Intersections
(S4)	$(A \cup B) \cup C = A \cup (B \cup C)$	Associativity of Unions
(S5)	$(A \cap B) \cap C = A \cap (B \cap C)$	Associativity of Intersections
(S6)	$(A \cup B)^c = A^c \cap B^c$	De Morgan's Law
(S7)	$(A \cap B)^c = A^c \cup B^c$	De Morgan's Law
(S8)	$A \cup \emptyset = A, \quad A \cap \emptyset = \emptyset$	Unions/Intersections with Empty Set
(S9)	$\Omega^c = \emptyset, \quad \emptyset^c = \Omega$	Sample Space & Empty Set are related
(S10)	$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$	Intersection Distributes over Union
(S11)	$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$	Union Distributes over Intersection

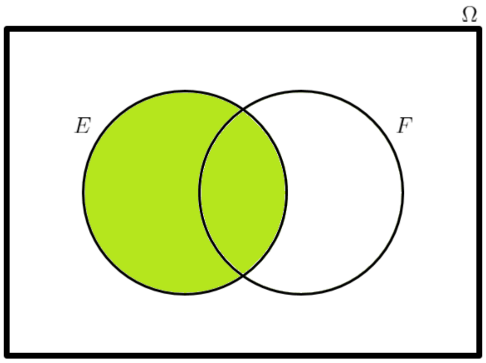
- **MUTUAL EXCLUSIVITY OF TWO EVENTS:**

  Events E, F are **mutually exclusive** (or **disjoint**) if they have no outcomes in common: $E \cap F = \emptyset$
- **MUTUAL EXCLUSIVITY OF THREE EVENTS:**

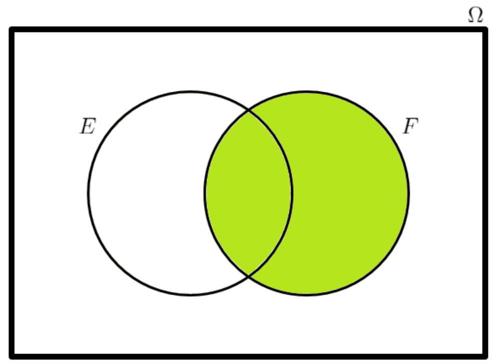
  Events E, F, G are **mutually exclusive** (or **pairwise disjoint**) if they have no outcomes in common:

$$E \cap F = \emptyset \quad \text{and} \quad E \cap G = \emptyset \quad \text{and} \quad F \cap G = \emptyset$$

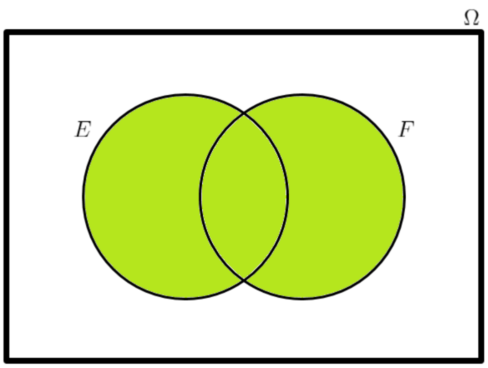
PROBABILITY: VENN DIAGRAMS OF EVENTS [DEVORE 2.1]



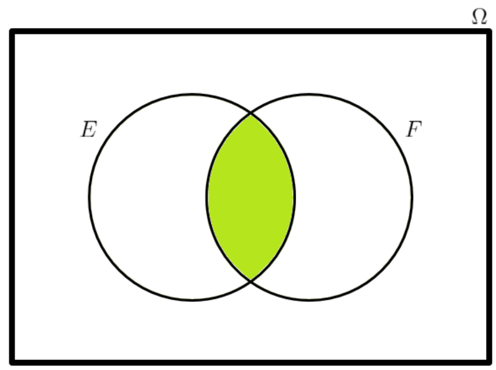
E



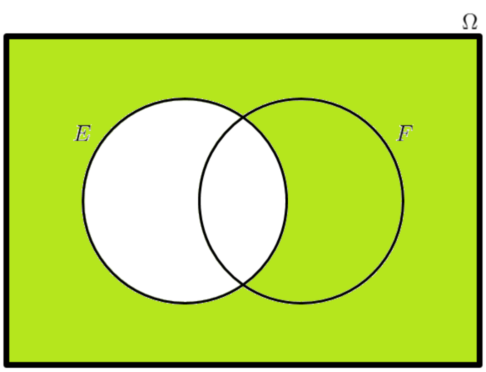
F



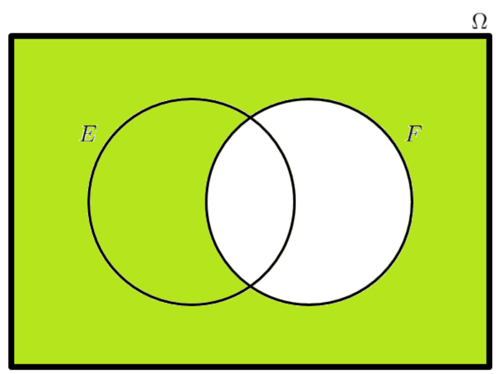
$E \cup F$



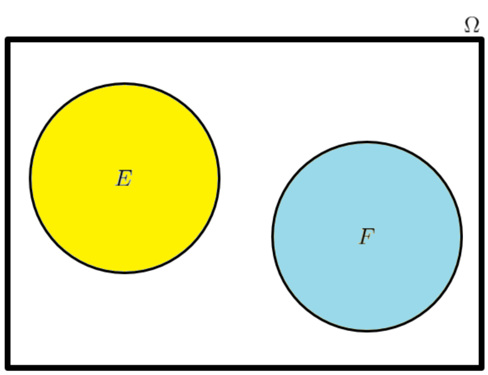
$E \cap F$



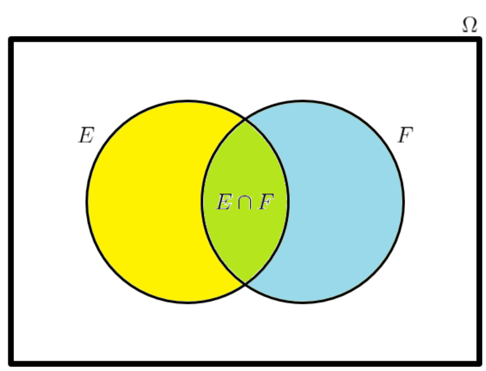
E^c



F^c



Mutually Exclusive Events



Not Mutually Exclusive

EX 2.1.1: Two fair 3-sided dice are rolled. Each die has the numbers 1,2,3 on its faces.

(a) Determine the sample space Ω for the experiment.

(b) Find the outcomes for event $E_1 \equiv$ "1^{st} die shows 2 and 2^{nd} die shows 1".

(c) Find the outcomes for event $E_2 \equiv$ "One die shows 2 and the other die shows 1".

(d) List the outcomes for event $E_3 \equiv$ "Both dice show 3".

(e) List the outcomes for event $E_4 \equiv$ "The two dice total 4".

(f) List the outcomes for event $E_5 \equiv$ "The two dice total an even number".

(g) Compute E_5^c. What does event E_5^c represent in the context of the experiment?

(h) Compute $E_3 \cup E_4$. What does event $E_3 \cup E_4$ represent in the context of the experiment?

(i) Compute $E_3 \cap E_5$. Interpret the result in the context of the experiment.

(j) Compute $E_3 \cap E_4$. Interpret the result in the context of the experiment.

(k) Compute $(E_3 \cup E_4)^c$. What does event $(E_3 \cup E_4)^c$ represent in the context of the experiment?

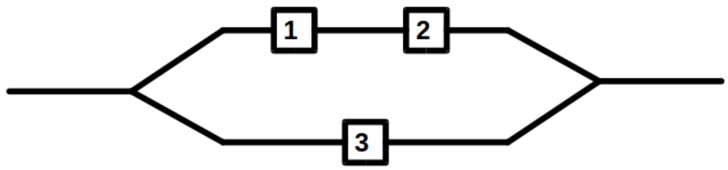
EX 2.1.2: Consider the following system of connected components shown below.

Components 1 & 2 are connected in series with component 3 connected to them in parallel.

For the entire system to function, either components 1 & 2 must function or component 3 must function.

The experiment consists of observing each component's condition and labeling it as either (S)uccess or (F)ailure.

So, for example, the outcome for components 1 & 2 functioning but 3 not functioning would be denoted as SSF.



(a) Determine the sample space Ω for the experiment.

(b) Find the outcomes for event $A \equiv$ "Component 1 functions".

(c) Find the outcomes for event $B \equiv$ "Component 2 functions".

(d) Find the outcomes for event $C \equiv$ "Component 3 functions".

(e) Find the outcomes for event $D \equiv$ "Exactly two components function".

(f) List the outcomes for event $E \equiv$ "At most two components function".

(g) List the outcomes for event $F \equiv$ "The entire system functions".

(h) Compute F^c. What does event F^c represent in the context of the experiment?

(i) Compute $A \cup B$. What does event $A \cup B$ represent in the context of the experiment?

(j) Compute $A \cap B$. What does event $A \cap B$ represent in the context of the experiment?

(k) Compute $(A \cup B)^c$. What does event $(A \cup B)^c$ represent in the context of the experiment?

(l) Compute $A \cap B^c \cap C$. What does event $A \cap B^c \cap C$ represent in the context of the experiment?