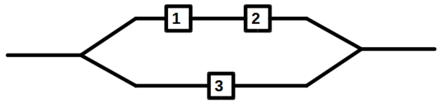
EX 2.2.1: Consider the following system of connected components shown below.

Components 1 & 2 are connected in series with component 3 connected to them in parallel.

For the entire system to function, either components 1 & 2 must function or component 3 must function.

The experiment consists of observing each component's condition and labeling it as either (S)uccess or (F)ailure.

So, for example, the outcome for components 1 & 2 functioning but 3 not functioning would be denoted as SSF.



(a) Determine the sample space Ω for the experiment.

$\Omega = (\text{The set of all possible outcomes}) = \boxed{\{FFF,\ FFS,\ FSF,\ FSS,\ SFF,\ SSF,\ SFS,\ SSS\}}$

(b) Find the probability that Component 1 functions.

Let event $E_1 \equiv$ "Component 1 functions" $= \{SFF,\ SSF,\ SFS,\ SSS\}$

Then $\mathbb{P}(E_1) = \dfrac{|E_1|}{|\Omega|} = \dfrac{(\#\text{ outomes in } E_1)}{(\#\text{ outcomes in } \Omega)} = \dfrac{4}{8} = \dfrac{1}{2} = \boxed{0.5}$

(c) Find the probability that Component 2 functions.

Let event $E_2 \equiv$ "Component 2 functions" $= \{FSF,\ FSS,\ SSF,\ SSS\}$

Then $\mathbb{P}(E_2) = \dfrac{|E_2|}{|\Omega|} = \dfrac{(\#\text{ outomes in } E_2)}{(\#\text{ outcomes in } \Omega)} = \dfrac{4}{8} = \dfrac{1}{2} = \boxed{0.5}$

(d) Find the probability that Component 3 functions.

Let event $E_3 \equiv$ "Component 3 functions" $= \{FFS,\ FSS,\ SFS,\ SSS\}$

Then $\mathbb{P}(E_3) = \dfrac{|E_3|}{|\Omega|} = \dfrac{(\#\text{ outomes in } E_3)}{(\#\text{ outcomes in } \Omega)} = \dfrac{4}{8} = \dfrac{1}{2} = \boxed{0.5}$

(e) Find the probability that the entire system functions.

Let event $E_4 \equiv$ "Entire system functions" $= \{SSF,\ FFS,\ SFS,\ FSS,\ SSS\}$

Then $\mathbb{P}(E_4) = \dfrac{|E_4|}{|\Omega|} = \dfrac{(\#\text{ outomes in } E_4)}{(\#\text{ outcomes in } \Omega)} = \dfrac{5}{8} = \boxed{0.625}$

(f) Find the probability that the entire system does not function.

$\mathbb{P}(E_4^c) = 1 - \mathbb{P}(E_4) = 1 - 0.625 = \boxed{0.375}$

(g) Find the probability that Component 1 functions and Component 2 functions.

Unfortunately, there's no direct formula for an intersection, so compute its probability via measures:

$\mathbb{P}(E_1 \cap E_2) = \dfrac{|E_1 \cap E_2|}{|\Omega|} = \dfrac{|\{SSF,\ SSS\}|}{|\Omega|} = \dfrac{2}{8} = \dfrac{1}{4} = \boxed{0.25}$

(h) Find the probability that Component 1 functions or Component 2 functions.

$\mathbb{P}(E_1 \cup E_2) = \mathbb{P}(E_1) + \mathbb{P}(E_2) - \mathbb{P}(E_1 \cap E_2) = 0.5 + 0.5 - 0.25 = \boxed{0.75}$

(i) Find the probability that neither Component 1 nor Component 2 function.

$\mathbb{P}\left[(E_1 \cup E_2)^c\right] = 1 - \mathbb{P}(E_1 \cup E_2) = 1 - 0.75 = \boxed{0.25}$

(j) Find the probability that Components 1 & 3 both function but not Component 2.

Unfortunately, there's no direct formula for an intersection, so compute its probability via measures:

$$E_1 \cap E_2^c \cap E_3 = (E_1 \cap E_2^c) \cap E_3 = (\{SFF, SSF, SFS, SSS\} \cap \{FFF, FFS, FSF, SFF, SFS\}) \cap \{FFS, FSS, SFS, SSS\}$$
$$= \{SFS\} \cap \{FFS, FSS, SFS, SSS\} = \{SFS\}$$

$\mathbb{P}(E_1 \cap E_2^c \cap E_3) = \dfrac{|E_1 \cap E_2^c \cap E_3|}{|\Omega|} = \dfrac{|\{SFS\}|}{|\Omega|} = \dfrac{1}{8} = \boxed{0.125}$

EX 2.2.2: In a high school with 200 students:

80 students are taking French
60 students are taking German
75 students are taking Spanish
18 students are taking French & German
13 students are taking German & Spanish
23 students are taking French & Spanish
4 students are taking French, German & Spanish

How many students are <u>not</u> taking French, German or Spanish?

First, realize that the experiment is determining which foreign language course(s), if any, a student takes.

Second, realize that the sample space Ω for the experiment is the 200 students.

<u>METHOD 1:</u> DIRECTLY APPLY PRINCIPLE OF INCLUSION-EXCLUSION

$$\mathbb{P}(F \cup G \cup S) = \mathbb{P}(F) + \mathbb{P}(G) + \mathbb{P}(S) - \mathbb{P}(F \cap G) - \mathbb{P}(G \cap S) - \mathbb{P}(F \cap S) + \mathbb{P}(F \cap G \cap S)$$

Multiply both sides by $|\Omega|$:

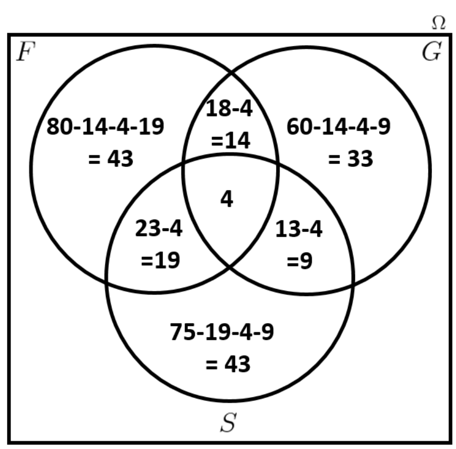
$$\begin{aligned} |F \cup G \cup S| &= |F| + |G| + |S| - |F \cap G| - |G \cap S| - |F \cap S| + |F \cap G \cap S| \\ &= 80 + 60 + 75 - 18 - 13 - 23 + 4 = 165 \end{aligned}$$

$\therefore$ (# Students not taking French, German or Spanish) $= |(F \cup G \cup S)^c| = |\Omega| - |F \cup G \cup S| = 200 - 165 = \boxed{35}$

<u>METHOD 2:</u> CONSTRUCT & POPULATE A CLEVER VENN DIAGRAM

(THIS IS EXTREMELY USEFUL IF INCLUSION-EXCLUSION FORMULA IS <u>NOT</u> PROVIDED)

Determine the measure of each disjoint piece of $F \cup G \cup S$ using a Venn Diagram working inside outward:



Now compute the measure of $F \cup G \cup S$ by simply adding the found disjoint measures:

$$|F \cup G \cup S| = 43 + 14 + 33 + 19 + 4 + 9 + 43 = 165$$

$\therefore$ (# Students not taking French, German or Spanish) $= |(F \cup G \cup S)^c| = |\Omega| - |F \cup G \cup S| = 200 - 165 = \boxed{35}$