PROBABILITY: AXIOMS, PROPERTIES, INTERPRETATIONS [DEVORE 2.2]

- **CHAIN OF UNIONS:** $\bigcup_{k=1}^{n} A_k = A_1 \cup A_2 \cup \cdots \cup A_{n-1} \cup A_n$ $\bigcup_{k=1}^{\infty} A_k = A_1 \cup A_2 \cup A_3 \cup \cdots$
- **PROBABILITY AXIOMS:** Let $E \subseteq \Omega$ be an event from sample space Ω of an experiment.

  Let $E_1, E_2, E_3, \ldots$ be an infinite collection of pairwise disjoint events. Then:

$(A1)$	$\mathbb{P}(E) \geq 0$	Probablity of an Event is Non-Negative
$(A2)$	$\mathbb{P}(\Omega) = 1$	Probability of Sample Space is always One
$(A3)$	$\mathbb{P}\left(\bigcup_{k=1}^{\infty} E_k\right) = \sum_{k=1}^{\infty} \mathbb{P}(E_k)$	Probabilty of Infinite Disjoint Union is a Sum

- **PROPERTIES OF PROBABILITY:** Let $E, F \subseteq \Omega$ be two events from sample space Ω of an experiment.

  Let $E_1, E_2, \ldots, E_n$ be a finite collection of pairwise disjoint events. Then:

$(P1)$	$\mathbb{P}(\emptyset) = 0$	Probability of Empty Set is always Zero
$(P2)$	$\mathbb{P}\left(\bigcup_{k=1}^{n} E_k\right) = \sum_{k=1}^{n} \mathbb{P}(E_k)$	Probabilty of Finite Disjoint Union is a Sum
$(P3)$	$\mathbb{P}(E^c) = 1 - \mathbb{P}(E)$	Probability of Complement
$(P4)$	$\mathbb{P}(E) \leq 1$	Probability is never greater than One
$(P5)$	$\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F) - \mathbb{P}(E \cap F)$	Probabilty of Union

- **PRINCIPLE OF INCLUSION-EXCLUSION:**

  Let $E, F, G \subseteq \Omega$ be three events from sample space Ω of an experiment. Then:

$$\mathbb{P}(E \cup F \cup G) = \mathbb{P}(E) + \mathbb{P}(F) + \mathbb{P}(G) - \mathbb{P}(E \cap F) - \mathbb{P}(E \cap G) - \mathbb{P}(F \cap G) + \mathbb{P}(E \cap F \cap G)$$

- **DEEP INTERPRETATION OF PROBABILITY:**

  All the above axioms & properties do not give a complete interpretation of probability!!

  The most intuitive interpretation is to treat probability as a relative frequency:

- **INTERPRETATION OF PROBABILITY:**

$$
\begin{array}{rcccccl}
 & & \mathbb{P}(E) & = & 0 & \Longrightarrow & \text{Event } E \text{ is impossible} \\
0 & < & \mathbb{P}(E) & < & 0.50 & \Longrightarrow & \text{Event } E \text{ is not likely to occur} \\
 & & \mathbb{P}(E) & = & 0.50 & \Longrightarrow & \text{Event } E \text{ has 50-50 chance of occurring} \\
0.50 & < & \mathbb{P}(E) & < & 1 & \Longrightarrow & \text{Event } E \text{ is likely to occur} \\
 & & \mathbb{P}(E) & = & 1 & \Longrightarrow & \text{Event } E \text{ is certain to occur}
\end{array}
$$

  - "There's a 30% chance of snow tomorrow." [$\mathbb{P}$(Snow tomorrow) $= 0.30$]
  - "25% of adults get seven hours of sleep." [$\mathbb{P}$(7 hrs of sleep) $= 0.25$]
  - "All dogs play fetch." [$\mathbb{P}$(Playing fetch) $= 1$]
  - "None of my cats catch mice." [$\mathbb{P}$(Catch mice) $= 0$]

MEASURE OF A SET:

The **measure** of a **countable set** is $|E| :=$ (# of elements in E)

The **measure** of a **1D set** is $|E| :=$ (Length of curve E)

The **measure** of a **2D set** is $|E| :=$ (Area of region E)

The **measure** of a **3D set** is $|E| :=$ (Volume of solid E)

The **measure** of the **empty set** is defined to be zero: $|\emptyset| := 0$

PROBABILITY: EQUALLY LIKELY OUTCOMES:

Let Ω be the sample space of an experiment with **equally likely outcomes**. Let E be an event of the experiment.

Then the **probability** of event E occurring is defined as: $\mathbb{P}(E) = \dfrac{|E|}{|\Omega|}$

i.e. The probability is the proportion of outcomes that comprise the event.

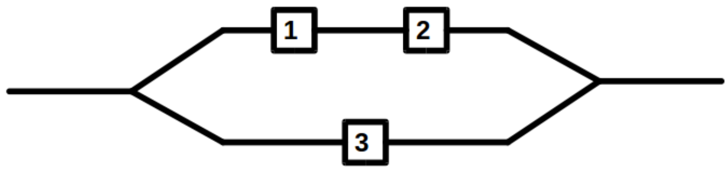
EX 2.2.1: Consider the following system of connected components shown below.

Components 1 & 2 are connected in series with component 3 connected to them in parallel.

For the entire system to function, either components 1 & 2 must function or component 3 must function.

The experiment consists of observing each component's condition and labeling it as either (S)uccess or (F)ailure.

So, for example, the outcome for components 1 & 2 functioning but 3 not functioning would be denoted as SSF.



(a) Determine the sample space Ω for the experiment.

(b) Find the probability that Component 1 functions.

(c) Find the probability that Component 2 functions.

(d) Find the probability that Component 3 functions.

(e) Find the probability that the entire system functions.

(f) Find the probability that the entire system does not function.

(g) Find the probability that Component 1 functions and Component 2 functions.

(h) Find the probability that Component 1 functions or Component 2 functions.

(i) Find the probability that neither Component 1 nor Component 2 function.

(j) Find the probability that Components 1 & 3 both function but not Component 2.

EX 2.2.2: In a high school with 200 students:

80 students are taking French
60 students are taking German
75 students are taking Spanish
18 students are taking French & German
13 students are taking German & Spanish
23 students are taking French & Spanish
4 students are taking French, German & Spanish

How many students are not taking French, German or Spanish?