EX 2.3.1: A library shelf has books from three subjects: 4 math books, 9 art books, and 5 chess books.

(a) If three books are randomly selected such that the subjects all differ, how many selections are possible?

This is a 3-stage experiment: Randomly select a math book, and then select an art book, and then select a chess book.

$\therefore$ (# Selections possible) = (# math books) × (# art books) × (# chess books) = $(4)(9)(5) = \boxed{180}$

(b) If two books are randomly selected such that the subjects differ, how many selections are possible?

This is a 2-stage experiment: Randomly select a book, and then select a book of a different subject.

Consider the events: $E_1 \equiv$ (Math book & Art book), $E_2 \equiv$ (Math book & Chess book), $E_3 \equiv$ (Art book & Chess book)

Then, an example outcome from each event is: $(M_2, A_8) \in E_1$, $(M_2, C_3) \in E_2$, $(A_7, C_2) \in E_3$

where $M_2 \equiv (2^{nd}$ math book), $A_8 \equiv (8^{th}$ art book), $C_3 \equiv (3^{rd}$ chess book), etc...

Moreover, events E_1, E_2, E_3 are all <u>pairwise disjoint</u> since each intersection contains no ordered pairs in common:

$$E_1 \cap E_2 = \emptyset, \quad E_1 \cap E_3 = \emptyset, \quad E_2 \cap E_3 = \emptyset$$

$\therefore$ (# Selections possible) $= |E_1 \cup E_2 \cup E_3| = |E_1| + |E_2| + |E_3| = (4)(9) + (9)(5) + (4)(5) = \boxed{101}$

EX 2.3.3: How many 5-letter words can be formed from the English alphabet? (The words need not make sense.)

This a 5-stage experiment:

Select a letter for 1^{st} position, and then select a letter for 2^{nd} position, ..., and then select a letter for 5^{th} position.

IF LETTERS CAN BE REUSED (i.e. DUPLICATES/REPETITION ALLOWED):

$\therefore$ (# Words) $= (26)(26)(26)(26)(26) = 26^5 = \boxed{11,881,376}$

IF LETTERS CANNOT BE REUSED (i.e. DUPLICATES/REPETITION NOT ALLOWED):

$\therefore$ (# Words) $= (26)(25)(24)(23)(22) = \boxed{7,893,600}$

—- OR —-

$$P_5^{26} = \frac{26!}{(26-5)!} = \frac{26!}{21!} = \frac{(26)(25)(24)(23)(22)(21)(20)\cdots(3)(2)(1)}{(21)(20)\cdots(3)(2)(1)} = (26)(25)(24)(23)(22) = \boxed{7,893,600}$$

EX 2.3.4: How many ways can one select ten apples from a bin of 25 identical apples?

Since the apples are all <u>identical</u>, **order does <u>not</u> matter**!

Therefore, the counting requires a **combination**:

$$(\#\text{ Ways}) = \binom{25}{10} = \frac{25!}{10!(25-10)!} = \frac{25!}{10!15!} = \frac{(25)(24)(23)(22)(21)(20)(19)(18)(17)(16)}{(10)(9)(8)(7)(6)(5)(4)(3)(2)(1)} = \boxed{3,268,760}$$

EX 2.3.6: How many ways can one arrange the letters of the word $MISSISSIPPI$?

In this context, **order <u>does</u> matter**! Thus, **permutation(s)** are necessary.

Moreover, the permutations have same # elements as given word. Thus, the permutations are **full permutations**.

Hence, the total full permutations (11-permutations) of $MISSISSIPPI$ is $P_{11}^{11} = \frac{11!}{(11-11)!} = \frac{11!}{0!} = \frac{11!}{1} = 11!$

But, <u>some of the letters are identical!!</u> Thus, irrelevant permutations must be divided out of the total permutations:

IDENTICAL LETTER(S)	# FULL PERMUTATIONS
The one M	1!
The four I's	4!
The four S's	4!
The two P's	2!

$$\therefore (\#\text{ Ways}) = \frac{11!}{1!4!4!2!} = \frac{(11)(10)(9)(8)(7)(6)(5)(4)(3)(2)(1)}{[(1)][(4)(3)(2)(1)][(4)(3)(2)(1)][(2)(1)]} = (11)(10)(9)(7)(5) = \boxed{34,650}$$