PROBABILITY: TUPLES, PERMUTATIONS, COMBINATIONS [DEVORE 2.3]

- **COUNTING TUPLES:** Given a set of k-tuples such that:

$$\begin{array}{lcll} \text{there are} & n_1 & \text{possible choices for the} & 1^{st} \text{ element,} \\ \text{there are} & n_2 & \text{possible choices for the} & 2^{nd} \text{ element,} \\ \text{there are} & n_3 & \text{possible choices for the} & 3^{rd} \text{ element,} \\ \vdots & \vdots & \vdots & \vdots \\ \text{there are} & n_{k-1} & \text{possible choices for the} & (k-1)^{st} \text{ element,} \\ \text{there are} & n_k & \text{possible choices for the} & k^{th} \text{ element.} \end{array}$$

  Then the # of possible **k-tuples** is: $\displaystyle\prod_{j=1}^{k} n_j = n_1 n_2 n_3 \cdots n_{k-1} n_k$

- **FACTORIALS:** Let k be a non-negative integer. Then: $k! := k(k-1)(k-2)\cdots(4)(3)(2)(1)$ $\qquad 0! := 1$

$$\begin{array}{rcccl} 0! & = & 1 & = & 1 \\ 1! & = & (1) & = & 1 \\ 2! & = & (2)(1) & = & 2 \\ 3! & = & (3)(2)(1) & = & 6 \\ 4! & = & (4)(3)(2)(1) & = & 24 \\ 5! & = & (5)(4)(3)(2)(1) & = & 120 \\ \vdots & & \vdots & & \vdots \end{array}$$

- **COUNTING PERMUTATIONS:** Let k, n be integers such that $0 \le k \le n$.

  Then the # of **k-permutations of an n-element set** is: $\qquad P_k^n := \dfrac{n!}{(n-k)!}$

- **FACTORIALS IN THE CONTEXT OF PERMUTATIONS:** $n!$ is the # of n-permutations of an n-element set.

- **COUNTING COMBINATIONS:** Let k, n be integers such that $0 \le k \le n$.

  Then the # of **k-combinations of an n-element set** is: $\qquad \dbinom{n}{k} := \dfrac{n!}{k!(n-k)!}$

EX 2.3.1: A library shelf has books from three subjects: 4 math books, 9 art books, and 5 chess books.

(a) If three books are randomly selected such that the subjects all differ, how many selections are possible?

(b) If two books are randomly selected such that the subjects differ, how many selections are possible?

EX 2.3.2: Thirty runners compete in a 10K race with three medals. How many ways can the medals be awarded?

EX 2.3.3: How many 5-letter words can be formed from the English alphabet? (The words need not make sense.)

EX 2.3.4: How many ways can one select ten apples from a bin of 25 identical apples?

EX 2.3.5: How many ways are there to appoint a committee of 4 women & 3 men from a group of 20 women & 25 men?

EX 2.3.6: How many ways can one arrange the letters of the word $MISSISSIPPI$?