PROBABILITY: INDEPENDENCE OF EVENTS [DEVORE 2.5]

- **INDEPENDENCE OF TWO EVENTS (DEFINITION):**

  Let $E, F \subseteq \Omega$ be two events from sample space Ω of an experiment.

  Then E & F are **independent** if $\mathbb{P}(E|F) = \mathbb{P}(E)$.

- **INDEPENDENCE OF TWO EVENTS (PROPERTY):**

  Events E & F are independent if and only if $\mathbb{P}(E \cap F) = \mathbb{P}(E) \cdot \mathbb{P}(F)$

- **INDEPENDENCE OF COMPLEMENTS OF TWO EVENTS:**

  Events E & F are independent if and only if E^c & F^c are independent.

  Events E & F are independent if and only if E^c & F are independent.

  Events E & F are independent if and only if E & F^c are independent.

- **INDEPENDENCE OF THREE EVENTS (DEFINITION):**

  Let $E_1, E_2, E_3 \subseteq \Omega$ be three events from sample space Ω of an experiment.

  Then E_1, E_2, E_3 are **(mutually) independent** if the following are all true:

$$\mathbb{P}(E_1 \cap E_2) = \mathbb{P}(E_1) \cdot \mathbb{P}(E_2)$$
$$\mathbb{P}(E_1 \cap E_3) = \mathbb{P}(E_1) \cdot \mathbb{P}(E_3)$$
$$\mathbb{P}(E_2 \cap E_3) = \mathbb{P}(E_2) \cdot \mathbb{P}(E_3)$$
$$\mathbb{P}(E_1 \cap E_2 \cap E_3) = \mathbb{P}(E_1) \cdot \mathbb{P}(E_2) \cdot \mathbb{P}(E_3)$$

Note that any of the mutually independent events E_1, E_2, E_3 can be replaced by their corresponding complements and still remain mutually independent:

$$\mathbb{P}(E_1 \cap E_2^c) = \mathbb{P}(E_1) \cdot \mathbb{P}(E_2^c), \quad \mathbb{P}(E_1^c \cap E_2) = \mathbb{P}(E_1^c) \cdot \mathbb{P}(E_2), \quad \mathbb{P}(E_1^c \cap E_2^c) = \mathbb{P}(E_1^c) \cdot \mathbb{P}(E_2^c), \quad \text{etc...}$$
$$\mathbb{P}(E_1 \cap E_2^c \cap E_3) = \mathbb{P}(E_1) \cdot \mathbb{P}(E_2^c) \cdot \mathbb{P}(E_3), \quad \mathbb{P}(E_1^c \cap E_2 \cap E_3^c) = \mathbb{P}(E_1^c) \cdot \mathbb{P}(E_2) \cdot \mathbb{P}(E_3^c), \quad \text{etc...}$$

- **INDEPENDENCE OF FOUR EVENTS (DEFINITION):**

  Let $E_1, E_2, E_3, E_4 \subseteq \Omega$ be four events from sample space Ω of an experiment.

  Then E_1, E_2, E_3, E_4 are **(mutually) independent** if the following are all true:

$$\mathbb{P}(E_1 \cap E_2) = \mathbb{P}(E_1) \cdot \mathbb{P}(E_2)$$
$$\mathbb{P}(E_1 \cap E_3) = \mathbb{P}(E_1) \cdot \mathbb{P}(E_3)$$
$$\mathbb{P}(E_1 \cap E_4) = \mathbb{P}(E_1) \cdot \mathbb{P}(E_4)$$
$$\mathbb{P}(E_2 \cap E_3) = \mathbb{P}(E_2) \cdot \mathbb{P}(E_3)$$
$$\mathbb{P}(E_2 \cap E_4) = \mathbb{P}(E_2) \cdot \mathbb{P}(E_4)$$
$$\mathbb{P}(E_3 \cap E_4) = \mathbb{P}(E_3) \cdot \mathbb{P}(E_4)$$
$$\mathbb{P}(E_1 \cap E_2 \cap E_3) = \mathbb{P}(E_1) \cdot \mathbb{P}(E_2) \cdot \mathbb{P}(E_3)$$
$$\mathbb{P}(E_1 \cap E_2 \cap E_4) = \mathbb{P}(E_1) \cdot \mathbb{P}(E_2) \cdot \mathbb{P}(E_4)$$
$$\mathbb{P}(E_1 \cap E_3 \cap E_4) = \mathbb{P}(E_1) \cdot \mathbb{P}(E_3) \cdot \mathbb{P}(E_4)$$
$$\mathbb{P}(E_2 \cap E_3 \cap E_4) = \mathbb{P}(E_2) \cdot \mathbb{P}(E_3) \cdot \mathbb{P}(E_4)$$
$$\mathbb{P}(E_1 \cap E_2 \cap E_3 \cap E_4) = \mathbb{P}(E_1) \cdot \mathbb{P}(E_2) \cdot \mathbb{P}(E_3) \cdot \mathbb{P}(E_4)$$

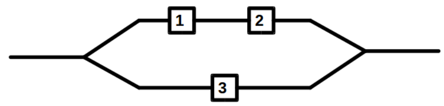
EX 2.5.1: Consider the following system of connected components shown below.

Components 1 & 2 are connected in series with component 3 connected to them in parallel.

For the entire system to function, either components 1 & 2 must function or component 3 must function.

The experiment consists of observing each component's condition and labeling it as either (S)uccess or (F)ailure.

So, for example, the outcome for components 1 & 2 functioning but 3 not functioning would be denoted as SSF.



(a) Determine the sample space Ω for the experiment.

(b) Are the events "Component 1 functions" & "Component 2 functions" independent? (Justify answer.)

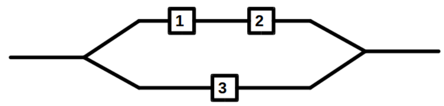
(c) Are the events "Component 1 functions" & "Exactly two components function" independent? (Justify answer.)

(d) Do the three components function independently of one another? (Justify answer.)

EX 2.5.2: Consider the following system of connected components shown below.

Components 1 & 2 are connected in series with component 3 connected to them in parallel.

For the entire system to function, either components 1 & 2 must function or component 3 must function.



Suppose the components work independently of one another. Moreover, suppose that:

Component 1 works 95% of the time, Component 2 works 97% of the time, and Component 3 works 91% of the time.

What is the probability that the entire system works?