EX 3.1.1: Consider the following experiment: Flip two fair coins and observe their top faces.

Let random variable $X \equiv$ (# Heads Observed)
Let random variable $Y \equiv$ (# Tails Observed)
Let random variable $Z \equiv$ (Is at least One Tail Observed? (1 = Yes, 0 = No))
Let random variable $W \equiv$ (# Heads Observed Minus # Tails Observed)

(a) List all the possible outcomes in the sample space Ω for the experiment.

$\Omega = \boxed{\{HH,\ HT,\ TH,\ TT\}}$

(b) For each outcome in the sample space Ω, determine the associated value of each random variable X, Y, Z, W.

$X(HH) = 2$	$Y(HH) = 0$	$Z(HH) = 0$	$W(HH) = 2 - 0 = 2$
$X(HT) = 1$	$Y(HT) = 1$	$Z(HT) = 1$	$W(HT) = 1 - 1 = 0$
$X(TH) = 1$	$Y(TH) = 1$	$Z(TH) = 1$	$W(TH) = 1 - 1 = 0$
$X(TT) = 0$	$Y(TT) = 2$	$Z(TT) = 1$	$W(TT) = 0 - 2 = -2$

(c) Determine the support of each random variable X, Y, Z, W for the experiment.

$\text{Supp}(X) = \boxed{\{0, 1, 2\}}$ $\text{Supp}(Y) = \boxed{\{0, 1, 2\}}$ $\text{Supp}(Z) = \boxed{\{0, 1\}}$ $\text{Supp}(W) = \boxed{\{-2, 0, 2\}}$

EX 3.1.2: Consider the following experiment: Repeatedly flip a fair coin and observe its top face until a tail occurs.

Let random variable $X \equiv$ (# Heads Observed)
Let random variable $Y \equiv$ (# Tails Observed)
Let random variable $Z \equiv$ (Is at least One Tail Observed? (1 = Yes, 0 = No))
Let random variable $W \equiv$ (# Heads Observed Minus # Tails Observed)

(a) List four possible outcomes in the sample space Ω for the experiment.

$\Omega = \boxed{\{T,\ HT,\ HHT,\ HHHT,\ \cdots\}}$ ⟵ Notice that the sample space is infinite (but still countable.)

(b) For the four outcomes in the sample space Ω listed in part (a), determine the associated value of each rv X, Y, Z, W.

$X(T) = 0$	$Y(T) = 1$	$Z(T) = 1$	$W(T) = 0 - 1 = -1$
$X(HT) = 1$	$Y(HT) = 1$	$Z(HT) = 1$	$W(HT) = 1 - 1 = 0$
$X(HHT) = 2$	$Y(HHT) = 1$	$Z(HHT) = 1$	$W(HHT) = 2 - 1 = 1$
$X(HHHT) = 3$	$Y(HHHT) = 1$	$Z(HHHT) = 1$	$W(HHHT) = 3 - 1 = 2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

(c) Determine the support of each random variable X, Y, Z, W for the experiment.

$\text{Supp}(X) = \boxed{\{0, 1, 2, 3, 4, \cdots\}}$ $\text{Supp}(Y) = \boxed{\{1\}}$ $\text{Supp}(Z) = \boxed{\{1\}}$ $\text{Supp}(W) = \boxed{\{-1, 0, 1, 2, 3, 4, \cdots\}}$