DISCRETE RANDOM VARIABLES: INTRODUCTION [DEVORE 3.1]

- **RANDOM VARIABLE (DEFINITION):** Given an experiment with sample space Ω. Then:

$$X \text{ is a } \textbf{random variable (r.v.)} \text{ for the experiment } \iff X : \Omega \to \mathbb{R}$$

i.e. X is a function that maps each outcome in the sample space to a real #.

NOTATION: Random variables are denoted by capital letters: X, Y, Z, U, V, W

Similar random variables have subscripts: $X_1, X_2, X_3, X_4, \ldots$

- **SUPPORT OF A RANDOM VARIABLE:** The **support** of a random variable is the **set of meaningful values**.

NOTATION: The support of random variable X is denoted as $\text{Supp}(X)$.

- **DISCRETE RANDOM VARIABLES:** X is a **discrete random variable** $\iff$ $\text{Supp}(X)$ is countable.

i.e. The meaningful values of X comprise a subset of integers $\mathbb{Z}$ or rationals $\mathbb{Q}$.

- **CONTINUOUS RANDOM VARIABLES:** X is a **continuous random variable** $\iff$ $\text{Supp}(X)$ is uncountable.

i.e. The meaningful values of X comprise an interval or union of intervals or $\mathbb{R}$.

NOTE: Continuous random variables will be explored in Chapter 4.

- **DISCRETE RANDOM VARIABLES (EXAMPLES):**

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)

Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables
$W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
$X \equiv$ If car has any available seats (1 = Yes, 0 = No)
$Y \equiv$ Number of available seats in car
$Z \equiv$ Difference in # of available and occupied seats

Then:

$$\begin{array}{llllll}
W(AAA)=1 & X(AAA)=1 & Y(AAA)=3 & Z(AAA)=3-0 & = & 3 \\
W(AAF)=0 & X(AAF)=1 & Y(AAF)=2 & Z(AAF)=2-1 & = & 1 \\
W(AFA)=1 & X(AFA)=1 & Y(AFA)=2 & Z(AFA)=2-1 & = & 1 \\
W(AFF)=0 & X(AFF)=1 & Y(AFF)=1 & Z(AFF)=1-2 & = & -1 \\
W(FAA)=1 & X(FAA)=1 & Y(FAA)=2 & Z(FAA)=2-1 & = & 1 \\
W(FAF)=0 & X(FAF)=1 & Y(FAF)=1 & Z(FAF)=1-2 & = & -1 \\
W(FFA)=1 & X(FFA)=1 & Y(FFA)=1 & Z(FFA)=1-2 & = & -1 \\
W(FFF)=0 & X(FFF)=0 & Y(FFF)=0 & Z(FFF)=0-3 & = & -3
\end{array}$$

Moreover, their supports are:

$$\begin{array}{lllll}
\text{Supp}(W) & = & \{0,1\} & \Longrightarrow & \text{Supp}(W) \text{ is countable} \Longrightarrow W \text{ is discrete} \\
\text{Supp}(X) & = & \{0,1\} & \Longrightarrow & \text{Supp}(X) \text{ is countable} \Longrightarrow X \text{ is discrete} \\
\text{Supp}(Y) & = & \{0,1,2,3\} & \Longrightarrow & \text{Supp}(Y) \text{ is countable} \Longrightarrow Y \text{ is discrete} \\
\text{Supp}(Z) & = & \{-3,-1,1,3\} & \Longrightarrow & \text{Supp}(Z) \text{ is countable} \Longrightarrow Z \text{ is discrete}
\end{array}$$

©2016 Josh Engwer – Revised February 11, 2016

EX 3.1.1: Consider the following experiment: Flip two fair coins and observe their top faces.

Let random variable	X	$\equiv$	(# Heads Observed)
Let random variable	Y	$\equiv$	(# Tails Observed)
Let random variable	Z	$\equiv$	(Is at least One Tail Observed? (1 = Yes, 0 = No))
Let random variable	W	$\equiv$	(# Heads Observed Minus # Tails Observed)

(a) List all the possible outcomes in the sample space Ω for the experiment.

(b) For each outcome in the sample space Ω, determine the associated value of each random variable X, Y, Z, W.

(c) Determine the support of each random variable X, Y, Z, W for the experiment.

EX 3.1.2: Consider the following experiment: Repeatedly flip a fair coin and observe its top face until a tail occurs.

Let random variable	X	$\equiv$	(# Heads Observed)
Let random variable	Y	$\equiv$	(# Tails Observed)
Let random variable	Z	$\equiv$	(Is at least One Tail Observed? (1 = Yes, 0 = No))
Let random variable	W	$\equiv$	(# Heads Observed Minus # Tails Observed)

(a) List four possible outcomes in the sample space Ω for the experiment.

(b) For the four outcomes in the sample space Ω listed in part (a), determine the associated value of each rv X, Y, Z, W.

(c) Determine the support of each random variable X, Y, Z, W for the experiment.

©2016 Josh Engwer – Revised February 11, 2016