DISCRETE R.V.'S: EXPECTED VALUE, VARIANCE, STD DEV [DEVORE 3.3]

- **EXPECTED VALUE (MEAN) OF A DISCRETE R.V.:** Let X be a **discrete** random variable with pmf $p_X(k)$.

  Then the **expected value** (AKA **mean**) of X is: $\mathbb{E}[X] := \sum_{k \in \mathrm{Supp}(X)} k \cdot p_X(k)$

  It's possible (but rare) that the expected value is **infinite**: $\mathbb{E}[X] = \infty$

  NOTATION: The expected value of X is alternatively denoted by μ_X.

- **EXPECTED VALUE OF A FUNCTION OF DISCRETE R.V.:** Let X be a **discrete** r.v. with pmf $p_X(k)$.

  Let $h(x)$ be a single-variable function. Then the **expected value** (AKA **mean**) of $h(X)$ is:

$$\mathbb{E}[h(X)] := \sum_{k \in \mathrm{Supp}(X)} h(k) \cdot p_X(k)$$

  It's possible (but rare) that the expected value is **infinite**: $\mathbb{E}[h(X)] = \pm\infty$

  NOTATION: The expected value of $h(X)$ is alternatively denoted by $\mu_{h(X)}$.

- **VARIANCE & STD DEV OF A DISCRETE R.V.:** Let X be a **discrete** r.v. with pmf $p_X(k)$ and mean μ_X.

  Then the **variance** of X is: $\mathbb{V}[X] := \mathbb{E}[(X - \mu_X)^2] = \sum_{k \in \mathrm{Supp}(X)} (k - \mu_X)^2 \cdot p_X(k)$

  Moreover, the **standard deviation** of X is: $\sigma_X := \sqrt{\mathbb{V}[X]}$

  NOTATION: The variance of X is alternatively denoted by σ_X^2 or $\mathrm{Var}(X)$.

- **EASIER FORMULA FOR VARIANCE:** $\mathbb{V}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$

- **LINEARITY OF DISCRETE EXPECTED VALUE (PART I):** $\mathbb{E}[aX + b] = a \cdot \mathbb{E}[X] + b$

- **LINEARITY OF DISCRETE EXPECTED VALUE (PART II):** $\mathbb{E}[a \cdot g(X) + b] = a \cdot \mathbb{E}[g(X)] + b$

- **SEMI-LINEARITY OF DISCRETE VARIANCE (PART I):** $\mathbb{V}[aX + b] = a^2 \cdot \mathbb{V}[X]$

- **SEMI-LINEARITY OF DISCRETE VARIANCE (PART II):** $\mathbb{V}[a \cdot g(X) + b] = a^2 \cdot \mathbb{V}[g(X)]$

- **SEMI-LINEARITY OF DISCRETE STANDARD DEVIATION (PART I):** $\sigma_{aX+b} = |a| \cdot \sigma_X$

- **SEMI-LINEARITY OF DISCRETE STANDARD DEVIATION (PART II):** $\sigma_{a \cdot g(X)+b} = |a| \cdot \sigma_{g(X)}$

EX 3.3.1: Consider the following experiment: Flip two fair coins and observe their top faces.

Let random variable $X \equiv$ (# Heads Observed)
Let random variable $Z \equiv$ (Is at least One Tail Observed? (1 = Yes, 0 = No))

Recall from the 3.2 Outline the pmf's of X and Z:

k	0	1	2
$p_X(k)$	1/4	1/2	1/4

k	0	1
$p_Z(k)$	1/4	3/4

(a) Compute the expected values of X and Z. Interpret the results.

(b) Compute the variances of X and Z.

(c) Compute the standard deviations of X and Z.

EX 3.3.2: A music store purchased four guitars at \$150 each from a wholesaler. The store will sell each guitar for \$350.
The wholesaler refuses to buy back any unsold guitars no matter how long they sat in the store.

Let r.v. $Y \equiv$ (# guitars sold) such that Y has the pmf:

k	0	1	2	3	4
$p_Y(k)$	0.20	0.10	0.30	0.15	0.25

What is the expected **profit** for the music store? [HINT: (Profit) := (Revenue) – (Cost)]

EX 3.3.3: A music store purchased four guitars at \$150 each from a wholesaler. The store will sell each guitar for \$350.
The wholesaler agrees to buy back any unsold guitars after six months for \$50 apiece.

Let r.v. $W \equiv$ (# guitars sold) such that W has the pmf:

k	0	1	2	3	4
$p_W(k)$	0.20	0.10	0.30	0.15	0.25

What is the expected **profit** for the music store? [HINT: (Profit) := (Revenue) – (Cost)]