EX 3.4.1: Suppose there are twenty plane flights from the Dallas airport to the Lubbock airport in a single day. Moreover, suppose that for every ten flights on a given day, three of them arrive late to the gate.

(a) What is the probability that exactly five flights are late in a day?

Let $X \equiv$ (# late flights in a day). Then, "Success" $\equiv$ "Flight is late" and $p \equiv \mathbb{P}(\text{Success}) = 3/10 = 0.3$

Therefore, $X \sim \text{Binomial}(n = 20, p = 0.3)$

<u>METHOD 1:</u> Compute probability using the **pmf**

$$\mathbb{P}(X = 5) = p_X(5; 20, 0.3) = \binom{20}{5} 0.3^5 (1 - 0.3)^{20-5} = \binom{20}{5} 0.3^5 0.7^{15} \approx \boxed{0.17886}$$

<u>METHOD 2:</u> Compute probability using the appropriate **cdf table**

$$\mathbb{P}(X = 5) = \mathbb{P}(X \le 5) - \mathbb{P}(X \le 4) = \text{Bi}(5; 20, 0.3) - \text{Bi}(4; 20, 0.3) \overset{LOOKUP}{\approx} 0.41637 - 0.23751 = \boxed{0.17886}$$

(b) What is the probability that at most five flights are late in a day?

It is far less work to lookup the appropriate **cdf table** than using the pmf:

$$\mathbb{P}(X \le 5) = \text{Bi}(5; 20, 0.3) \overset{LOOKUP}{\approx} \boxed{0.41637}$$

(c) What is the probability that at least five flights are late in a day?

It is far less work to lookup the appropriate **cdf table** than using the pmf:

$$\mathbb{P}(X \ge 5) = 1 - \mathbb{P}(X < 5) = 1 - \mathbb{P}(X \le 4) = 1 - \text{Bi}(4; 20, 0.3) \overset{LOOKUP}{\approx} 1 - 0.23751 = \boxed{0.76249}$$

(d) What is the expected number of late flights in a day?

$$\mathbb{E}[X] = np = (20)(0.3) = \boxed{6}$$

(e) What is the variance of the number of late flights in a day?

$$\mathbb{V}[X] = np(1-p) = (20)(0.3)(1 - 0.3) = (20)(0.3)(0.7) = \boxed{4.2}$$

(f) If Lubbock airport earns \$1000 for each early or on-time flight arrival and loses \$200 for each late arrival, what is the expected profit for Lubbock airport in a day?

$$\begin{aligned} \text{Let } h(X) \equiv (\text{Profit}) &= \$1000 \times (\text{\# Non-late flights each day}) - \$200 \times (\text{\# late flights each day}) \\ &= 1000(20 - X) - 200X \\ &= 20000 - 1200X \end{aligned}$$

$$\begin{aligned} \text{Then (Expected Profit)} &= \mathbb{E}[h(X)] \\ &= \mathbb{E}[20000 - 1200X] \\ &\overset{(*)}{=} 20000 - 1200 \cdot \mathbb{E}[X] \\ &= 20000 - (1200)(6) \\ &= \boxed{\$12800} \end{aligned}$$

$(*)$ Linearity of Expected Value: $\mathbb{E}[a \cdot X + b] = a \cdot \mathbb{E}[X] + b$