BERNOULLI & BINOMIAL RANDOM VARIABLES [DEVORE 3.4]

- **BERNOULLI TRIALS:**
  - Any random experiment where all possible outcomes are considered either a "Success" or "Failure" (but not both):
    - Flip a coin. (Success ≡ "Heads", Failure ≡ "Tails")
    - Roll a six-sided die. (Success ≡ "6", Failure ≡ "1,2,3,4,5 or 6")
    - Roll a six-sided die. (Success ≡ "odd #", Failure ≡ "even #")
    - Shake a mixed bag of almonds & cashews until a single piece falls out. (Success ≡ almond, Failure ≡ cashew)
    - Randomly select a person in a large busy conference.
      (Success ≡ person wearing a hat, Failure ≡ person not wearing a hat)
    - Randomly select a person in the 'Treatment' group of a medical trial.
      (Success ≡ treatment was effective, Failure ≡ treatment was not effective)
  - Any "Yes/No" question regarding some uncertain future event:
    - Is the next built widget defective? (Success ≡ "No", Failure ≡ "Yes")
    - Did the website get ≥ 1000 views today? (Success ≡ "Yes", Failure ≡ "No")
    - Is the newborn kitten female? (Success ≡ "Yes", Failure ≡ "No")

- **BERNOULLI RANDOM VARIABLES:** Model the result of a **Bernoulli Trial**:

Notation	$X \sim \text{Bernoulli}(p), \quad 0 < p < 1, \quad q := 1 - p$
Parameter(s)	$p \equiv \mathbb{P}(\text{Bernoulli Trial is a Success})$ $q \equiv \mathbb{P}(\text{Bernoulli Trial is a Failure})$
Support	$\text{Supp}(X) = \{0, 1\}$
Density (pmf)	$p_X(k; p) := p^k q^{1-k} = p^k (1-p)^{1-k}$
Mean	$\mathbb{E}[X] = p$
Variance	$\mathbb{V}[X] = pq = p(1-p)$
Model(s)	Result of One Bernoulli Trial: $1 \equiv \text{Success}, 0 \equiv \text{Failure}$
Assumption(s)	1. Random process has its sample space partitioned into Successes and Failures

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- **BINOMIAL RANDOM VARIABLES:** Model the **# successes** of n **independent** Bernoulli Trials:

Notation	$X \sim \text{Binomial}(n, p), \quad n \geq 1, \quad 0 < p < 1, \quad q := 1 - p$
Parameter(s)	$p \equiv \mathbb{P}(\text{Bernoulli Trial is a Success})$ $q \equiv \mathbb{P}(\text{Bernoulli Trial is a Failure})$
Support	$\text{Supp}(X) = \{0, 1, 2, \cdots, n-2, n-1, n\}$
Density (pmf)	$p_X(k; n, p) := \binom{n}{k} p^k q^{n-k} = \binom{n}{k} p^k (1-p)^{n-k}$
Mean	$\mathbb{E}[X] = np$
Variance	$\mathbb{V}[X] = npq = np(1-p)$
Model(s)	Result of One Bernoulli Trial: $1 \equiv \text{Success}, 0 \equiv \text{Failure}$
Assumption(s)	1. Random process comprises of n trials. 2. Trials are all identical & independent. 3. Random process has its sample space partitioned into Successes and Failures

SOME BINOMIAL CDF's

$$\mathbf{Bi}(x; n, p) := \sum_{k \le x} \binom{n}{k} p^k (1-p)^{n-k}$$

n = 5	**Success Probability** (p)										
x	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**
0	0.59049	0.32768	0.23730	0.16807	0.07776	0.03125	0.01024	0.00243	0.00098	0.00032	0.00001
1	0.91854	0.73728	0.63281	0.52822	0.33696	0.18750	0.08704	0.03078	0.01562	0.00672	0.00046
2	0.99144	0.94208	0.89648	0.83692	0.68256	0.50000	0.31744	0.16308	0.10352	0.05792	0.00856
3	0.99954	0.99328	0.98438	0.96922	0.91296	0.81250	0.66304	0.47178	0.36719	0.26272	0.08146
4	0.99999	0.99968	0.99902	0.99757	0.98976	0.96875	0.92224	0.83193	0.76270	0.67232	0.40951
5	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

n = 6	**Success Probability** (p)										
x	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**
0	0.53144	0.26214	0.17798	0.11765	0.04666	0.01562	0.00410	0.00073	0.00024	0.00006	0.00000
1	0.88574	0.65536	0.53394	0.42017	0.23328	0.10938	0.04096	0.01094	0.00464	0.00160	0.00005
2	0.98415	0.90112	0.83057	0.74431	0.54432	0.34375	0.17920	0.07047	0.03760	0.01696	0.00127
3	0.99873	0.98304	0.96240	0.92953	0.82080	0.65625	0.45568	0.25569	0.16943	0.09888	0.01585
4	0.99994	0.99840	0.99536	0.98906	0.95904	0.89062	0.76672	0.57983	0.46606	0.34464	0.11426
5	1.00000	0.99994	0.99976	0.99927	0.99590	0.98438	0.95334	0.88235	0.82202	0.73786	0.46856
6	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

n = 7	**Success Probability** (p)										
x	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**
0	0.47830	0.20972	0.13348	0.08235	0.02799	0.00781	0.00164	0.00022	0.00006	0.00001	0.00000
1	0.85031	0.57672	0.44495	0.32942	0.15863	0.06250	0.01884	0.00379	0.00134	0.00037	0.00001
2	0.97431	0.85197	0.75641	0.64707	0.41990	0.22656	0.09626	0.02880	0.01288	0.00467	0.00018
3	0.99727	0.96666	0.92944	0.87396	0.71021	0.50000	0.28979	0.12604	0.07056	0.03334	0.00273
4	0.99982	0.99533	0.98712	0.97120	0.90374	0.77344	0.58010	0.35293	0.24359	0.14803	0.02569
5	0.99999	0.99963	0.99866	0.99621	0.98116	0.93750	0.84137	0.67058	0.55505	0.42328	0.14969
6	1.00000	0.99999	0.99994	0.99978	0.99836	0.99219	0.97201	0.91765	0.86652	0.79028	0.52170
7	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

n = 8	**Success Probability** (p)										
x	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**
0	0.43047	0.16777	0.10011	0.05765	0.01680	0.00391	0.00066	0.00007	0.00002	0.00000	0.00000
1	0.81310	0.50332	0.36708	0.25530	0.10638	0.03516	0.00852	0.00129	0.00038	0.00008	0.00000
2	0.96191	0.79692	0.67854	0.55177	0.31539	0.14453	0.04981	0.01129	0.00423	0.00123	0.00002
3	0.99498	0.94372	0.88618	0.80590	0.59409	0.36328	0.17367	0.05797	0.02730	0.01041	0.00043
4	0.99957	0.98959	0.97270	0.94203	0.82633	0.63672	0.40591	0.19410	0.11382	0.05628	0.00502
5	0.99998	0.99877	0.99577	0.98871	0.95019	0.85547	0.68461	0.44823	0.32146	0.20308	0.03809
6	1.00000	0.99992	0.99962	0.99871	0.99148	0.96484	0.89362	0.74470	0.63292	0.49668	0.18690
7	1.00000	1.00000	0.99998	0.99993	0.99934	0.99609	0.98320	0.94235	0.89989	0.83223	0.56953
8	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SOME BINOMIAL CDF's

$$\mathbf{Bi}(x; n, p) := \sum_{k \le x} \binom{n}{k} p^k (1-p)^{n-k}$$

n = 9	**Success Probability** (p)										
x	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**
0	0.38742	0.13422	0.07508	0.04035	0.01008	0.00195	0.00026	0.00002	0.00000	0.00000	0.00000
1	0.77484	0.43621	0.30034	0.19600	0.07054	0.01953	0.00380	0.00043	0.00011	0.00002	0.00000
2	0.94703	0.73820	0.60068	0.46283	0.23179	0.08984	0.02503	0.00429	0.00134	0.00031	0.00000
3	0.99167	0.91436	0.83427	0.72966	0.48261	0.25391	0.09935	0.02529	0.00999	0.00307	0.00006
4	0.99911	0.98042	0.95107	0.90119	0.73343	0.50000	0.26657	0.09881	0.04893	0.01958	0.00089
5	0.99994	0.99693	0.99001	0.97471	0.90065	0.74609	0.51739	0.27034	0.16573	0.08564	0.00833
6	1.00000	0.99969	0.99866	0.99571	0.97497	0.91016	0.76821	0.53717	0.39932	0.26180	0.05297
7	1.00000	0.99998	0.99989	0.99957	0.99620	0.98047	0.92946	0.80400	0.69966	0.56379	0.22516
8	1.00000	1.00000	1.00000	0.99998	0.99974	0.99805	0.98992	0.95965	0.92492	0.86578	0.61258
9	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

n = 10	**Success Probability** (p)										
x	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**
0	0.34868	0.10737	0.05631	0.02825	0.00605	0.00098	0.00010	0.00001	0.00000	0.00000	0.00000
1	0.73610	0.37581	0.24403	0.14931	0.04636	0.01074	0.00168	0.00014	0.00003	0.00000	0.00000
2	0.92981	0.67780	0.52559	0.38278	0.16729	0.05469	0.01229	0.00159	0.00042	0.00008	0.00000
3	0.98720	0.87913	0.77588	0.64961	0.38228	0.17187	0.05476	0.01059	0.00351	0.00086	0.00001
4	0.99837	0.96721	0.92187	0.84973	0.63310	0.37695	0.16624	0.04735	0.01973	0.00637	0.00015
5	0.99985	0.99363	0.98027	0.95265	0.83376	0.62305	0.36690	0.15027	0.07813	0.03279	0.00163
6	0.99999	0.99914	0.99649	0.98941	0.94524	0.82812	0.61772	0.35039	0.22412	0.12087	0.01280
7	1.00000	0.99992	0.99958	0.99841	0.98771	0.94531	0.83271	0.61722	0.47441	0.32220	0.07019
8	1.00000	1.00000	0.99997	0.99986	0.99832	0.98926	0.95364	0.85069	0.75597	0.62419	0.26390
9	1.00000	1.00000	1.00000	0.99999	0.99990	0.99902	0.99395	0.97175	0.94369	0.89263	0.65132
10	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

n = 12	**Success Probability** (p)										
x	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**
0	0.28243	0.06872	0.03168	0.01384	0.00218	0.00024	0.00002	0.00000	0.00000	0.00000	0.00000
1	0.65900	0.27488	0.15838	0.08503	0.01959	0.00317	0.00032	0.00002	0.00000	0.00000	0.00000
2	0.88913	0.55835	0.39068	0.25282	0.08344	0.01929	0.00281	0.00021	0.00004	0.00000	0.00000
3	0.97436	0.79457	0.64878	0.49252	0.22534	0.07300	0.01527	0.00169	0.00039	0.00006	0.00000
4	0.99567	0.92744	0.84236	0.72366	0.43818	0.19385	0.05731	0.00949	0.00278	0.00058	0.00000
5	0.99946	0.98059	0.94560	0.88215	0.66521	0.38721	0.15821	0.03860	0.01425	0.00390	0.00005
6	0.99995	0.99610	0.98575	0.96140	0.84179	0.61279	0.33479	0.11785	0.05440	0.01941	0.00054
7	1.00000	0.99942	0.99722	0.99051	0.94269	0.80615	0.56182	0.27634	0.15764	0.07256	0.00433
8	1.00000	0.99994	0.99961	0.99831	0.98473	0.92700	0.77466	0.50748	0.35122	0.20543	0.02564
9	1.00000	1.00000	0.99996	0.99979	0.99719	0.98071	0.91656	0.74718	0.60932	0.44165	0.11087
10	1.00000	1.00000	1.00000	0.99998	0.99968	0.99683	0.98041	0.91497	0.84162	0.72512	0.34100
11	1.00000	1.00000	1.00000	1.00000	0.99998	0.99976	0.99782	0.98616	0.96832	0.93128	0.71757
12	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SOME BINOMIAL CDF's

$$\mathbf{Bi}(x; n, p) := \sum_{k \le x} \binom{n}{k} p^k (1-p)^{n-k}$$

n = 15	**Success Probability** (p)										
x	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**
0	0.20589	0.03518	0.01336	0.00475	0.00047	0.00003	0.00000	0.00000	0.00000	0.00000	0.00000
1	0.54904	0.16713	0.08018	0.03527	0.00517	0.00049	0.00003	0.00000	0.00000	0.00000	0.00000
2	0.81594	0.39802	0.23609	0.12683	0.02711	0.00369	0.00028	0.00001	0.00000	0.00000	0.00000
3	0.94444	0.64816	0.46129	0.29687	0.09050	0.01758	0.00193	0.00009	0.00001	0.00000	0.00000
4	0.98728	0.83577	0.68649	0.51549	0.21728	0.05923	0.00935	0.00067	0.00012	0.00001	0.00000
5	0.99775	0.93895	0.85163	0.72162	0.40322	0.15088	0.03383	0.00365	0.00079	0.00011	0.00000
6	0.99969	0.98194	0.94338	0.86886	0.60981	0.30362	0.09505	0.01524	0.00419	0.00078	0.00000
7	0.99997	0.99576	0.98270	0.94999	0.78690	0.50000	0.21310	0.05001	0.01730	0.00424	0.00003
8	1.00000	0.99922	0.99581	0.98476	0.90495	0.69638	0.39019	0.13114	0.05662	0.01806	0.00031
9	1.00000	0.99989	0.99921	0.99635	0.96617	0.84912	0.59678	0.27838	0.14837	0.06105	0.00225
10	1.00000	0.99999	0.99988	0.99933	0.99065	0.94077	0.78272	0.48451	0.31351	0.16423	0.01272
11	1.00000	1.00000	0.99999	0.99991	0.99807	0.98242	0.90950	0.70313	0.53871	0.35184	0.05556
12	1.00000	1.00000	1.00000	0.99999	0.99972	0.99631	0.97289	0.87317	0.76391	0.60198	0.18406
13	1.00000	1.00000	1.00000	1.00000	0.99997	0.99951	0.99483	0.96473	0.91982	0.83287	0.45096
14	1.00000	1.00000	1.00000	1.00000	1.00000	0.99997	0.99953	0.99525	0.98664	0.96482	0.79411
15	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

n = 20	**Success Probability** (p)										
x	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**
0	0.12158	0.01153	0.00317	0.00080	0.00004	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
1	0.39175	0.06918	0.02431	0.00764	0.00052	0.00002	0.00000	0.00000	0.00000	0.00000	0.00000
2	0.67693	0.20608	0.09126	0.03548	0.00361	0.00020	0.00001	0.00000	0.00000	0.00000	0.00000
3	0.86705	0.41145	0.22516	0.10709	0.01596	0.00129	0.00005	0.00000	0.00000	0.00000	0.00000
4	0.95683	0.62965	0.41484	0.23751	0.05095	0.00591	0.00032	0.00001	0.00000	0.00000	0.00000
5	0.98875	0.80421	0.61717	0.41637	0.12560	0.02069	0.00161	0.00004	0.00000	0.00000	0.00000
6	0.99761	0.91331	0.78578	0.60801	0.25001	0.05766	0.00647	0.00026	0.00003	0.00000	0.00000
7	0.99958	0.96786	0.89819	0.77227	0.41589	0.13159	0.02103	0.00128	0.00018	0.00002	0.00000
8	0.99994	0.99002	0.95907	0.88667	0.59560	0.25172	0.05653	0.00514	0.00094	0.00010	0.00000
9	0.99999	0.99741	0.98614	0.95204	0.75534	0.41190	0.12752	0.01714	0.00394	0.00056	0.00000
10	1.00000	0.99944	0.99606	0.98286	0.87248	0.58810	0.24466	0.04796	0.01386	0.00259	0.00001
11	1.00000	0.99990	0.99906	0.99486	0.94347	0.74828	0.40440	0.11333	0.04093	0.00998	0.00006
12	1.00000	0.99998	0.99982	0.99872	0.97897	0.86841	0.58411	0.22773	0.10181	0.03214	0.00042
13	1.00000	1.00000	0.99997	0.99974	0.99353	0.94234	0.74999	0.39199	0.21422	0.08669	0.00239
14	1.00000	1.00000	1.00000	0.99996	0.99839	0.97931	0.87440	0.58363	0.38283	0.19579	0.01125
15	1.00000	1.00000	1.00000	0.99999	0.99968	0.99409	0.94905	0.76249	0.58516	0.37035	0.04317
16	1.00000	1.00000	1.00000	1.00000	0.99995	0.99871	0.98404	0.89291	0.77484	0.58855	0.13295
17	1.00000	1.00000	1.00000	1.00000	0.99999	0.99980	0.99639	0.96452	0.90874	0.79392	0.32307
18	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998	0.99948	0.99236	0.97569	0.93082	0.60825
19	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99996	0.99920	0.99683	0.98847	0.87842
20	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SOME BINOMIAL CDF's $\quad \mathbf{Bi}(x;n,p) := \sum_{k\le x} \binom{n}{k} p^k (1-p)^{n-k}$

n = 25	**Success Probability** (p)										
x	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**
0	0.07179	0.00378	0.00075	0.00013	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
1	0.27121	0.02739	0.00702	0.00157	0.00005	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
2	0.53709	0.09823	0.03211	0.00896	0.00043	0.00001	0.00000	0.00000	0.00000	0.00000	0.00000
3	0.76359	0.23399	0.09621	0.03324	0.00237	0.00008	0.00000	0.00000	0.00000	0.00000	0.00000
4	0.90201	0.42067	0.21374	0.09047	0.00947	0.00046	0.00001	0.00000	0.00000	0.00000	0.00000
5	0.96660	0.61669	0.37828	0.19349	0.02936	0.00204	0.00005	0.00000	0.00000	0.00000	0.00000
6	0.99052	0.78004	0.56110	0.34065	0.07357	0.00732	0.00028	0.00000	0.00000	0.00000	0.00000
7	0.99774	0.89088	0.72651	0.51185	0.15355	0.02164	0.00121	0.00002	0.00000	0.00000	0.00000
8	0.99954	0.95323	0.85056	0.67693	0.27353	0.05388	0.00433	0.00010	0.00001	0.00000	0.00000
9	0.99992	0.98267	0.92867	0.81056	0.42462	0.11476	0.01317	0.00045	0.00004	0.00000	0.00000
10	0.99999	0.99445	0.97033	0.90220	0.58577	0.21218	0.03439	0.00178	0.00021	0.00001	0.00000
11	1.00000	0.99846	0.98927	0.95575	0.73228	0.34502	0.07780	0.00599	0.00092	0.00008	0.00000
12	1.00000	0.99963	0.99663	0.98253	0.84623	0.50000	0.15377	0.01747	0.00337	0.00037	0.00000
13	1.00000	0.99992	0.99908	0.99401	0.92220	0.65498	0.26772	0.04425	0.01073	0.00154	0.00000
14	1.00000	0.99999	0.99979	0.99822	0.96561	0.78782	0.41423	0.09780	0.02967	0.00555	0.00001
15	1.00000	1.00000	0.99996	0.99955	0.98683	0.88524	0.57538	0.18944	0.07133	0.01733	0.00008
16	1.00000	1.00000	0.99999	0.99990	0.99567	0.94612	0.72647	0.32307	0.14944	0.04677	0.00046
17	1.00000	1.00000	1.00000	0.99998	0.99879	0.97836	0.84645	0.48815	0.27349	0.10912	0.00226
18	1.00000	1.00000	1.00000	1.00000	0.99972	0.99268	0.92643	0.65935	0.43890	0.21996	0.00948
19	1.00000	1.00000	1.00000	1.00000	0.99995	0.99796	0.97064	0.80651	0.62172	0.38331	0.03340
20	1.00000	1.00000	1.00000	1.00000	0.99999	0.99954	0.99053	0.90953	0.78626	0.57933	0.09799
21	1.00000	1.00000	1.00000	1.00000	1.00000	0.99992	0.99763	0.96676	0.90379	0.76601	0.23641
22	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99957	0.99104	0.96789	0.90177	0.46291
23	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99995	0.99843	0.99298	0.97261	0.72879
24	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99987	0.99925	0.99622	0.92821

EX 3.4.1: Suppose there are twenty plane flights from the Dallas airport to the Lubbock airport in a single day. Moreover, suppose that for every ten flights on a given day, three of them arrive late to the gate.

(a) What is the probability that exactly five flights are late in a day?

(b) What is the probability that at most five flights are late in a day?

(c) What is the probability that at least five flights are late in a day?

(d) What is the expected number of late flights in a day?

(e) What is the variance of the number of late flights in a day?

(f) If Lubbock airport earns \$1000 for each early or on-time flight arrival and loses \$200 for each late arrival, what is the expected profit for Lubbock airport?