EX 3.6.1: A study of a bank's teller lines showed that from 10am to 1pm there is an average of six customers waiting. Assume that the number of customers waiting in line is modeled by a Poisson distribution.

(a) What is the probability that upon visiting the bank one day from 10am to 1pm that six customers are waiting?

Let $X \equiv$ (# Customers waiting in line from 10am to 1pm at bank) Then, $X \sim \text{Poisson}(\lambda = 6)$

METHOD 1: Compute probability using the **pmf**: $\mathbb{P}(X = 6) = p_X(6; 6) = \dfrac{e^{-6}6^6}{6!} \approx \boxed{0.16062}$

METHOD 2: Compute probability using the appropriate **cdf table**:

$$\mathbb{P}(X = 6) = \mathbb{P}(X \leq 6) - \mathbb{P}(X \leq 5) = \text{Pois}(6; 6) - \text{Pois}(5; 6) \overset{LOOKUP}{\approx} 0.60630 - 0.44568 = \boxed{0.16062}$$

(b) What is the probability that upon visiting the bank one day from 10am to 1pm that at least six customers are waiting?

$$\mathbb{P}(X \geq 6) = 1 - \mathbb{P}(X < 6) = 1 - \mathbb{P}(X \leq 5) = 1 - \text{Pois}(5; 6) \overset{LOOKUP}{\approx} 1 - 0.44568 = \boxed{0.55432}$$

(c) What is the probability that upon visiting the bank one day from 10am to 1pm that no customers are waiting?

$$\mathbb{P}(X = 0) = \mathbb{P}(X \leq 0) = \text{Pois}(0; 6) \overset{LOOKUP}{\approx} \boxed{0.00248}$$

EX 3.6.2: A study of a bank's teller lines indicated that all day two customers are expected to be waiting per hour. Assume that the number of customers waiting in line is modeled by a Poisson distribution.

(a) What is the probability that upon visiting the bank one day from 2pm to 7pm that six customers are waiting?

Let $Y \equiv$ (# Customers waiting in line from 2pm to 7pm at bank) Then, $Y \sim \text{Poisson}(\lambda = \underbrace{2}_{\alpha} \cdot \underbrace{(7-2)}_{\Delta t}) = \text{Poisson}(\lambda = 10)$

METHOD 1: Compute probability using the **pmf**: $\mathbb{P}(Y = 6) = p_Y(6; 10) = \dfrac{e^{-10}10^6}{6!} \approx \boxed{0.06306}$

METHOD 2: Compute probability using the appropriate Poisson **cdf table**:

$$\mathbb{P}(Y = 6) = \mathbb{P}(Y \leq 6) - \mathbb{P}(Y \leq 5) = \text{Pois}(6; 10) - \text{Pois}(5; 10) \overset{LOOKUP}{\approx} 0.13014 - 0.06709 = \boxed{0.06305}$$

(b) What is the probability that upon visiting the bank one day from 2pm to 7pm that at least six customers are waiting?

$$\mathbb{P}(Y \geq 6) = 1 - \mathbb{P}(Y < 6) = 1 - \mathbb{P}(Y \leq 5) = 1 - \text{Pois}(5; 10) \overset{LOOKUP}{\approx} 1 - 0.06709 = \boxed{0.93291}$$

(c) What is the probability that upon visiting the bank one day from 2pm to 7pm that no customers are waiting?

$$\mathbb{P}(Y = 0) = \mathbb{P}(Y \leq 0) = \text{Pois}(0; 10) \overset{LOOKUP}{\approx} \boxed{0.00005}$$

EX 3.6.3: The sales of Chevy Volt cars at a dealership follow a Poisson distribution with a mean of three cars sold per day.

(a) What is the probability that for the next four days exactly two Chevy Volts are sold in total?

Let $Z \equiv$ (# Chevy Volts sold in four days) Then, $Z \sim \text{Poisson}(\lambda = \underbrace{3}_{\alpha} \cdot \underbrace{4}_{\Delta t}) = \text{Poisson}(\lambda = 12)$

The provided Poisson cdf tables do not include $\lambda = 12$, so use **pmf** instead: $\mathbb{P}(Z = 2) = p_Z(2; 12) = \dfrac{e^{-12}12^2}{2!} \approx \boxed{0.00044}$

(b) What is the probability that for the next four days exactly two Chevy Volts are sold each day?

Let $Z_1 \equiv$ (# Chevy Volts sold in 1^{st} day), $Z_2 \equiv$ (# Chevy Volts sold in 2^{nd} day), $\ldots, Z_4 \equiv$ (# Chevy Volts sold in 4^{th} day)

Then, $Z_1, Z_2, Z_3, Z_4 \sim \text{Poisson}(\lambda = \underbrace{3}_{\alpha} \cdot \underbrace{1}_{\Delta t}) = \text{Poisson}(\lambda = 3)$

Assumption "# Arrivals during **disjoint** time periods are **independent**" $\implies Z_1, Z_2, Z_3, Z_4$ are **independent** of one another.

$$\begin{aligned}
\therefore\ \mathbb{P}(\text{Two Chevy Volts sold each day for four days}) &= \mathbb{P}(Z_1 = 2 \text{ and } Z_2 = 2 \text{ and } Z_3 = 2 \text{ and } Z_4 = 2) \\
&= \mathbb{P}(Z_1 = 2 \ \cap\ Z_2 = 2 \ \cap\ Z_3 = 2 \ \cap\ Z_4 = 2) \\
&\overset{IND}{=} \mathbb{P}(Z_1 = 2) \cdot \mathbb{P}(Z_2 = 2) \cdot \mathbb{P}(Z_3 = 2) \cdot \mathbb{P}(Z_4 = 2) \\
&\overset{(*)}{=} [\mathbb{P}(Z_1 = 2)]^4 = [p_{Z_1}(2; 3)]^4 = \left[\tfrac{e^{-3}3^2}{2!}\right]^4 \approx \boxed{0.00252}
\end{aligned}$$

$(*)$ This follows since the random variables Z_1, Z_2, Z_3, Z_4 are all **identical**.