POISSON RANDOM VARIABLES [DEVORE 3.6]

- **POISSON PROCESSES:** Here is a short list of **Poisson processes** in various applications:
  - Number of **arrivals** over a fixed **time period** Δt
    - # radioactive decays of $1\mu g$ of Iodine-123 in 1/1000 second
    - # phone calls a dispatcher received in 45 minutes
    - # emails an account received in two hours
    - # car accidents at a dangerous intersection in four weeks
    - # insurance claims from a given demographic in six months
    - # industrial accidents at a factory in five years
    - # wars started in a continent in three centuries
  - Number of **arrivals** over a fixed **length** ΔL
    - # mutations in a strand of DNA
    - # blemishes in a spool of copper wire
  - Number of **arrivals** over a fixed **area** ΔA
    - # chocolate chips in a large cookie
  - Number of **arrivals** over a fixed **volume** ΔV
    - # yeast cells used in brewing a glass of Guinness beer

- **POISSON RANDOM VARIABLES:** Model **Poisson Processes**:

Notation	$X \sim \text{Poisson}(\lambda), \quad \lambda > 0$
Parameter(s)	$\lambda = \alpha \Delta t$ s.t. $\lambda \equiv$ Expected/Average # Arrivals over entire Time Period Δt $\alpha \equiv$ Expected/Average # Arrivals per Unit Time (i.e. Expected Arrival Rate) $\Delta t \equiv$ Time period
Support	$\text{Supp}(X) = \{0, 1, 2, 3, 4, \cdots\}$
Density (pmf)	$p_X(k; \lambda) := \dfrac{e^{-\lambda}\lambda^k}{k!}$
Mean	$\mathbb{E}[X] = \lambda$
Variance	$\mathbb{V}[X] = \lambda$
Model(s)	Number of arrivals over a fixed time period Δt Number of arrivals over a fixed space $\Delta L, \Delta A$ or ΔV
Assumption(s)	$\mathbb{P}$(No arrivals during time period Δt) $\approx 1 - \alpha\Delta t$ $\mathbb{P}$(Exactly one arrival during time period Δt) $\approx \alpha\Delta t$ $\mathbb{P}$(More than one arrival during time period Δt) ≈ 0 # Arrivals during **disjoint** time periods are **independent**

SOME POISSON CDF's $\quad \textbf{Pois}(x;\lambda) := \sum_{k \le x} \frac{e^{-\lambda}\lambda^k}{k!}$

	Expected/Average # Arrivals over entire Time Period (λ)										
x	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**
0	0.90484	0.81873	0.77880	0.74082	0.67032	0.60653	0.54881	0.49659	0.47237	0.44933	0.40657
1	0.99532	0.98248	0.97350	0.96306	0.93845	0.90980	0.87810	0.84420	0.82664	0.80879	0.77248
2	0.99985	0.99885	0.99784	0.99640	0.99207	0.98561	0.97688	0.96586	0.95949	0.95258	0.93714
3	1.00000	0.99994	0.99987	0.99973	0.99922	0.99825	0.99664	0.99425	0.99271	0.99092	0.98654
4	1.00000	1.00000	0.99999	0.99998	0.99994	0.99983	0.99961	0.99921	0.99894	0.99859	0.99766
5	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99996	0.99991	0.99987	0.99982	0.99966
6	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99999	0.99998	0.99996
7	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

	Expected/Average # Arrivals over entire Time Period (λ)									
x	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0	0.36788	0.13534	0.04979	0.01832	0.00674	0.00248	0.00091	0.00034	0.00012	0.00005
1	0.73576	0.40601	0.19915	0.09158	0.04043	0.01735	0.00730	0.00302	0.00123	0.00050
2	0.91970	0.67668	0.42319	0.23810	0.12465	0.06197	0.02964	0.01375	0.00623	0.00277
3	0.98101	0.85712	0.64723	0.43347	0.26503	0.15120	0.08177	0.04238	0.02123	0.01034
4	0.99634	0.94735	0.81526	0.62884	0.44049	0.28506	0.17299	0.09963	0.05496	0.02925
5	0.99941	0.98344	0.91608	0.78513	0.61596	0.44568	0.30071	0.19124	0.11569	0.06709
6	0.99992	0.99547	0.96649	0.88933	0.76218	0.60630	0.44971	0.31337	0.20678	0.13014
7	0.99999	0.99890	0.98810	0.94887	0.86663	0.74398	0.59871	0.45296	0.32390	0.22022
8	1.00000	0.99976	0.99620	0.97864	0.93191	0.84724	0.72909	0.59255	0.45565	0.33282
9	1.00000	0.99995	0.99890	0.99187	0.96817	0.91608	0.83050	0.71662	0.58741	0.45793
10	1.00000	0.99999	0.99971	0.99716	0.98630	0.95738	0.90148	0.81589	0.70599	0.58304
11	1.00000	1.00000	0.99993	0.99908	0.99455	0.97991	0.94665	0.88808	0.80301	0.69678
12	1.00000	1.00000	0.99998	0.99973	0.99798	0.99117	0.97300	0.93620	0.87577	0.79156
13	1.00000	1.00000	1.00000	0.99992	0.99930	0.99637	0.98719	0.96582	0.92615	0.86446
14	1.00000	1.00000	1.00000	0.99998	0.99977	0.99860	0.99428	0.98274	0.95853	0.91654
15	1.00000	1.00000	1.00000	1.00000	0.99993	0.99949	0.99759	0.99177	0.97796	0.95126
16	1.00000	1.00000	1.00000	1.00000	0.99998	0.99983	0.99904	0.99628	0.98889	0.97296
17	1.00000	1.00000	1.00000	1.00000	0.99999	0.99994	0.99964	0.99841	0.99468	0.98572
18	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998	0.99987	0.99935	0.99757	0.99281
19	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99996	0.99975	0.99894	0.99655
20	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99991	0.99956	0.99841
21	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99997	0.99983	0.99930
22	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99993	0.99970
23	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998	0.99988
24	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99995
25	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998
26	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999
27	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

EX 3.6.1: A study of a bank's teller lines showed that from 10am to 1pm there is an average of six customers waiting. Assume that the number of customers waiting in line is modeled by a Poisson distribution.

(a) What is the probability that upon visiting the bank one day from 10am to 1pm that six customers are waiting?

(b) What is the probability that upon visiting the bank one day from 10am to 1pm that at least six customers are waiting?

(c) What is the probability that upon visiting the bank one day from 10am to 1pm that no customers are waiting?

EX 3.6.2: A study of a bank's teller lines indicated that all day two customers are expected to be waiting per hour. Assume that the number of customers waiting in line is modeled by a Poisson distribution.

(a) What is the probability that upon visiting the bank one day from 2pm to 7pm that six customers are waiting?

(b) What is the probability that upon visiting the bank one day from 2pm to 7pm that at least six customers are waiting?

(c) What is the probability that upon visiting the bank one day from 2pm to 7pm that no customers are waiting?

EX 3.6.3: The sales of Chevy Volt cars at a dealership follow a Poisson distribution with a mean of three cars sold per day.

(a) What is the probability that for the next four days exactly two Chevy Volts are sold in total?

(b) What is the probability that for the next four days exactly two Chevy Volts are sold each day?