EX 4.3.1: The amount of rainfall (in inches) in Houston during March follows a Uniform$(a = 1.52, b = 4.54)$ distribution. Let X represent the aforementioned rainfall amount.

(a) What is the pdf of X, $f_X(x)$?

$X \sim \text{Uniform}(a = 1.52, b = 4.54)$

$$f_X(x) = \begin{cases} \frac{1}{b-a} & \text{, if } a \le x \le b \\ 0 & \text{, otherwise} \end{cases} = \begin{cases} \frac{1}{4.54-1.52} & \text{, if } 1.52 \le x \le 4.54 \\ 0 & \text{, otherwise} \end{cases} \quad \therefore \quad f_X(x) = \boxed{\begin{cases} \frac{1}{3.02} & \text{, if } 1.52 \le x \le 4.54 \\ 0 & \text{, otherwise} \end{cases}}$$

(b) What is the cdf of X, $F_X(x)$?

$$F_X(x) = \begin{cases} 0 & \text{, if } x < 1.52 \\ \int_{-\infty}^{x} \frac{1}{3.02}\, dt & \text{, if } 1.52 \le x \le 4.54 \\ 1 & \text{, if } 4.54 < x \end{cases} \quad \therefore \quad F_X(x) = \boxed{\begin{cases} 0 & \text{, if } x < 1.52 \\ \frac{x-1.52}{3.02} & \text{, if } 1.52 \le x \le 4.54 \\ 1 & \text{, if } 4.54 < x \end{cases}}$$

$$\int_{-\infty}^{x} \frac{1}{3.02}\, dt = \int_{1.52}^{x} \frac{1}{3.02}\, dt = \left[\frac{t}{3.02}\right]_{t=1.52}^{t=x} \overset{FTC}{=} \frac{x}{3.02} - \frac{1.52}{3.02} = \frac{x-1.52}{3.02}$$

(c) What is the expected amount of rainfall in Houston during March?

$$\mathbb{E}[X] = \frac{b+a}{2} = \frac{4.54+1.52}{2} = \boxed{\mathbf{3.03} \text{ inches}}$$

(d) What is the variance of the amount of rainfall in Houston during March?

$$\mathbb{V}[X] = \frac{(b-a)^2}{12} = \frac{(4.54-1.52)^2}{12} = \boxed{\mathbf{0.76003} \text{ inches}^2}$$

(e) What is the standard deviation of the amount of rainfall in Houston during March?

$$\sigma_X = \sqrt{\mathbb{V}[X]} = \sqrt{0.76003} \approx \boxed{\mathbf{0.87180} \text{ inches}}$$

(f) What is the probability that Houston receives at most 2 inches of rainfall during March of next year?

$$\mathbb{P}(X \le 2) = F_X(2) = \frac{2-1.52}{3.02} \approx \boxed{\mathbf{0.15894}}$$

(g) What is the probability that Houston receives between 2 and 3 inches of rainfall during March of next year?

$$\mathbb{P}(2 < X < 3) = \mathbb{P}(X \le 3) - \mathbb{P}(X \le 2) = F_X(3) - F_X(2) = \frac{3-1.52}{3.02} - \frac{2-1.52}{3.02} = \frac{3-2}{3.02} \approx \boxed{\mathbf{0.33113}}$$

(h) What is the probability that Houston receives at least 2.5 inches of rainfall during March of next year?

$$\mathbb{P}(X \ge 2.5) = 1 - \mathbb{P}(X < 2.5) = 1 - \mathbb{P}(X \le 2.5) = 1 - F_X(2.5) = 1 - \frac{2.5-1.52}{3.02} \approx \boxed{\mathbf{0.67550}}$$

(i) What is the probability that Houston receives between 0.5 and 4 inches of rainfall during March of next year?

$$\mathbb{P}(0.5 < X < 4) = \mathbb{P}(X \le 4) - \mathbb{P}(X \le 0.5) = F_X(4) - F_X(0.5) = \frac{4-1.52}{3.02} - 0 \approx \boxed{\mathbf{0.82119}}$$

EX 4.3.2: Consider the **standard normal distribution**: Let random variable $Z \sim \text{Normal}(\mu = 0, \sigma = 1)$

(a) **Using the provided standard normal cdf table**, compute $\Phi(1.8)$.

$\Phi(1.8) \overset{LOOKUP}{\approx} \boxed{\mathbf{0.96407}}$

(b) **Using the provided standard normal cdf table**, compute $\Phi(-1.8)$.

$\Phi(-1.8) = 1 - \Phi(1.8) \overset{LOOKUP}{\approx} 1 - 0.96407 = \boxed{\mathbf{0.03593}}$

(c) **Using the provided standard normal cdf table**, compute $\mathbb{P}(Z \leq 2.47)$.

$\mathbb{P}(Z \leq 2.47) = \Phi(2.47) \overset{LOOKUP}{\approx} \boxed{\mathbf{0.99324}}$

(d) **Using the provided standard normal cdf table**, compute $\mathbb{P}(Z \geq 2.47)$.

$\mathbb{P}(Z \geq 2.47) = 1 - \mathbb{P}(Z < 2.47) = 1 - \mathbb{P}(Z \leq 2.47) = 1 - \Phi(2.47) \overset{LOOKUP}{\approx} 1 - 0.99324 = \boxed{\mathbf{0.00676}}$

(e) **Using the provided standard normal cdf table**, compute $\mathbb{P}(1.15 < Z \leq 2.47)$.

$\mathbb{P}(1.15 < Z \leq 2.47) = \mathbb{P}(Z \leq 2.47) - \mathbb{P}(Z \leq 1.15) = \Phi(2.47) - \Phi(1.15) \overset{LOOKUP}{\approx} 0.99324 - 0.87493 = \boxed{\mathbf{0.11831}}$

(f) **Using the provided standard normal cdf table**, compute $\mathbb{P}(Z > -2.3)$.

$\mathbb{P}(Z > -2.3) = 1 - \mathbb{P}(Z \leq -2.3) = 1 - \Phi(-2.3) = 1 - [1 - \Phi(2.3)] = \Phi(2.3) \overset{LOOKUP}{\approx} \boxed{\mathbf{0.98928}}$

(g) **Using the provided standard normal cdf table**, compute $\mathbb{P}(-2.5 < Z < -2.3)$.

$$\begin{aligned}
\mathbb{P}(-2.5 < Z < -2.3) &= \mathbb{P}(Z \leq -2.3) - \mathbb{P}(Z \leq -2.5) \\
&= \Phi(-2.3) - \Phi(-2.5) \\
&= [1 - \Phi(2.3)] - [1 - \Phi(2.5)] \\
&= \Phi(2.5) - \Phi(2.3) \\
&\overset{LOOKUP}{\approx} 0.99379 - 0.98928 \\
&= \boxed{\mathbf{0.00451}}
\end{aligned}$$

(h) **Using the provided standard normal cdf table**, determine the 57^{th} percentile of Z, $z_{0.57}$.

(Interpolate if necessary, which means find the closest entry in the table.)

The closest table entries to 0.57 are:

$$\begin{aligned}
\Phi(0.17) \approx 0.56749 &\implies |0.57 - 0.56749| = |0.00251| = 0.00251 \\
\Phi(0.18) \approx 0.57142 &\implies |0.57 - 0.57142| = |-0.00142| = 0.00142
\end{aligned}$$

$\therefore \quad z_{0.57} \overset{REV\ LOOKUP}{\approx} \boxed{\mathbf{0.18}}$

(i) **Using the provided standard normal cdf table**, determine the 13^{th} percentile of Z, $z_{0.13}$.

(Interpolate if necessary, which means find the closest entry in the table.)

$z_{0.13} = -z_{1-0.13} = -z_{0.87} \overset{REV\ LOOKUP}{\approx} \boxed{\mathbf{-1.13}}$

The closest table entries to 0.87 are:

$$\begin{aligned}
\Phi(1.12) \approx 0.86864 &\implies |0.87 - 0.86864| = |0.00136| = 0.00136 \\
\Phi(1.13) \approx 0.87076 &\implies |0.87 - 0.87076| = |-0.00076| = 0.00076
\end{aligned}$$

EX 4.3.3: The weights of cans of pineapples follow a normal distribution with a mean of 1000 g and std deviation of 50 g.

(a) What is the probability of a can of pineapples weighing more than 1075 grams?

$$\mathbb{P}(X > 1075) = \mathbb{P}\left(Z > \frac{1075 - 1000}{50}\right) = \mathbb{P}(Z > 1.5) = 1 - \mathbb{P}(Z \le 1.5) = 1 - \Phi(1.5) \overset{LOOKUP}{\approx} 1 - 0.93319 = \boxed{\mathbf{0.06681}}$$

(b) What is the probability of a can of pineapples weighing between 970 and 1140 grams?

$$\begin{aligned}
\mathbb{P}(970 \le X \le 1140) &= \mathbb{P}\left(\frac{970 - 1000}{50} \le Z \le \frac{1140 - 1000}{50}\right) \\
&= \mathbb{P}(-0.6 \le Z \le 2.8) \\
&= \mathbb{P}(Z \le 2.8) - \mathbb{P}(Z \le -0.6) \\
&= \Phi(2.8) - \Phi(-0.6) \\
&= \Phi(2.8) - [1 - \Phi(0.6)] \\
&\overset{LOOKUP}{\approx} 0.99744 - [1 - 0.72575] = \boxed{\mathbf{0.72319}}
\end{aligned}$$

(c) What is the probability of a can of pineapples weighing less than 905 grams?

$$\mathbb{P}(X < 905) = \mathbb{P}\left(Z < \frac{905 - 1000}{50}\right) = \mathbb{P}(Z \le -1.9) = \Phi(-1.9) = 1 - \Phi(1.9) \overset{LOOKUP}{\approx} 1 - 0.97128 = \boxed{\mathbf{0.02872}}$$

(d) What is the probability of a can of pineapples weighing within 10 grams of 960 grams?

$$\begin{aligned}
\mathbb{P}(|X - 960| \le 10) &= \mathbb{P}(-10 \le X - 960 \le 10) \\
&= \mathbb{P}(950 \le X \le 970) = \mathbb{P}\left(\frac{950 - 1000}{50} \le Z \le \frac{970 - 1000}{50}\right) \\
&= \mathbb{P}(-1 \le Z \le -0.6) \\
&= \mathbb{P}(Z \le -0.6) - \mathbb{P}(Z \le -1) \\
&= \Phi(-0.6) - \Phi(-1) \\
&= [1 - \Phi(0.6)] - [1 - \Phi(1)] \\
&= \Phi(1) - \Phi(0.6) \\
&\overset{LOOKUP}{\approx} 0.84134 - 0.72575 = \boxed{\mathbf{0.11559}}
\end{aligned}$$

(e) What is the probability of a can of pineapples weighing within two standard deviations of the mean?

$$\begin{aligned}
\mathbb{P}(|X - \mu_X| \le 2\sigma_X) &= \mathbb{P}(-2\sigma \le X - \mu \le 2\sigma) = \mathbb{P}\left(-2 \le \frac{X - \mu}{\sigma} \le 2\right) \\
&= \mathbb{P}(-2 \le Z \le 2) \\
&= \mathbb{P}(Z \le 2) - \mathbb{P}(Z \le -2) \\
&= \Phi(2) - \Phi(-2) \\
&= \Phi(2) - [1 - \Phi(2)] \\
&= 2 \cdot \Phi(2) - 1 \\
&\overset{LOOKUP}{\approx} (2)(0.97725) - 1 = \boxed{\mathbf{0.95450}}
\end{aligned}$$

(f) What is the probability of a can of pineapples weighing outside one standard deviation of the mean?

$$\begin{aligned}
\mathbb{P}(|X - \mu_X| > \sigma_X) &= \mathbb{P}(X - \mu_X < -\sigma_X \quad \text{or} \quad X - \mu_X > \sigma_X) \\
&= \mathbb{P}(X - \mu < -\sigma \;\cup\; X - \mu > \sigma) \\
&\overset{PD}{=} \mathbb{P}(X - \mu < -\sigma) + \mathbb{P}(X - \mu > \sigma) = \mathbb{P}\left(\frac{X - \mu}{\sigma} < -1\right) + \mathbb{P}\left(\frac{X - \mu}{\sigma} > 1\right) \\
&= \mathbb{P}(Z < -1) + \mathbb{P}(Z > 1) \\
&= \mathbb{P}(Z \le -1) + [1 - \mathbb{P}(Z \le 1)] \\
&= \Phi(-1) + [1 - \Phi(1)] \\
&= [1 - \Phi(1)] + [1 - \Phi(1)] \\
&= 2 \cdot [1 - \Phi(1)] \\
&\overset{LOOKUP}{\approx} 2 \cdot [1 - 0.84134] = \boxed{\mathbf{0.31732}}
\end{aligned}$$