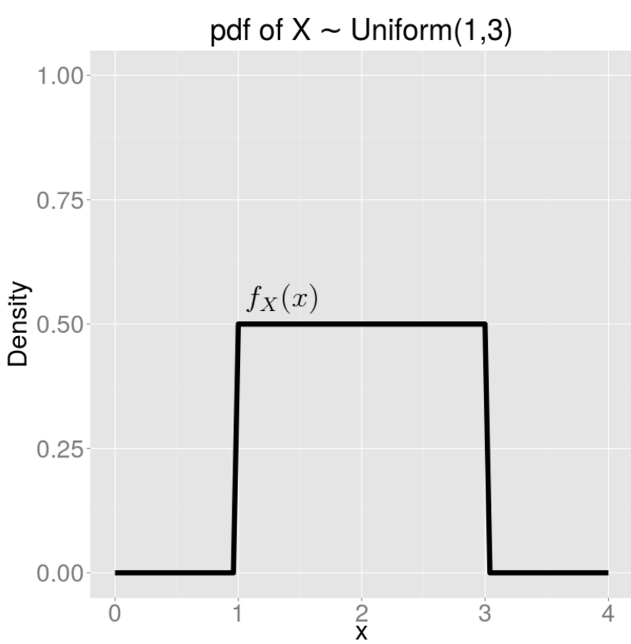
UNIFORM RANDOM VARIABLES [DEVORE 4.3]

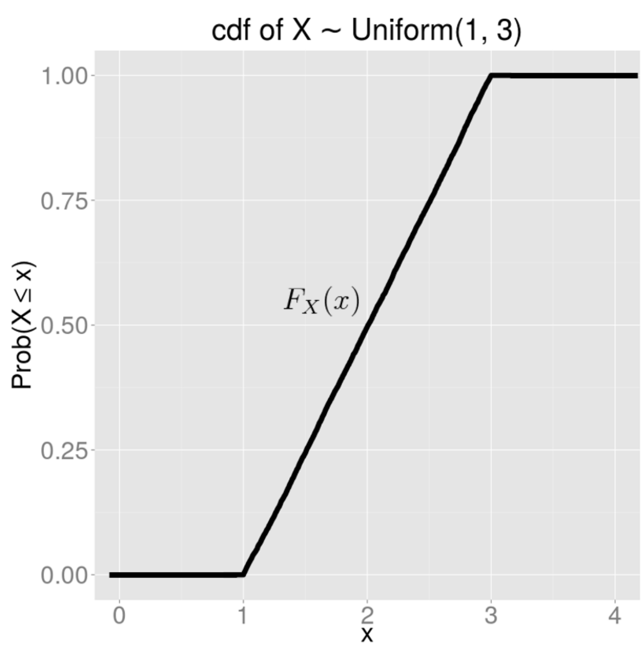
- **UNIFORM RANDOM VARIABLES:** **Uniform** random variables reasonably model:
  - Waiting times for a single arrival that repeats over a small time period:
    - Waiting time for a bus arrival over a 10-min interval
    - Waiting time for a train arrival over a 1-hour interval
  - Randomly-chosen point on a line segment
  - Random generation of numbers
  - Very Small Measurement Errors:
    - Time-measuring test equipment is accurate to within 0.05 μsec
    - Quantization error in analog-to-digital signal conversion (ADC)

- **UNIFORM RANDOM VARIABLES (SUMMARY):**

Notation	$X \sim \text{Uniform}(a, b), \quad a < b$
Parameter(s)	$a, b \in \mathbb{R}$
Support	$\text{Supp}(X) = [a, b]$
pdf	$f_X(x; a, b) = \dfrac{1}{b-a}$
Mean	$\mathbb{E}[X] = \dfrac{1}{2}(b+a)$
Variance	$\mathbb{V}[X] = \dfrac{1}{12}(b-a)^2$
Model(s)	Random-number generation Very small measurement errors

- **UNIFORM DENSITY PLOTS (PDF & CDF):**





NORMAL RANDOM VARIABLES [DEVORE 4.3]

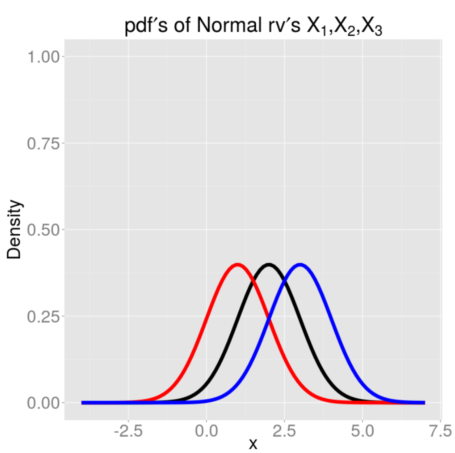
- **NORMAL RANDOM VARIABLES:** **Normal** random variables reasonably model:
  - First midterm scores of a lecture hall class of students
  - Grade point averages of college students
  - Heights of people at a large busy conference
  - Monthly rainfall totals in a humid climate
  - Measurement errors
  - Brownian motion of particles suspended in fluid

  Normal distributions are often used to develop later statistical methods, many of which will be encountered later.

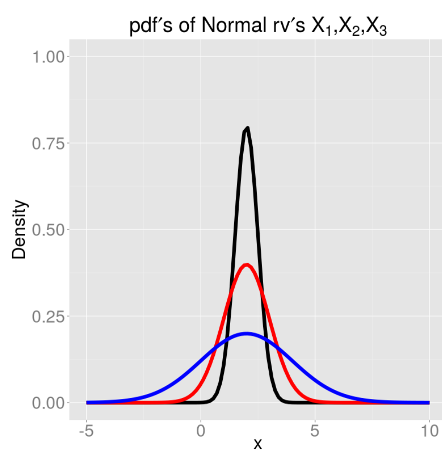
- **NORMAL RANDOM VARIABLES (SUMMARY):**

Notation	$X \sim \text{Normal}(\mu, \sigma^2), \quad \mu \in \mathbb{R},\ \sigma^2 > 0$
Parameter(s)	$\mu \equiv$ Mean $\sigma^2 \equiv$ Variance
Support	$\text{Supp}(X) = (-\infty, \infty)$
pdf	$f_X(x; \mu, \sigma) = \dfrac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)}$
cdf	$\Phi\left(\dfrac{x-\mu}{\sigma}\right) \quad \leftarrow$ (Φ is **std normal** cdf)
Mean	$\mathbb{E}[X] = \mu$
Variance	$\mathbb{V}[X] = \sigma^2$
Model(s)	Exam Scores, Heights, Measurement Errors
Assumption(s)	1. Very large population size.

- **HOW CHANGING μ & σ AFFECT A NORMAL DISTRIBUTION'S LOCATION & SHAPE:**



$X_1 \sim \text{Normal}(1, 1)$ (Red Curve)
$X_2 \sim \text{Normal}(2, 1)$ (Black Curve)
$X_3 \sim \text{Normal}(3, 1)$ (Blue Curve)

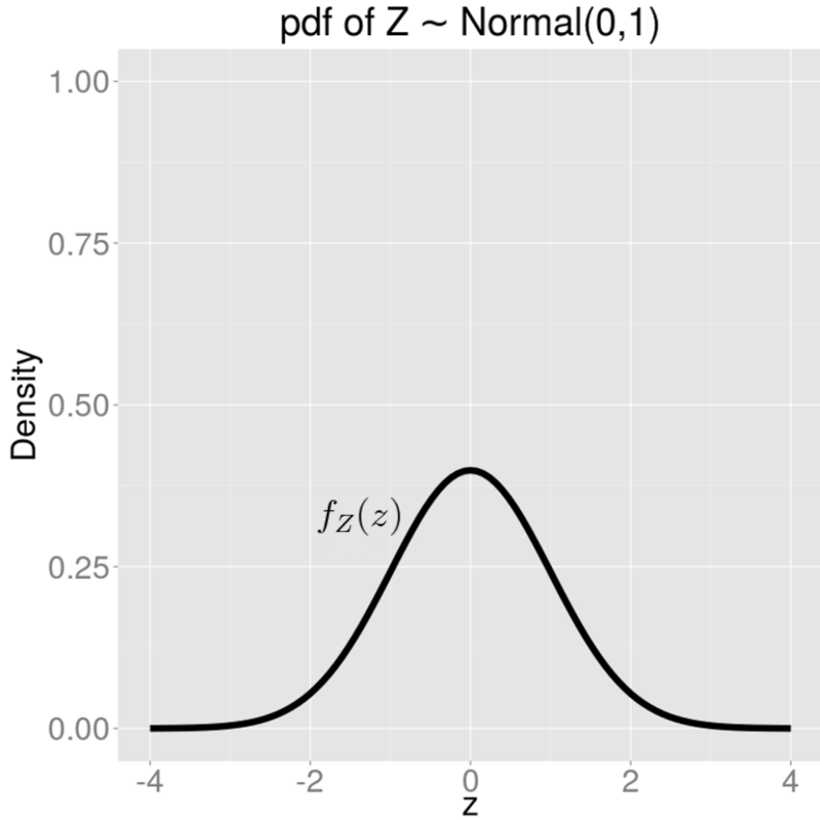


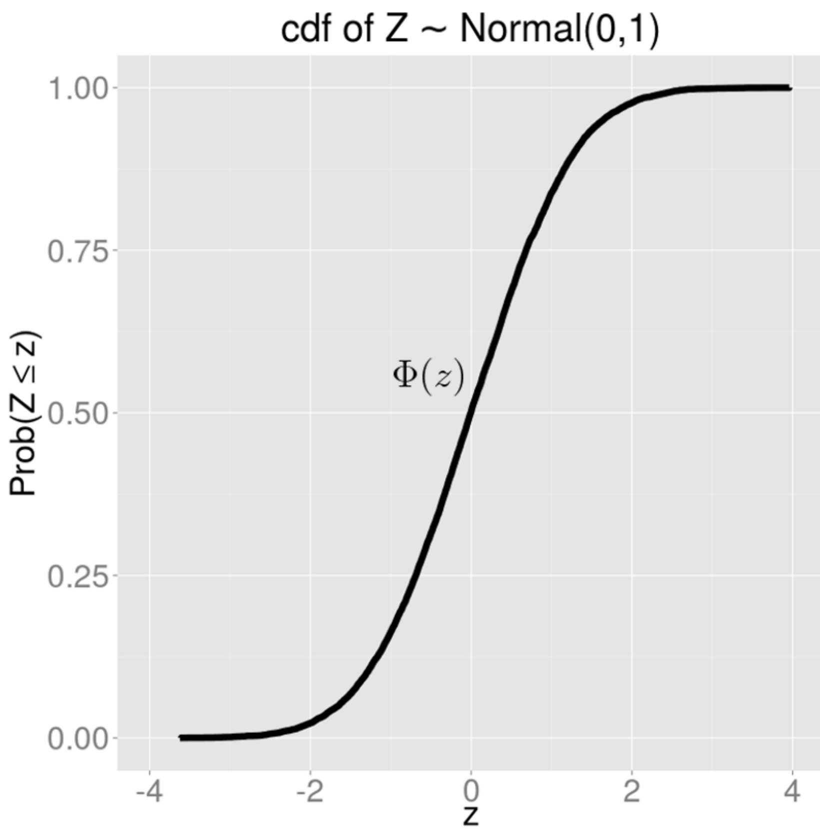
$X_1 \sim \text{Normal}(1, 0.25)$ (Black Curve)
$X_2 \sim \text{Normal}(1, 1)$ (Red Curve)
$X_3 \sim \text{Normal}(1, 4)$ (Blue Curve)

- **PROPERTIES OF NORMAL DISTRIBUTIONS:** Let $X \sim \text{Normal}(\mu, \sigma^2)$. Then:
  - The density curve is **unimodal, bell-shaped, and not skewed**.
  - The density curve is **symmetric** about its **mean** μ.
  - The mean & median of X are equal: $\widetilde{\mu}_X = \mu$

STANDARD NORMAL CDF $\Phi(z) := \mathbb{P}(Z \leq z)$

z	**+0.00**	**+0.01**	**+0.02**	**+0.03**	**+0.04**	**+0.05**	**+0.06**	**+0.07**	**+0.08**	**+0.09**
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997
4.0	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998	0.99998	0.99998	0.99998
4.1	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99999	0.99999
4.2	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999
4.3	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999
4.4	0.99999	0.99999	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000


pdf of Z ~ Normal(0,1)
Density
1.00
0.75
0.50
0.25
0.00
$f_Z(z)$
-4
-2
0
2
4
z


cdf of Z ~ Normal(0,1)
Prob(Z ≤ z)
1.00
0.75
0.50
0.25
0.00
$\Phi(z)$
-4
-2
0
2
4
z

USING STD NORMAL CDF $\Phi(z)$ TO COMPUTE PROBABILITIES [DEVORE 4.3]

z	+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53980	0.54380	0.54776	0.55172	0.55567	0.55966	0.56360	0.56749	0.57142	0.57535
0.2	0.57930	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
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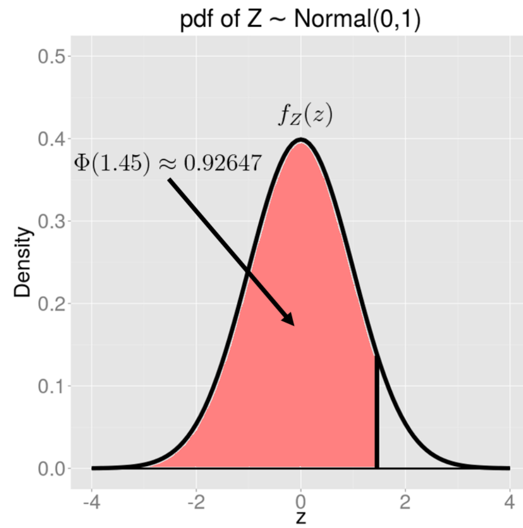
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408



TI-82/83/84+	normalcdf(-1E99,1.45,0,1)	2nd → VARS
TI-86	nmcdf(-1E99,1.45,0,1)	2nd → MATH
TI-89	Normal cdf	APPS → Stats
TI-36X Pro	Normalcdf	2nd → data
MATLAB	normcdf(1.45)	(Stats Toolbox)
R	pnorm(1.45)	
Python	scipy.stats.norm.cdf(1.45)	(Needs SciPy)

$$\Phi(-1.23) = 1 - \Phi(1.23) \approx 1 - 0.89065 = 0.10935$$

z	+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53980	0.54380	0.54776	0.55172	0.55567	0.55966	0.56360	0.56749	0.57142	0.57535
0.2	0.57930	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
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0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
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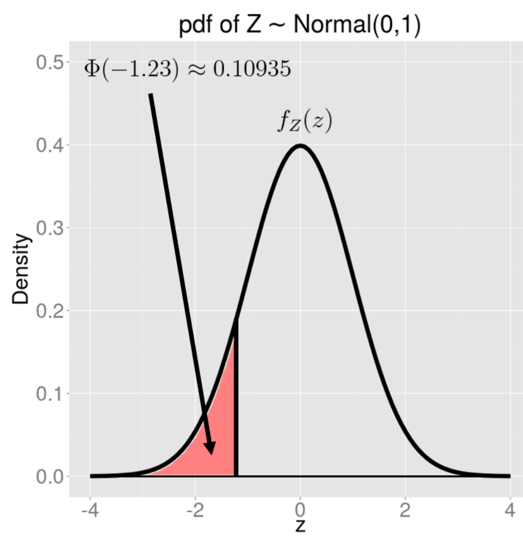
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408



TI-82/83/84+	normalcdf(-1E99,-1.23,0,1)	2nd → VARS
TI-86	nmcdf(-1E99,-1.23,0,1)	2nd → MATH
TI-89	Normal cdf	APPS → Stats
TI-36X Pro	Normalcdf	2nd → data
MATLAB	normcdf(-1.23)	(Stats Toolbox)
R	pnorm(-1.23)	
Python	scipy.stats.norm.cdf(-1.23)	(Needs SciPy)

IMPORTANT: The $\Phi(z)$ table **will be provided on exams**, so computing $\Phi(z)$ via calculator/software is not required.

EX 4.3.1: The amount of rainfall (in inches) in Houston during March follows a Uniform($a = 1.52, b = 4.54$) distribution. Let X represent the aforementioned rainfall amount.

(a) What is the pdf of X, $f_X(x)$?

(b) What is the cdf of X, $F_X(x)$?

(c) What is the expected amount of rainfall in Houston during March?

(d) What is the standard deviation of the amount of rainfall in Houston during March?

(e) What is the variance of the amount of rainfall in Houston during March?

(f) What is the probability that Houston receives at most 2 inches of rainfall during March of next year?

(g) What is the probability that Houston receives between 2 and 3 inches of rainfall during March of next year?

(h) What is the probability that Houston receives at least 2.5 inches of rainfall during March of next year?

(i) What is the probability that Houston receives between 0.5 and 4 inches of rainfall during March of next year?

EX 4.3.2: Consider the **standard normal distribution**: Let random variable $Z \sim \text{Normal}(\mu = 0, \sigma = 1)$

(a) **Using the provided standard normal cdf table**, compute $\Phi(1.8)$.

(b) **Using the provided standard normal cdf table**, compute $\Phi(-1.8)$.

(c) **Using the provided standard normal cdf table**, compute $\mathbb{P}(Z \leq 2.47)$.

(d) **Using the provided standard normal cdf table**, compute $\mathbb{P}(Z \geq 2.47)$.

(e) **Using the provided standard normal cdf table**, compute $\mathbb{P}(1.15 < Z \leq 2.47)$.

(f) **Using the provided standard normal cdf table**, compute $\mathbb{P}(Z > -2.3)$.

(g) **Using the provided standard normal cdf table**, compute $\mathbb{P}(-2.5 < Z < -2.3)$.

(h) **Using the provided standard normal cdf table**, determine the 57^{th} percentile of Z, $z_{0.57}$.
(Interpolate if necessary, which means find the closest entry in the table.)

(i) **Using the provided standard normal cdf table**, determine the 13^{th} percentile of Z, $z_{0.13}$.
(Interpolate if necessary, which means find the closest entry in the table.)

EX 4.3.3: The weights of cans of pineapples follow a normal distribution with a mean of 1000 g and std deviation of 50 g.

(a) What is the probability of a can of pineapples weighing more than 1075 grams?

(b) What is the probability of a can of pineapples weighing between 970 and 1140 grams?

(c) What is the probability of a can of pineapples weighing less than 905 grams?

(d) What is the probability of a can of pineapples weighing within 10 grams of 960 grams?

(e) What is the probability of a can of pineapples weighing within two standard deviations of the mean?

(f) What is the probability of a can of pineapples weighing outside one standard deviation of the mean?