EX 7.2.1: Consider the population of all college student heights and the average height μ (in feet.)

(a) Suppose a sample of size ($n = 50$) is taken from the population.

Moreover, the sample mean $\overline{x} = 5.1$ ft and the sample std deviation $s = 1.2$ ft.

Construct the approximate 90% and 95% confidence intervals for μ.

$$\overline{x} \pm z^*_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Approximate 90% CI for μ: $5.1 \pm \mathbf{1.64} \cdot \frac{1.2}{\sqrt{50}} \implies 5.1 \pm 0.2783 \implies (5.1 - 0.2783,\ 5.1 + 0.2783) = \boxed{(\mathbf{4.8217},\ \mathbf{5.3783})}$

Approximate 95% CI for μ: $5.1 \pm \mathbf{1.96} \cdot \frac{1.2}{\sqrt{50}} \implies 5.1 \pm 0.3326 \implies (5.1 - 0.3326,\ 5.1 + 0.3326) = \boxed{(\mathbf{4.7674},\ \mathbf{5.4326})}$

(b) Suppose a sample of size ($n = 100$) is taken from the population.

Moreover, the sample mean $\overline{x} = 5.5$ ft and the sample std deviation $s = 1.4$ ft.

Construct the approximate 90% and 95% confidence intervals for μ.

$$\overline{x} \pm z^*_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Approximate 90% CI for μ: $5.5 \pm \mathbf{1.64} \cdot \frac{1.4}{\sqrt{100}} \implies 5.5 \pm 0.2296 \implies (5.5 - 0.2296,\ 5.1 + 0.2296) = \boxed{(\mathbf{5.2704},\ \mathbf{5.7296})}$

Approximate 95% CI for μ: $5.5 \pm \mathbf{1.96} \cdot \frac{1.4}{\sqrt{100}} \implies 5.5 \pm 0.2744 \implies (5.5 - 0.2744,\ 5.1 + 0.2744) = \boxed{(\mathbf{5.2523},\ \mathbf{5.7477})}$

(c) Which of the four confidence intervals from parts (a) & (b) has the most precision?

The CI with the most precision has the shortest width: $\boxed{(\mathbf{5.2704},\ \mathbf{5.7296})}$

(d) What is the minimum sample size n needed to construct an approx. 95% CI for estimating μ to within 0.25 foot?

$w = 0.25$, $z^*_{\alpha/2} = 1.96$, Best guess for $s = \frac{1.2 + 1.4}{2} = 1.3$

$$\therefore \quad n = \left\lceil \left(\frac{z^*_{\alpha/2} \cdot s}{w} \right)^2 \right\rceil = \left\lceil \left(\frac{(1.96)(1.3)}{0.25} \right)^2 \right\rceil = \lceil 103.876864 \rceil = \boxed{\mathbf{104}}$$

(e) What is the minimum sample size n needed to construct an approx. 95% CI for estimating μ to within 0.10 feet?

$w = 0.10$, $z^*_{\alpha/2} = 1.96$, Best guess for $s = \frac{1.2 + 1.4}{2} = 1.3$

$$\therefore \quad n = \left\lceil \left(\frac{z^*_{\alpha/2} \cdot s}{w} \right)^2 \right\rceil = \left\lceil \left(\frac{(1.96)(1.3)}{0.10} \right)^2 \right\rceil = \lceil 649.2304 \rceil = \boxed{\mathbf{650}}$$

EX 7.2.2: A dollar store owner wants to determine the proportion of customers who pay with a credit card.

(a) One hundred customers are surveyed with the result that thirty of them paid with a credit card.

Construct approx 90% and 95% Wilson Score CI's for the proportion of all customers who pay with credit card.

$n = 100$, "Success" $\equiv$ "Customer pays with credit card", $\widehat{p} = \dfrac{\#\text{ Successes in Sample}}{\text{Sample Size}} = \dfrac{30}{100} = 0.3 \implies \widehat{q} = 1 - \widehat{p} = 0.7$

$$\frac{n\widehat{p} + 0.5(z^*_{\alpha/2})^2}{n + (z^*_{\alpha/2})^2} \pm z^*_{\alpha/2} \cdot \frac{\sqrt{n\widehat{p}\widehat{q} + 0.25(z^*_{\alpha/2})^2}}{n + (z^*_{\alpha/2})^2}$$

90% Wilson Score CI for p: $\dfrac{(100)(0.3) + (0.5)(\mathbf{1.64})^2}{100 + (\mathbf{1.64})^2} \pm (\mathbf{1.64}) \cdot \dfrac{\sqrt{(100)(0.3)(0.7) + (0.25)(\mathbf{1.64})^2}}{100 + (\mathbf{1.64})^2} \implies 0.3052 \pm 0.0743$

$$\implies (0.3052 - 0.0743,\ 0.3052 + 0.0743) = \boxed{(\mathbf{0.2309},\ \mathbf{0.3795})}$$

95% Wilson Score CI for p: $\dfrac{(100)(0.3) + (0.5)(\mathbf{1.96})^2}{100 + (\mathbf{1.96})^2} \pm (\mathbf{1.96}) \cdot \dfrac{\sqrt{(100)(0.3)(0.7) + (0.25)(\mathbf{1.96})^2}}{100 + (\mathbf{1.96})^2} \implies 0.3074 \pm 0.0885$

$$\implies (0.3074 - 0.0885,\ 0.3074 + 0.0885) = \boxed{(\mathbf{0.2189},\ \mathbf{0.3959})}$$

(b) Two hundred customers are surveyed with the result that forty of them paid with a credit card.

Construct approx 90% and 95% Wilson Score CI's for the proportion of all customers who pay with credit card.

$n = 200$, "Success" $\equiv$ "Customer pays with credit card", $\widehat{p} = \dfrac{\#\text{ Successes in Sample}}{\text{Sample Size}} = \dfrac{40}{200} = 0.2 \implies \widehat{q} = 1 - \widehat{p} = 0.8$

$$\frac{n\widehat{p} + 0.5(z^*_{\alpha/2})^2}{n + (z^*_{\alpha/2})^2} \pm z^*_{\alpha/2} \cdot \frac{\sqrt{n\widehat{p}\widehat{q} + 0.25(z^*_{\alpha/2})^2}}{n + (z^*_{\alpha/2})^2}$$

90% Wilson Score CI for p: $\dfrac{(200)(0.2) + (0.5)(\mathbf{1.64})^2}{200 + (\mathbf{1.64})^2} \pm (\mathbf{1.64}) \cdot \dfrac{\sqrt{(200)(0.2)(0.8) + (0.25)(\mathbf{1.64})^2}}{200 + (\mathbf{1.64})^2} \implies 0.2040 \pm 0.0462$

$$\implies (0.2040 - 0.0462,\ 0.2040 + 0.0462) = \boxed{(\mathbf{0.1578},\ \mathbf{0.2502})}$$

95% Wilson Score CI for p: $\dfrac{(200)(0.2) + (0.5)(\mathbf{1.96})^2}{200 + (\mathbf{1.96})^2} \pm (\mathbf{1.96}) \cdot \dfrac{\sqrt{(200)(0.2)(0.8) + (0.25)(\mathbf{1.96})^2}}{200 + (\mathbf{1.96})^2} \implies 0.2057 \pm 0.0552$

$$\implies (0.2057 - 0.0552,\ 0.2057 + 0.0552) = \boxed{(\mathbf{0.1505},\ \mathbf{0.2609})}$$

HOW TO EFFECTIVELY ENTER THE PREVIOUS EXPRESSIONS ON CALCULATOR:

$\dfrac{(200)(0.2) + (0.5)(\mathbf{1.96})^2}{200 + (\mathbf{1.96})^2}$

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((200*0.2) + (0.5*1.96^2)) / (200+1.96^2)
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$\dfrac{\sqrt{(200)(0.2)(0.8) + (0.25)(\mathbf{1.96})^2}}{200 + (\mathbf{1.96})^2}$

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SQRT((200*0.2*0.8) + (0.25*1.96^2)) / (200+1.96^2)
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