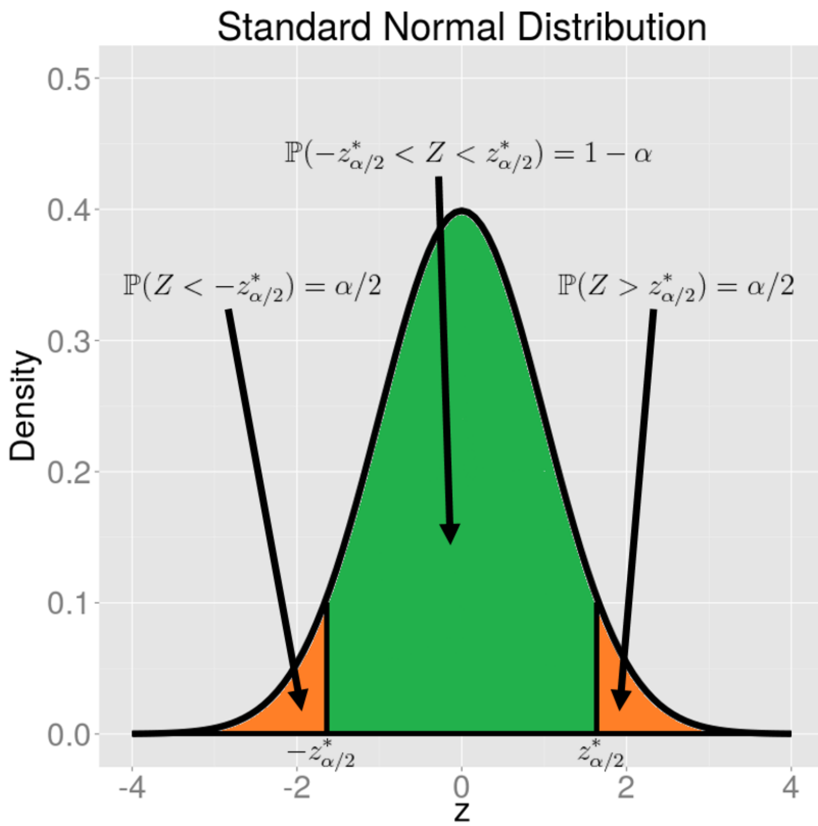
1-SAMPLE INFERENCE: LARGE-SAMPLE CI'S FOR MEAN/PROPORTION [DEVORE 7.2]

- **z CRITICAL VALUES:**

  $z^*_{\alpha/2}$ is called a z **critical value** of the std normal dist s.t. its upper-tail probability is $\alpha/2$: $\mathbb{P}(Z > z^*_{\alpha/2}) = \alpha/2$

  IMPORTANT: Do not confuse z critical value $z^*_{\alpha/2}$ with the z percentile $z_{\alpha/2}$: $\mathbb{P}(Z \leq z_{\alpha/2}) = \alpha/2$

  Finally, notice that $z^*_{\alpha/2}$ is always **positive**.



Common z critical values:

	90% CI	95% CI	99% CI
$z^*_{\alpha/2}$	1.64	1.96	2.58

- **LARGE-SAMPLE CI FOR POPULATION MEAN:**

  Given any population with mean μ. Let $x_1, \ldots, x_n$ be a large sample ($n > 40$) taken from the population.

  Then the $100(1-\alpha)\%$ **large-sample CI for** μ is approximately

$$\left(\overline{x} - z^*_{\alpha/2} \cdot \frac{s}{\sqrt{n}},\ \ \overline{x} + z^*_{\alpha/2} \cdot \frac{s}{\sqrt{n}}\right) \quad \text{OR WRITTEN MORE COMPACTLY} \quad \overline{x} \pm z^*_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

  If a **half-width** of w is desired for $100(1-\alpha)\%$ CI yielding $(\overline{x} - w, \overline{x} + w)$, then the minimum sample size required is

$$n = \left\lceil \left(\frac{z^*_{\alpha/2} \cdot s}{w}\right)^2 \right\rceil, \quad \text{where } s \text{ is the "best guess" for the sample std deviation.}$$

- **WILSON SCORE CI FOR POPULATION PROPORTION:**

  Given any population with proportion p of some "success." Let $x_1, \ldots, x_n$ be a sample taken from the population.

  Then the $100(1-\alpha)\%$ **Wilson score CI for** p is approximately

$$\frac{n\widehat{p} + 0.5(z^*_{\alpha/2})^2}{n + (z^*_{\alpha/2})^2} \pm z^*_{\alpha/2} \cdot \frac{\sqrt{n\widehat{p}\widehat{q} + 0.25(z^*_{\alpha/2})^2}}{n + (z^*_{\alpha/2})^2}$$

  where $\widehat{p} := \dfrac{X}{n} \equiv \dfrac{\#\text{ Successes in Sample}}{\text{Sample Size}}$ and $\widehat{q} := 1 - \widehat{p}$

EX 7.2.1: Consider the population of all college student heights and the average height μ (in feet.)

(a) Suppose a sample of size $(n = 50)$ is taken from the population.

Moreover, the sample mean $\overline{x} = 5.1$ ft and the sample std deviation $s = 1.2$ ft.

Construct the approximate 90% and 95% confidence intervals for μ.

(b) Suppose a sample of size $(n = 100)$ is taken from the population.

Moreover, the sample mean $\overline{x} = 5.5$ ft and the sample std deviation $s = 1.4$ ft.

Construct the approximate 90% and 95% confidence intervals for μ.

(c) Which of the four confidence intervals from parts (a) & (b) has the most precision?

(d) What is the minimum sample size n needed to construct an approx. 95% CI for estimating μ to within 0.25 foot?

(e) What is the minimum sample size n needed to construct an approx. 95% CI for estimating μ to within 0.10 feet?

EX 7.2.2: A dollar store owner wants to determine the proportion of customers who pay with a credit card.

(a) One hundred customers are surveyed with the result that thirty of them paid with a credit card.

Construct approx 90% and 95% Wilson Score CI's for the proportion of all customers who pay with credit card.

(b) Two hundred customers are surveyed with the result that forty of them paid with a credit card.

Construct approx 90% and 95% Wilson Score CI's for the proportion of all customers who pay with credit card.