EX 7.3.1: Consider the normal population of all college student heights and the average height μ (in feet.)

(a) Suppose a sample of size $(n = 5)$ is taken from the population.

Moreover, the sample mean $\overline{x} = 5.1$ ft and the sample std deviation $s = 1.2$ ft.

Construct the 90% and 95% confidence intervals for μ.

$$\overline{x} \pm t^*_{n-1,\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

90% CI for μ: $5.1 \pm t^*_{4,0.05} \cdot \frac{1.2}{\sqrt{5}} \implies 5.1 \pm \mathbf{2.132} \cdot \frac{1.2}{\sqrt{5}} \implies 5.1 \pm 1.1442 \implies (5.1 - 1.1442,\ 5.1 + 1.1442) = \boxed{(\mathbf{3.9558},\ \mathbf{6.2442})}$

95% CI for μ: $5.1 \pm t^*_{4,0.025} \cdot \frac{1.2}{\sqrt{5}} \implies 5.1 \pm \mathbf{2.776} \cdot \frac{1.2}{\sqrt{5}} \implies 5.1 \pm 1.4898 \implies (5.1 - 1.4898,\ 5.1 + 1.4898) = \boxed{(\mathbf{3.6102},\ \mathbf{6.5898})}$

(b) Suppose a sample of size $(n = 10)$ is taken from the population.

Moreover, the sample mean $\overline{x} = 5.5$ ft and the sample std deviation $s = 1.4$ ft.

Construct the 90% and 95% confidence intervals for μ.

$$\overline{x} \pm t^*_{n-1,\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

90% CI for μ: $5.5 \pm t^*_{9,0.05} \cdot \frac{1.4}{\sqrt{10}} \implies 5.5 \pm \mathbf{1.833} \cdot \frac{1.4}{\sqrt{10}} \implies 5.5 \pm 0.8115 \implies (5.5 - 0.8115,\ 5.5 + 0.8115) = \boxed{(\mathbf{4.6885},\ \mathbf{6.3115})}$

95% CI for μ: $5.5 \pm t^*_{9,0.025} \cdot \frac{1.4}{\sqrt{10}} \implies 5.5 \pm \mathbf{2.262} \cdot \frac{1.4}{\sqrt{10}} \implies 5.5 \pm 1.0014 \implies (5.5 - 1.0014,\ 5.5 + 1.0014) = \boxed{(\mathbf{4.4986},\ \mathbf{6.5014})}$

(c) Which of the four confidence intervals from parts (a) & (b) has the most precision?

The CI with the most precision has the shortest width: $\boxed{(\mathbf{4.6885},\ \mathbf{6.3115})}$