GOSSET'S t DISTRIBUTION (AKA STUDENT'S t) [DEVORE 7.3]

• GOSSET'S t DISTRIBUTION (SUMMARY):

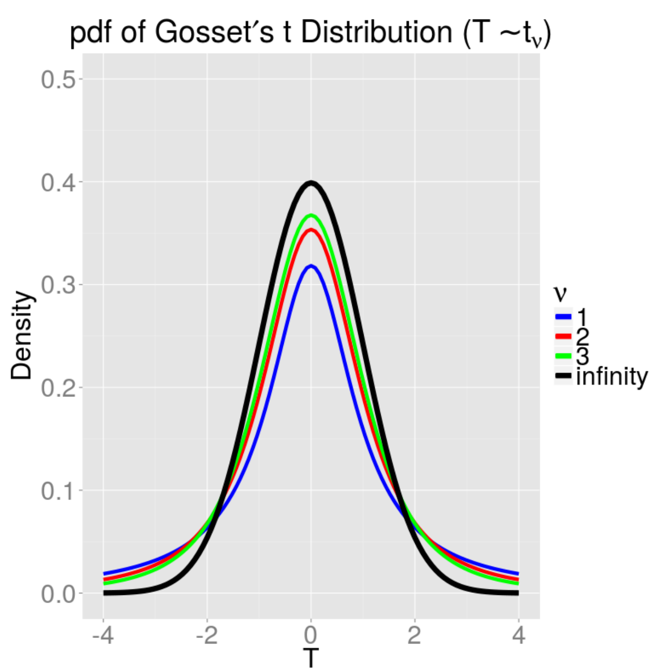
Notation	$T \sim t_\nu$
Parameter(s)	$\nu \equiv \#$ Degrees of Freedom $(\nu = 1, 2, 3, 4, \cdots)$
Support	$\mathrm{Supp}(T) = (-\infty, \infty)$
pdf	$f_T(t;\nu) := \dfrac{\Gamma((\nu+1)/2)}{\sqrt{\pi\nu}\cdot\Gamma(\nu/2)} \cdot \dfrac{1}{[1+(t^2/\nu)]^{(v+1)/2}}$
cdf	$\Phi_t(t;\nu) = \dfrac{\Gamma((\nu+1)/2)}{\sqrt{\pi\nu}\cdot\Gamma(\nu/2)} \displaystyle\int_{-\infty}^{t} \dfrac{1}{[1+(\tau^2/\nu)]^{(v+1)/2}}\, d\tau$
Mean	$\mathbb{E}[T] = +\infty$, for $\nu = 1$ $\mathbb{E}[T] = 0$, for $\nu > 1$
Variance	$\mathbb{V}[T] = +\infty$, for $\nu = 1, 2$ $\mathbb{V}[T] = \nu/(\nu-2)$, for $\nu > 2$
Model(s)	(Used exclusively for Statistical Inference)

ν is the lowercase Greek letter "nu" τ is the lowercase Greek letter "tau"

• GOSSET'S t DISTRIBUTION (PROPERTIES):

- t_ν is symmetric, bell-shaped and centered at zero.
- t_ν is more spread out than the standard normal curve.
- The spread of the t_ν curve decreases as ν increases.
- As $\nu \to \infty$, the t_ν curves approaches the standard normal curve.

• GOSSET'S t DISTRIBUTION (PLOTS):



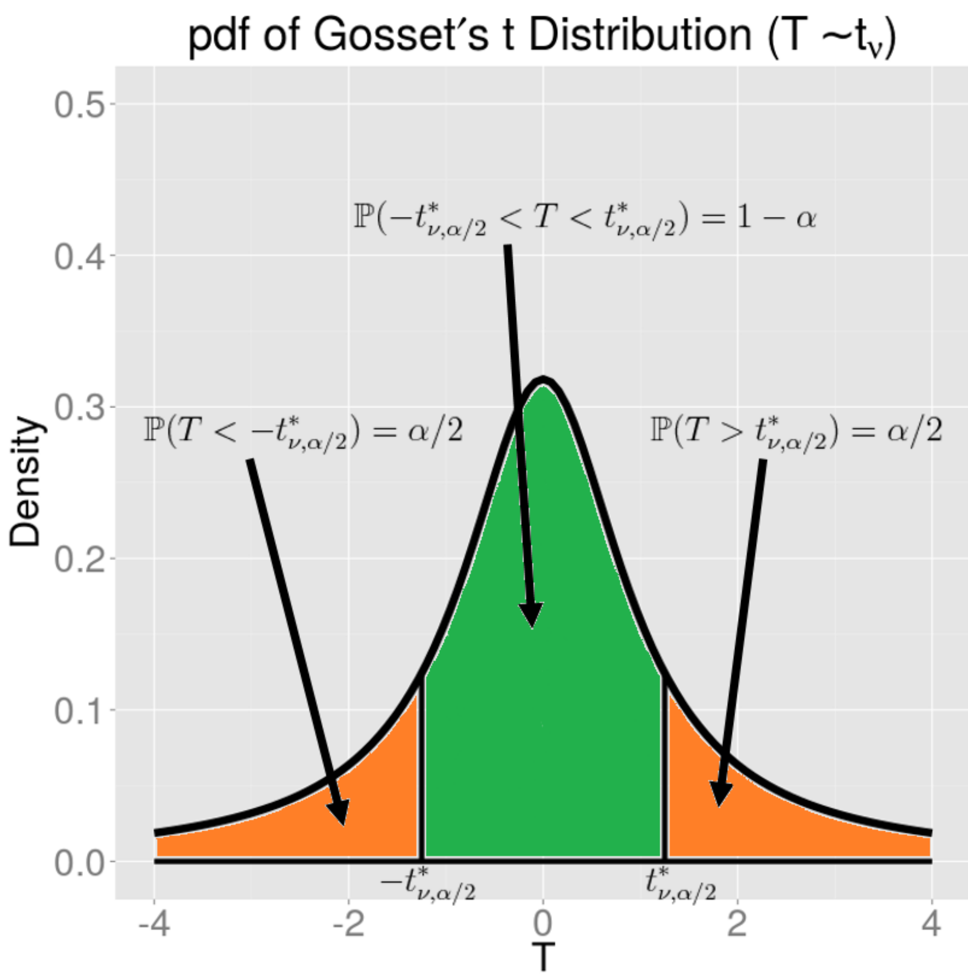
The black curve is the **Standard Normal curve**.

1-SAMPLE INFERENCE: SMALL-SAMPLE CI'S FOR MEAN [DEVORE 7.3]

- **t CRITICAL VALUES (DEFINITION):** $t^*_{\nu,\alpha/2}$ is called a t **critical value** of the t distribution with ν df's s.t. its upper-tail probability is exactly its subscript value $\alpha/2$: $\mathbb{P}(T > t^*_{\nu,\alpha/2}) = \alpha/2$

  IMPORTANT: Do not confuse t critical value $t^*_{\nu,\alpha/2}$ with the t percentile $t_{\nu,\alpha/2}$: $\mathbb{P}(T \le t_{\nu,\alpha/2}) = \alpha/2$

  Finally, notice that $t^*_{\nu,\alpha/2}$ is always **positive**.



- **A STATISTICS RELATED TO t DISTRIBUTION:** Let $X_1, \dots, X_n \overset{iid}{\sim} \text{Normal}(\mu, \sigma^2)$ population. Then:

$$\frac{\overline{X} - \mu}{S/\sqrt{n}} \sim t_{n-1}$$

- **SMALL-SAMPLE CI FOR NORMAL POPULATION MEAN μ :**

  Given a normal population with unknown mean μ and std dev σ.

  Let $x_1, \dots, x_n$ be a small sample taken from the population.

  Then the $100(1-\alpha)\%$ **small-sample CI for** μ is

$$\left(\overline{x} - t^*_{n-1,\alpha/2} \cdot \frac{s}{\sqrt{n}},\quad \overline{x} + t^*_{n-1,\alpha/2} \cdot \frac{s}{\sqrt{n}}\right)$$

— OR WRITTEN MORE COMPACTLY —

$$\overline{x} \pm t^*_{n-1,\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

GOSSET'S t CRITICAL VALUES, $t^*_{\nu,\alpha/2}$ $\quad \mathbb{P}(T > t^*_{\nu,\alpha/2}) = \alpha/2$

ν	**90% CI** ($\alpha/2 = 0.05$)	**95% CI** ($\alpha/2 = 0.025$)	**99% CI** ($\alpha/2 = 0.005$)
1	6.314	12.706	63.657
2	2.920	4.303	9.925
3	2.353	3.182	5.841
4	2.132	2.776	4.604
5	2.015	2.571	4.032
6	1.943	2.447	3.707
7	1.895	2.365	3.499
8	1.860	2.306	3.355
9	1.833	2.262	3.250
10	1.812	2.228	3.169
11	1.796	2.201	3.106
12	1.782	2.179	3.055
13	1.771	2.160	3.012
14	1.761	2.145	2.977
15	1.753	2.131	2.947
16	1.746	2.120	2.921
17	1.740	2.110	2.898
18	1.734	2.101	2.878
19	1.729	2.093	2.861
20	1.725	2.086	2.845
21	1.721	2.080	2.831
22	1.717	2.074	2.819
23	1.714	2.069	2.807
24	1.711	2.064	2.797
25	1.708	2.060	2.787
26	1.706	2.056	2.779
27	1.703	2.052	2.771
28	1.701	2.048	2.763
29	1.699	2.045	2.756
30	1.697	2.042	2.750
31	1.696	2.040	2.744
32	1.694	2.037	2.738
33	1.692	2.035	2.733
34	1.691	2.032	2.728
35	1.690	2.030	2.724
36	1.688	2.028	2.719
37	1.687	2.026	2.715
38	1.686	2.024	2.712
39	1.685	2.023	2.708
40	1.684	2.021	2.704
60	1.671	2.000	2.660
120	1.658	1.980	2.617

$\boxed{\textbf{EX 7.3.1:}}$ Consider the <u>normal</u> population of all college student heights and the average height μ (in feet.)

(a) Suppose a sample of size ($n = 5$) is taken from the population.

Moreover, the sample mean $\overline{x} = 5.1$ ft and the sample std deviation $s = 1.2$ ft.

Construct the approximate 90% and 95% confidence intervals for μ.

(b) Suppose a sample of size ($n = 10$) is taken from the population.

Moreover, the sample mean $\overline{x} = 5.5$ ft and the sample std deviation $s = 1.4$ ft.

Construct the approximate 90% and 95% confidence intervals for μ.

(c) Which of the four confidence intervals from parts (a) & (b) has the most precision?