EX 7.4.1: Consider the <u>normal</u> population of all college student heights and the height variance σ^2 (in feet^2.)

(a) Suppose a sample of size $(n = 5)$ is taken from the population.

Moreover, the sample mean $\overline{x} = 5.1$ ft and the sample std deviation $s = 1.2$ ft.

Construct the 90% and 95% confidence intervals for σ^2 & σ.

90% CI for σ^2: $$\left(\frac{(n-1)s^2}{\chi^{2*}_{n-1,\alpha/2}}, \frac{(n-1)s^2}{\chi^{2*}_{n-1,1-\alpha/2}}\right) = \left(\frac{(5-1)(1.2^2)}{\chi^{2*}_{4,0.05}}, \frac{(5-1)(1.2^2)}{\chi^{2*}_{4,0.95}}\right) = \left(\frac{5.76}{9.488}, \frac{5.76}{0.711}\right) = \boxed{(\mathbf{0.6071},\ \mathbf{8.1013})}\ \text{ft}^2$$

95% CI for σ^2: $$\left(\frac{(n-1)s^2}{\chi^{2*}_{n-1,\alpha/2}}, \frac{(n-1)s^2}{\chi^{2*}_{n-1,1-\alpha/2}}\right) = \left(\frac{(5-1)(1.2^2)}{\chi^{2*}_{4,0.025}}, \frac{(5-1)(1.2^2)}{\chi^{2*}_{4,0.975}}\right) = \left(\frac{5.76}{11.143}, \frac{5.76}{0.484}\right) = \boxed{(\mathbf{0.5169},\ \mathbf{11.9008})}\ \text{ft}^2$$

90% CI for σ: $$\left(\sqrt{\frac{(n-1)s^2}{\chi^{2*}_{n-1,\alpha/2}}}, \sqrt{\frac{(n-1)s^2}{\chi^{2*}_{n-1,1-\alpha/2}}}\right) = \left(\sqrt{\frac{5.76}{9.488}}, \sqrt{\frac{5.76}{0.711}}\right) = \boxed{(\mathbf{0.7792},\ \mathbf{2.8463})}\ \text{ft}$$

95% CI for σ: $$\left(\sqrt{\frac{(n-1)s^2}{\chi^{2*}_{n-1,\alpha/2}}}, \sqrt{\frac{(n-1)s^2}{\chi^{2*}_{n-1,1-\alpha/2}}}\right) = \left(\sqrt{\frac{5.76}{11.143}}, \sqrt{\frac{5.76}{0.484}}\right) = \boxed{(\mathbf{0.7190},\ \mathbf{3.4498})}\ \text{ft}$$

(b) Suppose a sample of size $(n = 10)$ is taken from the population.

Moreover, the sample mean $\overline{x} = 5.5$ ft and the sample std deviation $s = 1.4$ ft.

Construct the 90% and 95% confidence intervals for σ^2 & σ.

90% CI for σ^2: $$\left(\frac{(n-1)s^2}{\chi^{2*}_{n-1,\alpha/2}}, \frac{(n-1)s^2}{\chi^{2*}_{n-1,1-\alpha/2}}\right) = \left(\frac{(10-1)(1.4^2)}{\chi^{2*}_{9,0.05}}, \frac{(10-1)(1.4^2)}{\chi^{2*}_{9,0.95}}\right) = \left(\frac{17.64}{16.919}, \frac{17.64}{3.325}\right) = \boxed{(\mathbf{1.0426},\ \mathbf{5.3053})}\ \text{ft}^2$$

95% CI for σ^2: $$\left(\frac{(n-1)s^2}{\chi^{2*}_{n-1,\alpha/2}}, \frac{(n-1)s^2}{\chi^{2*}_{n-1,1-\alpha/2}}\right) = \left(\frac{(10-1)(1.4^2)}{\chi^{2*}_{9,0.025}}, \frac{(10-1)(1.4^2)}{\chi^{2*}_{9,0.975}}\right) = \left(\frac{17.64}{19.023}, \frac{17.64}{2.700}\right) = \boxed{(\mathbf{0.9273},\ \mathbf{6.5333})}\ \text{ft}^2$$

90% CI for σ: $$\left(\sqrt{\frac{(n-1)s^2}{\chi^{2*}_{n-1,\alpha/2}}}, \sqrt{\frac{(n-1)s^2}{\chi^{2*}_{n-1,1-\alpha/2}}}\right) = \left(\sqrt{\frac{17.64}{16.919}}, \sqrt{\frac{17.64}{3.325}}\right) = \boxed{(\mathbf{1.0211},\ \mathbf{2.3033})}\ \text{ft}$$

95% CI for σ: $$\left(\sqrt{\frac{(n-1)s^2}{\chi^{2*}_{n-1,\alpha/2}}}, \sqrt{\frac{(n-1)s^2}{\chi^{2*}_{n-1,1-\alpha/2}}}\right) = \left(\sqrt{\frac{17.64}{19.023}}, \sqrt{\frac{17.64}{2.700}}\right) = \boxed{(\mathbf{0.9630},\ \mathbf{2.5560})}\ \text{ft}$$

(c) Which of the four confidence intervals for σ^2 from parts (a) & (b) has the most precision?

The CI for σ^2 with the most precision has the <u>shortest width</u>: $\boxed{(\mathbf{1.0426},\ \mathbf{5.3053})}$ ft^2
