OVERVIEW OF HYPOTHESIS TESTS [DEVORE 8.1]

- **NULL & ALTERNATIVE HYPOTHESES:**

  The **null hypothesis**, denoted H_0, is the claim initially assumed to be true.

  The **alternative hypothesis**, denoted H_A, is a claim that contradicts H_0.

  A **hypothesis test** uses sample data & probability to decide whether the null hypothesis is a reasonable statement.

  If sample evidence strongly contradicts H_0, then the null hypothesis will be rejected in favor of the alternative hypothesis.

  If sample evidence does not strongly contradict H_0, then it's reasonable to continue to believe that H_0 is still plausible.

  i.e. THINK: "Innocent until proven guilty." i.e. THINK: The Scientific Method

  The two possible conclusions from a hypothesis test are: "reject H_0" OR "accept H_0"

  NOTE: Sometimes one says "fail to reject H_0" instead of "accept H_0".

- **THE CORRECT FORMAT FOR HYPOTHESIS TESTS:** Hypothesis tests must conform to the following rules:
  - Population parameter(s) must be involved. Statistics must not be involved.
  - The null hypothesis H_0 must only involve equality. The alternative hypothesis H_A must not include equality.
  - The asserted value in H_0 should also appear in H_A.

- **TYPE I & TYPE II ERRORS:**

  A **Type I error** (AKA **false positive**) consists of rejecting null hypothesis H_0 when it is true.

  A **Type II error** (AKA **false negative**) consists of not rejecting null hypothesis H_0 when it is false.

$$\alpha = \mathbb{P}(\text{Type I Error}) = \mathbb{P}(\text{Rejecting } H_0 \text{ when } H_0 \text{ is true})$$
$$\beta = \mathbb{P}(\text{Type II Error}) = \mathbb{P}(\text{Accepting } H_0 \text{ when } H_0 \text{ is false})$$

	Decide to Reject H_0	Decide to Accept H_0
H_0 is actually true	**Type I Error**	**(Correct Decision)**
H_A is actually true	**(Correct Decision)**	**Type II Error**

Based on the scenario, one error type may be more serious than the other.

- **P-VALUES:**

  Given random sample $\mathbf{X} := (X_1, X_2, \ldots, X_n)$ of a population with parameter θ.

  Suppose a sample $\mathbf{x} := (x_1, x_2, \ldots, x_n)$ is taken from the population.

  Finally, let $W(\mathbf{X}; \theta_0)$ be the **test statistic** for null hypothesis $H_0: \theta = \theta_0$.

  The **P-value** is the probability of obtaining a value of the test statistic at least as contradictory to H_0 as the value computed from the sample, all while assuming that the null hypothesis is true:

$$\begin{array}{l} H_0: \theta = \theta_0 \\ H_A: \theta > \theta_0 \end{array} \implies \text{P-value} := \mathbb{P}\left(W(\mathbf{X};\theta_0) \geq W(\mathbf{x};\theta_0) \text{ assuming } H_0 \text{ is true}\right)$$

$$\begin{array}{l} H_0: \theta = \theta_0 \\ H_A: \theta < \theta_0 \end{array} \implies \text{P-value} := \mathbb{P}\left(W(\mathbf{X};\theta_0) \leq W(\mathbf{x};\theta_0) \text{ assuming } H_0 \text{ is true}\right)$$

$$\begin{array}{l} H_0: \theta = \theta_0 \\ H_A: \theta \neq \theta_0 \end{array} \implies \text{P-value} := \begin{pmatrix} \text{Requires distribution type of population} \\ \text{Will be encountered in the rest of Ch8} \end{pmatrix}$$

- **SIGNIFICANCE LEVELS:**

  A conclusion is reached in a hypothesis test for θ by choosing a **significance level** α that is reasonably close to zero.

  If P-value $\leq \alpha$, then H_0 will be rejected in favor of H_A

  If P-value $> \alpha$, then H_0 will be accepted (still considered plausible)

  The significance levels used in practice are: $\alpha = 0.05$, $\alpha = 0.01$, $\alpha = 0.001$

  Recall from earlier that $\alpha := \mathbb{P}(\text{Type I Error})$

  Also, α is the same α used in $100(1-\alpha)\%$ confidence intervals.

  i.e. The lower the α-level, the more skeptical the decision maker is.