SMALL-SAMPLE t-TEST ABOUT NORMAL POP. MEAN μ [DEVORE 8.3]

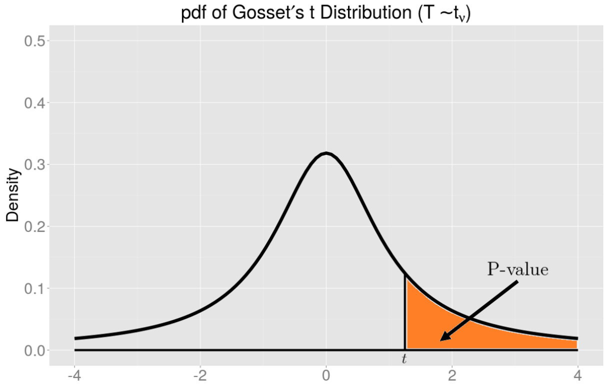
• SMALL-SAMPLE t-TEST ABOUT NORMAL POPULATION MEAN μ (SUMMARY):

Population:	Normal Population with std dev σ unknown
Random Sample:	$\mathbf{X} := (X_1, X_2, \dots, X_n)$
Realized Sample:	$\mathbf{x} := (x_1, x_2, \dots, x_n)$
Test Statistic $W(\mathbf{X}; \mu_0)$	$T = \dfrac{\overline{X} - \mu_0}{S/\sqrt{n}}$
Test Statistic Value $W(\mathbf{x}; \mu_0)$	$t = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$

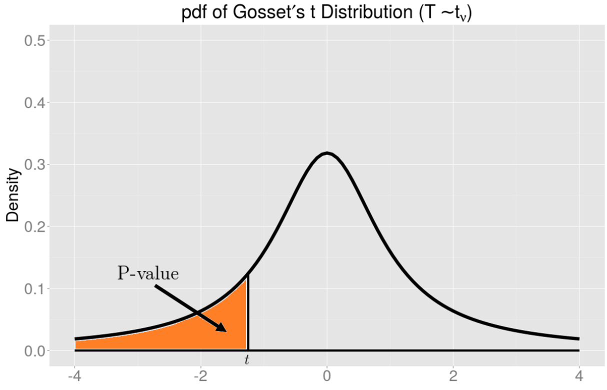
HYPOTHESIS TEST:	**P-VALUE DETERMINATION:**
$H_0: \mu = \mu_0$ vs. $H_A: \mu > \mu_0$	P-value $= \mathbb{P}(T \geq t) = 1 - \Phi_t(t; \nu = n-1)$
$H_0: \mu = \mu_0$ vs. $H_A: \mu < \mu_0$	P-value $= \mathbb{P}(T \leq t) = \Phi_t(t; \nu = n-1)$
$H_0: \mu = \mu_0$ vs. $H_A: \mu \neq \mu_0$	P-value $= \mathbb{P}(\lvert T\rvert \geq \lvert t\rvert) = 2 \cdot [1 - \Phi_t(\lvert t\rvert; n-1)]$

Decision Rule:
If P-value $\leq \alpha$ then reject H_0 in favor of H_A
If P-value $> \alpha$ then accept H_0 (i.e. fail to reject H_0)

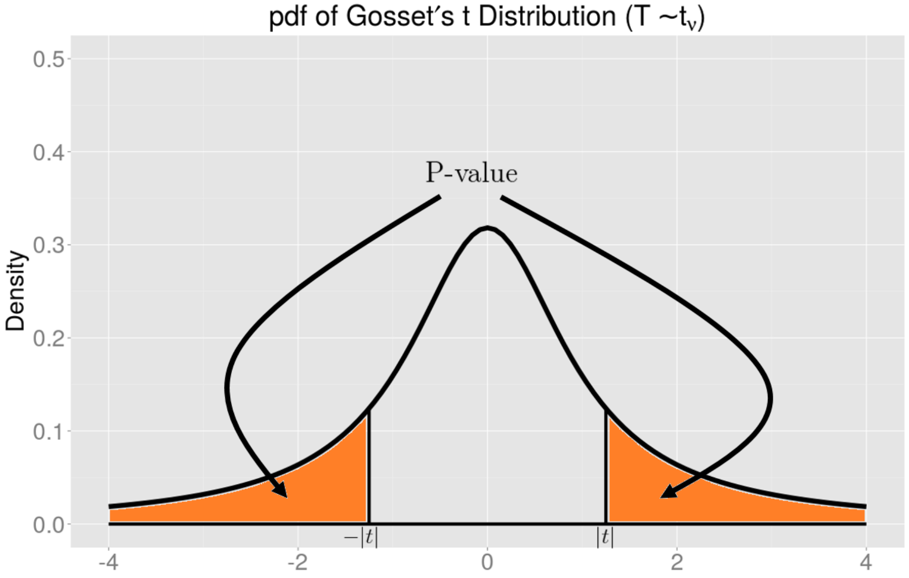
• SMALL-SAMPLE t-TEST ABOUT NORMAL POPULATION MEAN μ (P-VALUES VISUALIZED):



$H_0: \mu = \mu_0$
$H_A: \mu > \mu_0$



$H_0: \mu = \mu_0$
$H_A: \mu < \mu_0$



$H_0: \mu = \mu_0$
$H_A: \mu \neq \mu_0$

GOSSET'S t CDF TABLE FOR $\Phi_t(t;\nu)$

	DEGREES OF FREEDOM (ν)														
t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
0.0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.1	0.532	0.535	0.537	0.537	0.538	0.538	0.538	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539
0.2	0.563	0.570	0.573	0.574	0.575	0.576	0.576	0.577	0.577	0.577	0.577	0.578	0.578	0.578	0.578
0.3	0.593	0.604	0.608	0.610	0.612	0.613	0.614	0.614	0.615	0.615	0.615	0.615	0.616	0.616	0.616
0.4	0.621	0.636	0.642	0.645	0.647	0.648	0.649	0.650	0.651	0.651	0.652	0.652	0.652	0.652	0.653
0.5	0.648	0.667	0.674	0.678	0.681	0.683	0.684	0.685	0.685	0.686	0.687	0.687	0.687	0.688	0.688
0.6	0.672	0.695	0.705	0.710	0.713	0.715	0.716	0.717	0.718	0.719	0.720	0.720	0.721	0.721	0.721
0.7	0.694	0.722	0.733	0.739	0.742	0.745	0.747	0.748	0.749	0.750	0.751	0.751	0.752	0.752	0.753
0.8	0.715	0.746	0.759	0.766	0.770	0.773	0.775	0.777	0.778	0.779	0.780	0.780	0.781	0.781	0.782
0.9	0.733	0.768	0.783	0.790	0.795	0.799	0.801	0.803	0.804	0.805	0.806	0.807	0.808	0.808	0.809
1.0	0.750	0.789	0.804	0.813	0.818	0.822	0.825	0.827	0.828	0.830	0.831	0.831	0.832	0.833	0.833
1.1	0.765	0.807	0.824	0.833	0.839	0.843	0.846	0.848	0.850	0.851	0.853	0.854	0.854	0.855	0.856
1.2	0.779	0.823	0.842	0.852	0.858	0.862	0.865	0.868	0.870	0.871	0.872	0.873	0.874	0.875	0.876
1.3	0.791	0.838	0.858	0.868	0.875	0.879	0.883	0.885	0.887	0.889	0.890	0.891	0.892	0.893	0.893
1.4	0.803	0.852	0.872	0.883	0.890	0.894	0.898	0.900	0.902	0.904	0.905	0.907	0.908	0.908	0.909
1.5	0.813	0.864	0.885	0.896	0.903	0.908	0.911	0.914	0.916	0.918	0.919	0.920	0.921	0.922	0.923
1.6	0.822	0.875	0.896	0.908	0.915	0.920	0.923	0.926	0.928	0.930	0.931	0.932	0.933	0.934	0.935
1.7	0.831	0.884	0.906	0.918	0.925	0.930	0.934	0.936	0.938	0.940	0.941	0.943	0.944	0.944	0.945
1.8	0.839	0.893	0.915	0.927	0.934	0.939	0.943	0.945	0.947	0.949	0.950	0.951	0.952	0.953	0.954
1.9	0.846	0.901	0.923	0.935	0.942	0.947	0.950	0.953	0.955	0.957	0.958	0.959	0.960	0.961	0.962
2.0	0.852	0.908	0.930	0.942	0.949	0.954	0.957	0.960	0.962	0.963	0.965	0.966	0.967	0.967	0.968
2.1	0.859	0.915	0.937	0.948	0.955	0.960	0.963	0.966	0.967	0.969	0.970	0.971	0.972	0.973	0.973
2.2	0.864	0.921	0.942	0.954	0.960	0.965	0.968	0.971	0.972	0.974	0.975	0.976	0.977	0.977	0.978
2.3	0.869	0.926	0.948	0.959	0.965	0.969	0.973	0.975	0.977	0.978	0.979	0.980	0.981	0.981	0.982
2.4	0.874	0.931	0.952	0.963	0.969	0.973	0.976	0.978	0.980	0.981	0.982	0.983	0.984	0.985	0.985
2.5	0.879	0.935	0.956	0.967	0.973	0.977	0.980	0.982	0.983	0.984	0.985	0.986	0.987	0.987	0.988
2.6	0.883	0.939	0.960	0.970	0.976	0.980	0.982	0.984	0.986	0.987	0.988	0.988	0.989	0.990	0.990
2.7	0.887	0.943	0.963	0.973	0.979	0.982	0.985	0.986	0.988	0.989	0.990	0.990	0.991	0.991	0.992
2.8	0.891	0.946	0.966	0.976	0.981	0.984	0.987	0.988	0.990	0.991	0.991	0.992	0.992	0.993	0.993
2.9	0.894	0.949	0.969	0.978	0.983	0.986	0.989	0.990	0.991	0.992	0.993	0.993	0.994	0.994	0.995
3.0	0.898	0.952	0.971	0.980	0.985	0.988	0.990	0.991	0.993	0.993	0.994	0.994	0.995	0.995	0.996
3.1	0.901	0.955	0.973	0.982	0.987	0.989	0.991	0.993	0.994	0.994	0.995	0.995	0.996	0.996	0.996
3.2	0.904	0.957	0.975	0.984	0.988	0.991	0.992	0.994	0.995	0.995	0.996	0.996	0.997	0.997	0.997
3.3	0.906	0.960	0.977	0.985	0.989	0.992	0.993	0.995	0.995	0.996	0.996	0.997	0.997	0.997	0.998
3.4	0.909	0.962	0.979	0.986	0.990	0.993	0.994	0.995	0.996	0.997	0.997	0.997	0.998	0.998	0.998
3.5	0.911	0.964	0.980	0.988	0.991	0.994	0.995	0.996	0.997	0.997	0.998	0.998	0.998	0.998	0.998
3.6	0.914	0.965	0.982	0.989	0.992	0.994	0.996	0.997	0.997	0.998	0.998	0.998	0.998	0.999	0.999
3.7	0.916	0.967	0.983	0.990	0.993	0.995	0.996	0.997	0.998	0.998	0.998	0.998	0.999	0.999	0.999
3.8	0.918	0.969	0.984	0.990	0.994	0.996	0.997	0.997	0.998	0.998	0.999	0.999	0.999	0.999	0.999
3.9	0.920	0.970	0.985	0.991	0.994	0.996	0.997	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999
4.0	0.922	0.971	0.986	0.992	0.995	0.996	0.997	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999

GOSSET'S t CDF TABLE FOR $\Phi_t(t;\nu)$

	DEGREES OF FREEDOM (ν)														
t	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
0.0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.1	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539
0.2	0.578	0.578	0.578	0.578	0.578	0.578	0.578	0.578	0.578	0.578	0.578	0.579	0.579	0.579	0.579
0.3	0.616	0.616	0.616	0.616	0.616	0.616	0.617	0.617	0.617	0.617	0.617	0.617	0.617	0.617	0.617
0.4	0.653	0.653	0.653	0.653	0.653	0.653	0.653	0.654	0.654	0.654	0.654	0.654	0.654	0.654	0.654
0.5	0.688	0.688	0.688	0.689	0.689	0.689	0.689	0.689	0.689	0.689	0.689	0.689	0.690	0.690	0.690
0.6	0.722	0.722	0.722	0.722	0.722	0.723	0.723	0.723	0.723	0.723	0.723	0.723	0.723	0.723	0.723
0.7	0.753	0.753	0.754	0.754	0.754	0.754	0.754	0.755	0.755	0.755	0.755	0.755	0.755	0.755	0.755
0.8	0.782	0.783	0.783	0.783	0.783	0.784	0.784	0.784	0.784	0.784	0.785	0.785	0.785	0.785	0.785
0.9	0.809	0.810	0.810	0.810	0.811	0.811	0.811	0.811	0.811	0.812	0.812	0.812	0.812	0.812	0.812
1.0	0.834	0.834	0.835	0.835	0.835	0.836	0.836	0.836	0.836	0.837	0.837	0.837	0.837	0.837	0.837
1.1	0.856	0.857	0.857	0.857	0.858	0.858	0.858	0.859	0.859	0.859	0.859	0.859	0.860	0.860	0.860
1.2	0.876	0.877	0.877	0.878	0.878	0.878	0.879	0.879	0.879	0.879	0.880	0.880	0.880	0.880	0.880
1.3	0.894	0.895	0.895	0.895	0.896	0.896	0.896	0.897	0.897	0.897	0.897	0.898	0.898	0.898	0.898
1.4	0.910	0.910	0.911	0.911	0.912	0.912	0.912	0.913	0.913	0.913	0.913	0.914	0.914	0.914	0.914
1.5	0.923	0.924	0.925	0.925	0.925	0.926	0.926	0.926	0.927	0.927	0.927	0.927	0.928	0.928	0.928
1.6	0.935	0.936	0.936	0.937	0.937	0.938	0.938	0.938	0.939	0.939	0.939	0.939	0.940	0.940	0.940
1.7	0.946	0.946	0.947	0.947	0.948	0.948	0.948	0.949	0.949	0.949	0.949	0.950	0.950	0.950	0.950
1.8	0.955	0.955	0.956	0.956	0.957	0.957	0.957	0.958	0.958	0.958	0.958	0.958	0.959	0.959	0.959
1.9	0.962	0.963	0.963	0.964	0.964	0.964	0.965	0.965	0.965	0.965	0.966	0.966	0.966	0.966	0.966
2.0	0.969	0.969	0.970	0.970	0.970	0.971	0.971	0.971	0.972	0.972	0.972	0.972	0.972	0.973	0.973
2.1	0.974	0.975	0.975	0.975	0.976	0.976	0.976	0.977	0.977	0.977	0.977	0.977	0.978	0.978	0.978
2.2	0.979	0.979	0.979	0.980	0.980	0.980	0.981	0.981	0.981	0.981	0.982	0.982	0.982	0.982	0.982
2.3	0.982	0.983	0.983	0.984	0.984	0.984	0.984	0.985	0.985	0.985	0.985	0.985	0.985	0.986	0.986
2.4	0.986	0.986	0.986	0.987	0.987	0.987	0.987	0.988	0.988	0.988	0.988	0.988	0.988	0.988	0.989
2.5	0.988	0.989	0.989	0.989	0.989	0.990	0.990	0.990	0.990	0.990	0.990	0.991	0.991	0.991	0.991
2.6	0.990	0.991	0.991	0.991	0.991	0.992	0.992	0.992	0.992	0.992	0.992	0.993	0.993	0.993	0.993
2.7	0.992	0.992	0.993	0.993	0.993	0.993	0.993	0.994	0.994	0.994	0.994	0.994	0.994	0.994	0.994
2.8	0.994	0.994	0.994	0.994	0.994	0.995	0.995	0.995	0.995	0.995	0.995	0.995	0.995	0.996	0.996
2.9	0.995	0.995	0.995	0.995	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.997
3.0	0.996	0.996	0.996	0.996	0.996	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
3.1	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.998	0.998	0.998	0.998	0.998	0.998	0.998
3.2	0.997	0.997	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998
3.3	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999
3.4	0.998	0.998	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.6	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.7	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000
3.8	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.9	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4.0	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

GOSSET'S t CDF TABLE FOR $\Phi_t(t;\nu)$

	DEGREES OF FREEDOM (ν)									
t	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
0.0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.1	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540
0.2	0.579	0.579	0.579	0.579	0.579	0.579	0.579	0.579	0.579	0.579
0.3	0.617	0.617	0.617	0.617	0.617	0.617	0.617	0.617	0.617	0.617
0.4	0.654	0.654	0.654	0.654	0.654	0.654	0.654	0.654	0.654	0.654
0.5	0.690	0.690	0.690	0.690	0.690	0.690	0.690	0.690	0.690	0.690
0.6	0.724	0.724	0.724	0.724	0.724	0.724	0.724	0.724	0.724	0.724
0.7	0.755	0.756	0.756	0.756	0.756	0.756	0.756	0.756	0.756	0.756
0.8	0.785	0.785	0.785	0.785	0.785	0.786	0.786	0.786	0.786	0.786
0.9	0.812	0.813	0.813	0.813	0.813	0.813	0.813	0.813	0.813	0.813
1.0	0.837	0.838	0.838	0.838	0.838	0.838	0.838	0.838	0.838	0.838
1.1	0.860	0.860	0.860	0.860	0.861	0.861	0.861	0.861	0.861	0.861
1.2	0.880	0.881	0.881	0.881	0.881	0.881	0.881	0.881	0.881	0.881
1.3	0.898	0.899	0.899	0.899	0.899	0.899	0.899	0.899	0.899	0.899
1.4	0.914	0.914	0.915	0.915	0.915	0.915	0.915	0.915	0.915	0.915
1.5	0.928	0.928	0.928	0.929	0.929	0.929	0.929	0.929	0.929	0.929
1.6	0.940	0.940	0.940	0.941	0.941	0.941	0.941	0.941	0.941	0.941
1.7	0.950	0.951	0.951	0.951	0.951	0.951	0.951	0.951	0.951	0.952
1.8	0.959	0.959	0.959	0.960	0.960	0.960	0.960	0.960	0.960	0.960
1.9	0.967	0.967	0.967	0.967	0.967	0.967	0.967	0.967	0.968	0.968
2.0	0.973	0.973	0.973	0.973	0.973	0.973	0.974	0.974	0.974	0.974
2.1	0.978	0.978	0.978	0.978	0.978	0.979	0.979	0.979	0.979	0.979
2.2	0.982	0.982	0.983	0.983	0.983	0.983	0.983	0.983	0.983	0.983
2.3	0.986	0.986	0.986	0.986	0.986	0.986	0.986	0.986	0.987	0.987
2.4	0.989	0.989	0.989	0.989	0.989	0.989	0.989	0.989	0.989	0.989
2.5	0.991	0.991	0.991	0.991	0.991	0.991	0.992	0.992	0.992	0.992
2.6	0.993	0.993	0.993	0.993	0.993	0.993	0.993	0.993	0.993	0.994
2.7	0.994	0.995	0.995	0.995	0.995	0.995	0.995	0.995	0.995	0.995
2.8	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996
2.9	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
3.0	0.997	0.997	0.997	0.997	0.998	0.998	0.998	0.998	0.998	0.998
3.1	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998
3.2	0.998	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.3	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.4	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.5	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.6	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
3.7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

EX 8.3.1: Consider the normal population of all US college student heights and the average height μ (in feet.)

The last US census revealed that the average college student height was 5.4 ft.

Suppose a sample of size ($n = 25$) is taken from the population.

Moreover, the sample mean $\overline{x} = 5.1$ ft and the sample std deviation $s = 1.2$ ft.

Does the sample data suggest that the average student height nowadays has decreased??

(Use significance level $\alpha = 0.05$)

(a) State the appropriate null hypothesis H_0 & alternative hypothesis H_A.

(b) Compute the appropriate test statistic value for this hypothesis test.

(c) Compute the resulting P-value.

(d) Make the appropriate decision.

EX 8.3.2: Jim has a well on his land from which he draws well water.

For the ten years he lived there, the well water tasted fine, meaning its pH was 7.0.

However, recently he noticed the well water tastes slightly alkaline (pH above 7.0).

So, he draws 12 buckets of water on different days, at different times & independently of each other.

He measures the pH level of each bucket and determines that the sample mean is 7.6 and sample variance is 5.2.

Does the data suggest that the average pH level of the well water is more alkaline??

(Use significance level $\alpha = 0.01$)

(a) State the appropriate null hypothesis H_0 & alternative hypothesis H_A.

(b) Compute the appropriate test statistic value for this hypothesis test.

(c) Compute the resulting P-value.

(d) Make the appropriate decision.

EX 8.3.3: Jerry has a well on his land from which he draws well water.

For the twenty years he lived there, the well water tasted fine, meaning it has a neutral pH (pH $= 7.0$).

Recently he noticed the water sometimes tastes alkaline (pH above 7.0) and sometimes acidic (pH below 7.0).

So, he draws 33 buckets of water on different days, at different times & independently of each other.

He measures the pH level of each bucket and determines that the sample mean is 6.7 and sample std dev is 1.13.

Does the data confirm Jerry's suspicion that the average pH level of the well water is not neutral??

(Use significance level $\alpha = 0.05$)

(a) State the appropriate null hypothesis H_0 & alternative hypothesis H_A.

(b) Compute the appropriate test statistic value for this hypothesis test.

(c) Compute the resulting P-value.

(d) Make the appropriate decision.
