EX 8.4.1: A long-time dollar store owner determined 10 years ago that 38% of customers paid with a credit card.
Recently, he looked backed at the payment methods of 80 customers from last quarter's records.
He discovered that 51 of those customers purchased with a credit card.
Does the sample data suggest that more customers are paying with a credit card these days??
(Use significance level $\alpha = 0.01$)

(a) State the appropriate null hypothesis H_0 & alternative hypothesis H_A.

$$H_0 : \ p = 0.38$$
$$H_A : \ p > 0.38$$

(b) Compute the appropriate test statistic value for this hypothesis test.

$$z = \frac{\widehat{p} - p_0}{\sqrt{p_0 q_0 / n}} = \frac{(51/80) - 0.38}{\sqrt{(0.38)(0.62)/80}} \approx \boxed{\mathbf{4.74498}} \qquad (q_0 := 1 - p_0 = 1 - 0.38 = 0.62)$$

(c) Compute the resulting P-value.

$$\text{P-value} = \mathbb{P}(Z \geq z) = 1 - \Phi(4.74) \overset{LOOKUP}{\approx} 1 - 1 = \boxed{\mathbf{0}}$$

Note that the P-value is not exactly zero, but it is zero to five decimal places. In reality, P-value $\approx 1.0686 \times 10^{-6}$

(d) Make the appropriate decision.

Since P-value $\approx 0 \leq 0.01 = \alpha$, **Reject H_0 in favor of H_A**

The sample evidence is compelling enough to conclude that it's plausible that more customers pay with a credit card.

EX 8.4.2: A long-time dollar store owner determined 5 years ago that 21 out of 50 customers paid with a credit card.
Recently, he looked backed at the payment methods of 120 customers from last quarter's records.
He discovered that fifty-five of those customers purchased with a credit card.
Does the sample data suggest that the proportion of credit card paying customers has changed nowadays??
(Use significance level $\alpha = 0.05$)

(a) State the appropriate null hypothesis H_0 & alternative hypothesis H_A.

$$H_0 : \ p = 21/50 \quad \text{OR EQUIVALENTLY} \quad H_0 : \ p = 0.42$$
$$H_A : \ p \neq 21/50 \qquad\qquad H_A : \ p \neq 0.42$$

(b) Compute the appropriate test statistic value for this hypothesis test.

$$z = \frac{\widehat{p} - p_0}{\sqrt{p_0 q_0 / n}} = \frac{(55/120) - 0.42}{\sqrt{(0.42)(0.58)/120}} \approx \boxed{\mathbf{0.85080}} \qquad (q_0 := 1 - p_0 = 1 - 0.42 = 0.58)$$

(c) Compute the resulting P-value.

$$\text{P-value} = 2 \cdot [1 - \Phi(|z|)] = 2 \cdot [1 - \Phi(0.85)] \overset{LOOKUP}{\approx} 2 \cdot [1 - 0.80234] = \boxed{\mathbf{0.39532}}$$

(d) Make the appropriate decision.

Since P-value $= 0.39532 > 0.05 = \alpha$, **Accept (or Fail to Reject) H_0**

There is not enough compelling evidence from the data to support the claim that
the proportion of credit card paying customers has changed nowadays.